

IMAGE REGISTRATION BY GENETIC SEARCH

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Abstract

When comparing two digital images of a moving physical object, it may be necessary to find a transformation from one image into the other. Motion more complicated than rotation and translation requires a description involving several parameters. The determination of the transformation can be described as a search for an optimal point in parameter space. The genetic search method has been applied to this optimization problem.

I. Introduction

In order to compare two grey level images of a scene taken at different times or from different vantage points it is often necessary to determine a transformation which will map one image into the other. This transformation, or "registration," problem, is important in such diverse fields as aerial photography [10,17,18,24,23] and medical imaging [1,14,16,19]. Approaches to determining the unknown transformation necessary to register two images may be divided into those which require human guidance in matching pairs of "control points" [16,24,23] and those which provide automatic registration. Within the category of automatic registration are methods which rely on previously recognized features (e.g., straight lines) [11,13,17,18] and those which work directly with image intensity [1,5,6,10,19,2]. The work described in this paper falls into this latter category. General introductions to the field of image registration and extensive bibliographies may be found in [5,11,13].

We present here a technique for registering images which takes advantage of recent progress in the field of function optimization. By specifying both a parameterized class of image transformations and a particular measure for the difference, or "distance," between the two images, we cast the registration problem into the problem of searching a parameter

space for the minimum distance transformation. This method is standard [10,13,20,24] except that the search technique selected here, the "genetic search," has heretofore not been applied to image registration.

The primary application of this work is expected to be in X-ray, gamma ray, or nuclear magnetic resonance imaging (NMR), but the techniques presented apply to any image modality in which the image intensity corresponds to the spatial distribution of some density function and to any image pair in which the predominant change between images is due to motion of the objects in the scene, the camera, or both. The class of transformations which we consider includes elastic motion as well as rotation and translation.

Section II states the problem in more specific terms; section III gives a brief introduction to the genetic search technique; section IV provides some of the implementation details; and section V presents some experimental results.

II. The Problem

A pair of images--the original image and the target image--of the same scene are acquired at two successive times by means of the same image acquisition modality. Differences between the two images are assumed to be caused predominantly by motion--both elastic motion of the object or objects in the scene and displacement of the point of view. It is desired to determine a transformation which will map the original image into the target image.

This work applies to images whose intensity is proportional to some physical density, such as mass. This class of images includes, for example, two dimensional X-ray or gamma ray projections and three dimensional images obtained through computer assisted tomography or NMR. These "density" images have the property that the intensity is conserved. The density may move from one point to another but it is neither created nor destroyed

(except at the edges). Such images obey the conservation relation [10],

$$(1) \int_{R_1} \text{im1}(x_1) dx_1 = \int_{R_2} \text{im2}(x_2) dx_2$$

where im1 and im2 represent the original image and the target image, x_1 and x_2 represent vectors specifying positions in im1 and im2, respectively, R_1 represents some arbitrary image region and R_2 is the transformation of R_1 produced by motion. From elementary vector analysis [10] if $x_1(x_2)$ is a function specifying the correspondence between points in im1 and im2 and, if we define

$$(2) \text{im3}(x_2) = |J| * \text{im1}(x_1(x_2))$$

where J is the Jacobian of the transformation, which is, for example, in two dimensions

$$(\partial x_1 / \partial x_2)(\partial y_1 / \partial y_2) - (\partial x_1 / \partial y_2)(\partial y_1 / \partial x_2)$$

with x_i and y_i representing components of the vector x_i , then we have for all x_2 that

$$(3) \text{im2}(x_2) = \text{im3}(x_2)$$

The problem to be solved here is to find $x_1(x_2)$ such that Eq. (3) is true or at least approximately true. Our method for two dimensional images is (a) to limit $x_1(x_2)$ to be of the form

$$(4a) x_1 = \sum_{i,j=0}^N a_{ij} x_2^i y_2^j$$

$$(4b) y_1 = \sum_{i,j=0}^N b_{ij} x_2^i y_2^j$$

(b) to define a non-negative function $d(\text{im2}, \text{im3})$ which indicates the "distance" between im2 and im3 and which equals zero only if Eq. (3) is satisfied; and (c) to search for the set $\{a_{ij}, b_{ij}\}$ which minimizes $d(\text{im2}, \text{im3})$.

