Physical implementations of Quantum Information Processing Devices (QIPD)

Note Title 12/6/2006

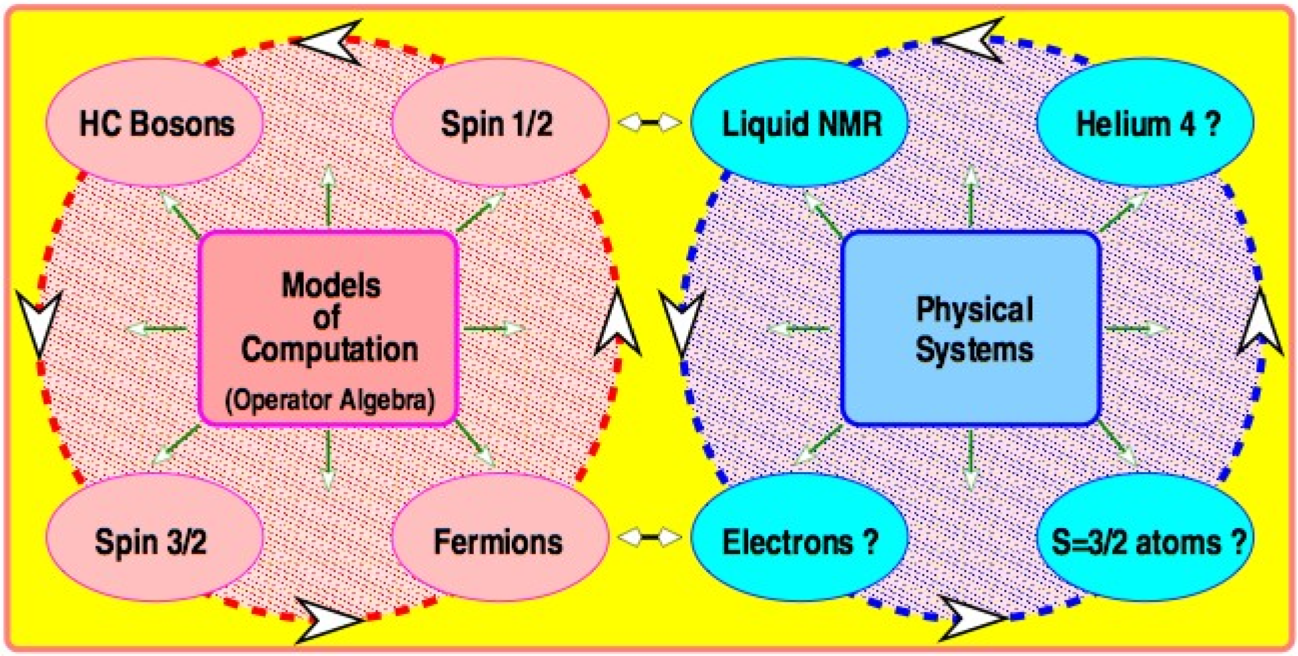
Building an scalable QIPD is not an easy task

I will assume a QIPD made out of qubits using the standard model of quantum computation

- What are the physical requirements to build a QIPD? (experimental)

- What are the current technologies that have been pursued?

Models of Computation and Physical Systems


HC Bosons
Spin 1/2
Models of Computation (Operator Algebra)
Spin 3/2
Fermions
Liquid NMR
Helium 4 ?
Physical Systems
Electrons ?
S=3/2 atoms ?

Theoretical T Division

Los Alamos NATIONAL LABORATORY

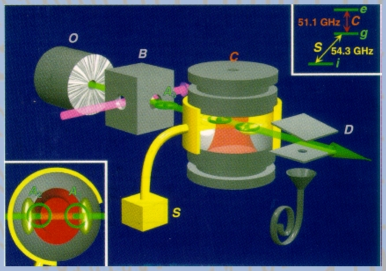
Will a quantum information processing device rescue us?