The Jacobian arises in density images because of the dimming which results from stretching and the brightening which results from contraction. If J is omitted, i.e., set equal to 1 in Eq. (2), there is often no solution to Eq. (3). In general an approximate solution obtained by using $J = 1$ can be expected to be incorrect for density images in those regions where there is a large degree of stretching or contraction. Because they are not properly "density" images and hence do not obey Eq. (1), images of terrain taken by satellite and other images

of surfaces do not obey Eq. (2) but may in some circumstances [23] be expected to obey Eq. (2) with the Jacobian omitted.

The transformation of an image according to Eqs (4a,b), commonly called "polynomial warping" [24], is a standard [14] technique which has been applied to both satellite images [24] and X-ray images [1,20]. This class of transformations has been selected because it is simple and because its analytical form permits direct differentiation in the calculation of the Jacobian. The size of N, limited to 1 in this work, determines the complexity of the motion considered. The N = 1 case includes as special cases translation, rotation, scaling, and all other affine transformations. The functions selected as distance functions are described in section IV below.

The use of Eqs. (2) and (3) along with parts (a) and (b) of the method serves to reduce the problem of determining the image transformation to a search within a specified multi-dimensional parameter space for a point which minimizes the chosen distance function. The minimization is made difficult by the inevitable presence of many local minima [11]. Because of its excellent performance in finding a global minimum in the presence of local minima, we have applied the "genetic search" technique, described in section III below, to this problem.

III. Genetic Search

Genetic algorithms are search procedures based on principles distilled from natural adaptive systems [16]. These algorithms have been applied recently to extremely difficult search problems in several AI domains [21,22]. This work is the first application of these techniques to image processing problems. Genetic algorithms are probabilistic global optimization procedures which, according to several experimental studies [4,5,9], are able to locate very good approximate solutions in extremely large search spaces with a reasonable amount of computational effort. Genetic algorithms consistently outperform both gradient techniques and various forms of random search on difficult optimizations involving discontinuous, noisy, high-dimensional, and multimodal objective functions. The remainder of this section gives a brief overview of genetic algorithms. For a more detailed discussion, see [9].

Genetic algorithms are iterative procedures which maintain a population P of candidate solutions:

$$P(t) = \langle v_1(t), v_2(t), \dots, v_N(t) \rangle$$

For the image registration problem, each structure v_i represents a vector of parameters which describes an image transformation. A general sketch of the genetic algorithm follows:

```
t ← 0;
initialize P(t);
evaluate P(t);
repeat
  t ← t+1;
  select P(t);
  recombine P(t);
  evaluate P(t);
until termination condition is satisfied;
```

Figure 1.
A Genetic Algorithm

The initial population can be chosen heuristically or randomly. During each iteration step, each structure in the current population $P(t)$ is evaluated. The structures in the population $P(t+1)$ are chosen from the population $P(t)$ by a randomized selection procedure that ensures that the expected number of times a structure is chosen is proportional to that structure's performance, relative to the rest of the population. For example, if the performance of v_i is twice as good as the average performance of the structures in $P(t)$, then v_i is expected to appear twice in population $P(t+1)$. In order to search other points in the search space, some variation is introduced into the resulting population by means of idealized genetic recombination operators. The most important recombination operator is called "crossover", under which two structures exchange portions of their binary representation. This exchange is implemented by choosing a crossover point at random and exchanging the segments to the right of this point. For example, let

$$v_1 = 100|01010, \text{ and}$$

$$v_2 = 010|10100.$$

and suppose that the crossover point has been chosen as indicated by the vertical bar. The resulting structures would be

$$v'_1 = 100|10100 \quad \text{and}$$

$$v'_2 = 010|01010.$$

The vectors resulting from crossover thus share some combinations of features of the original vectors, and also introduce some new combinations of features. For a detailed analysis of genetic search, see [16]. Termination may be triggered by finding an acceptable approximate solution to the problem, by fixing the total number of structure evaluations, or some other application dependent criterion.

IV. Implementation

This research employs GENESIS, a software system which facilitates the use of the genetic algorithm for solving search problems [13]. GENESIS is written in the C programming language and runs under the UNIX* operating system. Since the genetic algorithm is task independent, the only task dependent tailoring required is to provide an evaluation procedure, which returns a numerical rating when given a particular point in the search space. This section describes the evaluation procedure used for the image registration problem.

As mentioned in section III, genetic algorithms operate on binary strings which, in our case, denote image transformations. There are three steps involved in the evaluation: 1) interpret the binary string as a spatial transformation; 2) apply the resulting transformation to the original image; 3) measure the distance between the resulting transformed image and the target image. Each of these steps will now be described.