- 1981 — Quantum computer concept R. Feynman
- 1994 — Quantum computer power P. Shor
- 2007? — Quantum simulators
- 2030? — Quantum computer technology

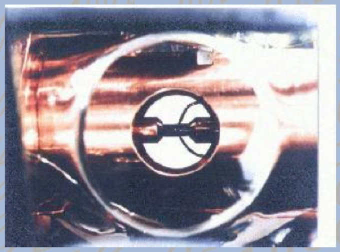
"DiVincenzo commandments"
- *I. Scalable set of qubits*
- *II. Easy initialization*
- *III. Universal operations*
- *IV. Long decoherence time*
- *V. Easy final readout*

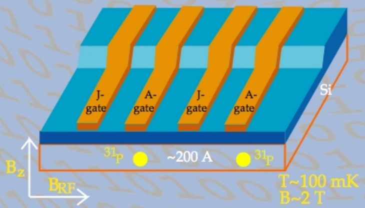
2030??

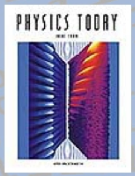
Devices for Quantum Information Processing



- Atom traps
- Cavity QED
- Electron floating on helium
- Electron trapped by surface acoustic waves
- LANL P — Ion traps
- LANL CSS-T — Nuclear Magnetic Resonance
- LANL CSS-T — Quantum Optics
- Quantum dots
- LANL MST — Solid state
- LANL CSS-MST-T — Spintronics
- Superconducting Josephson junctions







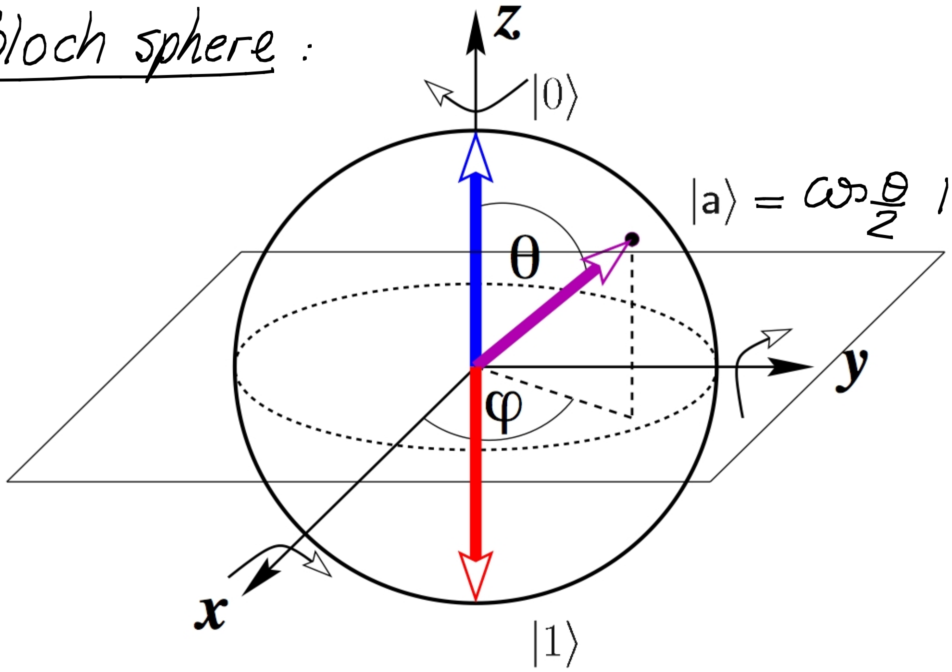
Physical Requirements

Elementary unit: qubit (two-level system)

↑ ——— $|0\rangle$

↓ ——— $|1\rangle$

Bloch sphere:



$$|a\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}\,|1\rangle$$

(1) What physical forms they may take on ?
(It must be robust)

(2) Can we make them evolve ? → physical control

(3) Can we prepare an initial state ? → initialization

(4) Can we readout ? → Quantum measurement

Fix Experimental constraints !

Why is it so hard to build a QC?