Each binary string is interpreted as a vector of fields, each field representing one coefficient in the spatial transformation. Although there are several ways to map a binary string into a real-valued coefficient, our experience suggests that a Gray code interpretation performs considerably better than the more natural techniques. That is, each binary field is mapped to the index of that pattern in a Gray code. The resulting integer is then scaled to obtain a real-valued coefficient.

The transformation corresponding the binary string is then applied to the original image, using a pixel filling algorithm [8].

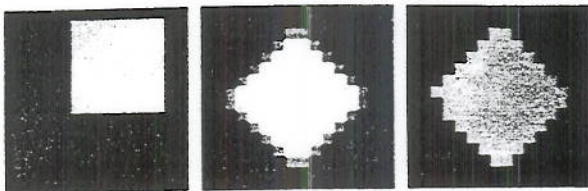
Finally, the resulting transformed image is compared with the target image, and a distance measure is obtained. The distance function has a significant impact on the performance of the genetic search. We have experimented with three different distance functions. The first is the sum of the absolute values of pixel differences. The second is the sum of squares of the pixel differences. The third, and most promising, distance function is called "pixel averaging", and consists of summing average pixel differences over recursively smaller subimages. This distance function apparently allows the genetic algorithm to concentrate initially on finding transformations which provide a good match with respect to global

*UNIX is a Trademark of Bell Laboratories.

features, while focusing later on finer detail.

V. Results

This section describes the results of two early experiments. The first example consists of an artificial image which undergoes translation and rotation in the presence of varying contrast. The original image, figure 2(a), consists of a bright square against a dark background. The target image, figure 2(b), consists of a rotation and a slight stretching of the original image, resulting in a diamond shape against a dark background. Also, to test the importance of varying contrast, the contrast was increased. The genetic search was used to find the transformation from the original image to the target image. The resulting transformed image is shown in figure 2(c).



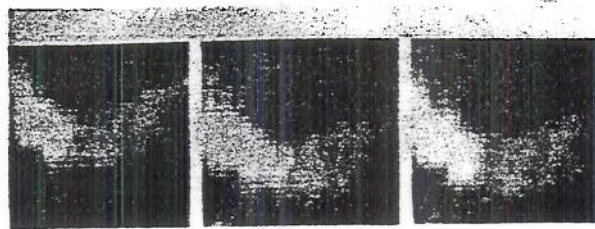
(a) (b) (c)

Figure 2.
Artificial Images

Despite the contrast variation the transformed image is nearly optimal; it disagrees in shape with the target image at just one pixel. This nearly optimal transformation was found after approximately 50000 transformation trials, but very good approximate transformations were found after fewer than 3000 trials. (In all experiments, 8 bits were allocated to each of the 8 coefficients of the transformation, yielding a search space of 2^{64} (or more than 10^{18}) possible transformations.) From this example and others, it appears that genetic search can adequately handle simple translations and rotations, even in the presence of variation in contrast.

The second example consists of two images digitized from 35mm X-ray film. The images, taken at a 1/30 second interval, show a coronary artery injected with a contrast enhancing fluid, figure 3(a), which undergoes translation, rotation, and stretching, figure 3(b). After approximately 10000 transformation trials, the genetic search found a transformation

which yielded the image in figure 3(c).



(a) (b) (c)

Figure 3.
X-ray Images

An analysis of its shape and intensity indicates that the transformed image, figure 3(c), is a very good approximation of the target image, figure 3(b).

VI. Conclusions

The use of the genetic algorithm to search for image transformations appears to hold considerable promise. Although our experiments to date have been encouraging, we acknowledge the need to reduce the computation time of the search procedure. Work is in progress on the application of Barnea's technique[2] to the evaluation procedure. We are also exploring the reduction in computer time to be gained via parallel processing. The genetic search routines can be easily decomposed into parallel search routines by evaluating each structure in a given population in parallel; and in the evaluation routine itself, the distance measurement between the transformed image and the target image can be more efficiently performed on an array processor. With these opportunities in mind, we are encouraged to continue studying the power of the genetic algorithms in searching for optimal image transformations.

References

1. Barber, D. C., "Automatic Alignment of Radionuclide Images," *Phys. Med. Biol.* Vol. 27(3), pp.387-96 (1982).
2. Barnea, Daniel I. and Silverman, Harvey F., "A Class of Algorithms for Fast Digital Image Registration," *IEEE Trans. Comp.* Vol. 21(2), pp.179-86 (Feb 1972).

3. Bethke, A. D., Genetic algorithms as function optimizers, Ph. D. Thesis, Dept. Computer and Communication Sciences, Univ. of Michigan (1981).
4. Brindle, A., Genetic algorithms for function optimization, Ph. D. Thesis, Computer Science Dept., Univ. of Alberta (1981).
5. Broit, Chaim, Optimal Registration of Deformed Images, Ph. D. thesis, Computer and Info. Sci., Univ. of Pennsylvania (1981).
6. Burr, D. J., "A Dynamic Model for Image Registration," Proc. IEEE Comput. Soc. Conf. Patt. Recog. and Image Process., pp.17-24 (Aug 6-8, 1979).
7. Castleman, Kenneth R., Digital Image Processing, Prentice-Hall, Inc., Englewood Cliffs, N J (1979).
8. DeJong, K. A., "Adaptive system design: a genetic approach," IEEE Trans. Sys., Man, and Cyber. Vol. SMC-10(9), pp.566-574 (Sept. 1980).
9. Desloge, Edward A., Statistical Physics, Holt, Rinehart, and Winston, Inc., New York (1966).
10. Frei, Werner, Shibata, T., and Chen, C. C., "Fast Matching of Non-stationary Images with False Fix Protection," Proc. 5th Intl. Conf. Patt. Recog. Vol. 1, pp.208-12, IEEE Computer Society (Dec 1-4, 1980).
11. Goshtasby, Ardesir, A Symbolically-assisted Approach to Digital Image Registration with Application in Computer Vision, Ph. D. thesis, Computer Science, Michigan State Univ. (1983).
12. Grefenstette, J. J., "A user's guide to GENESIS," Tech. Report CS-83-11, Computer Science Dept., Vanderbilt Univ. (August 1983).
13. Hall, Ernest L., Computer Image Processing and Recognition, Academic Press, Inc., New York (1979).
14. Hohne, K. H. and Bohm, M., "The Processing and Analysis of Radiographic Image Sequences," Proc. 6th Intl. Conf. Patt. Recog. Vol. 2, pp.884-897, IEEE Computer Society Press (Oct 19-22, 1982).
15. Holland, J. H., Adaptation in Natural and Artificial Systems, Univ. Michigan Press, Ann Arbor (1975).
16. Kinsey, J. H. and Vannellii, B. D., "Applic. of Digit. Image Change Detection to Diagn. and Follow-up of Cancer Involving the Lungs," Proc. Soc. Photo-optical Instrum. Eng. Vol. 70, pp.99-112, Society of Photo-optical Instr. Eng. (1975).
17. Little, James J., "Automatic Registration of Landsat MSS Images to Digital Elevation Models," Proc. Workshop Computer Vision: Representation and Control, pp.178-84, IEEE Computer Science Press (Aug 23-25, 1982).
18. Medioni, Gerard G., "Matching Regions in Aerial Images," Proc. Comp. Vision and Patt. Recog., pp.364-65, IEEE Computer Society Press (Jun 19-23, 1983).
19. Potel, Michael J. and Gustafson, David E., "Motion Correction for Digital Subtraction Angiography," IEEE Proc. 5th An. Conf. Eng. in Med. Biol. Soc., pp.166-9 (Sep 1983).
20. Price, R. R., Lindstrom, D. P., Hillis, S., Friesinger, G. C., and Brill, A. B., "Analytical Techniques for Image Superposition," Proc. 5th Symp. on Sharing of Comp. Programs and Tech. in Nuc. Med., pp.241-50 (1975).
21. Rendell, L. A., "A doubly layered, genetic penetrance learning system," Proc. National Conf. on AI, pp.343-347 (1983).
22. Smith, S. F., "Flexible learning of problem solving heuristics through adaptive search," Proc. of 8th IJCAI (August, 1983).
23. Van Wie, Peter and Stein, Maurice, "A Landsat Digital Image Rectification System," IEEE Trans. on Geoscience Electronics Vol. GE-15(3), pp.130-7 (July, 1977).
24. Wong, R. Y., "Image Sensor Transformations," IEEE Trans. Syst. Man Cyber. Vol. SMC-7(12), pp.836-41 (Dec. 1977).