Who is the enemy? → Decoherence:
(coupling to the external world)

↓ dilemma:

QC must be isolated but at the same time it needs to be accessible to perform computation

A qualitative figure of merit that characterizes how good a QC is, is the ratio:

$$\#\text{ops} = \lambda = \frac{\tau_Q}{\tau_G}$$

τ_Q → time the system remains coherent

τ_G → time it takes to perform a gate operatio

System	τ_Q [sec]	τ_G [sec]	λ
Nuclear spin	$10^{2} - 10^{8}$	$10^{-3} - 10^{-6}$	$10^{5} - 10^{14}$
Electron spin	10^{-3}	10^{-7}	10^{4}
Ion Trap	10^{-1}	10^{-14}	10^{13}
e^- - Au	10^{-8}	10^{-14}	10^{6}
e^- - GaAs	10^{-10}	10^{-13}	10^{3}
Quantum dot	10^{-6}	10^{-9}	10^{3}
Optical cavity	10^{-5}	10^{-14}	10^{9}
Microwave "	1	10^{-4}	10^{4}

(1) Representation : It must be robust

figure of merit : minimum lifetime of arbitrary superpositions

(2) Control : - It must be universal

single qubit rotations + 2 qubit interactions

- Ability to address 1 and 2 qubits (it is not obvious)

figure of merit : τ_G and fidelity

(3) Preparation of initial state :

Classicaly it is quite easy : set switches

Quantum mechanically it is a challenge.

- A fundamental question is what is the set of states of a Hilbert space that can be prepared with polynomial complexity

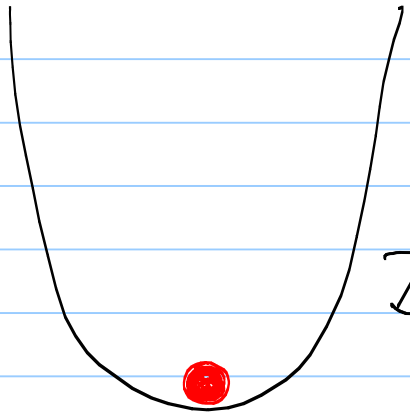
- A practical question is how do we prepare pure states? (finite T)

figure of merit : fidelity, entropy (pure states have 0 entropy)

(4) Quantum measurement : Wavefunction collapse (strong measurement)

figure of merit : Signal to noise ratio

Harmonic oscillator QC



$$H = \frac{p^2}{\underset{\downarrow \text{mass}}{2m}} + \frac{1}{2} m\omega^2 x^2 \;\; ; \;\; [x,p] = i\hbar$$

Define $\begin{cases} x = \sqrt{\dfrac{\hbar}{2m\omega}}\,(a+a^\dagger) \\ p = \sqrt{\dfrac{m\hbar\omega}{2}}\,\dfrac{(a-a^\dagger)}{i} \end{cases} \Rightarrow [a,a^\dagger] = 1$

$\Rightarrow \quad H = \hbar\omega\left(a^\dagger a + 1/2\right)$ (will drop the 1/2)

Eigenstates : $|n\rangle$: $\begin{cases} a^\dagger a\,|n\rangle = n\,|n\rangle \\ a^\dagger\,|n\rangle = \sqrt{n+1}\;|n+1\rangle \\ a\,|n\rangle = \sqrt{n}\;|n-1\rangle \end{cases}$

$(n \in \mathbb{Z})$

$$\Rightarrow \quad |n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}\,|0\rangle$$

Assumptions :
- We can perfectly prepare any state
- " " " perform projective measurem.
- System is closed

CNOT : $\begin{cases} |00\rangle_L \rightarrow |00\rangle_L \\ |01\rangle_L \rightarrow |01\rangle_L \\ |10\rangle_L \rightarrow |11\rangle_L \\ |11\rangle_L \rightarrow |10\rangle_L \end{cases}$

Encoding: $|00\rangle_L = |0\rangle$; $|10\rangle_L = (|4\rangle + |1\rangle)/\sqrt{2}$

$|01\rangle_L = |2\rangle$; $|11\rangle_L = (|4\rangle - |1\rangle)/\sqrt{2}$

↓

depends on the nature of the physical Hamiltonian

$$e^{-iHT}|n\rangle = e^{-i\hbar\omega n T}|n\rangle \quad ; \text{ choose } T = \frac{\pi}{\hbar\omega}$$

$$\Rightarrow \quad e^{-iHT}|n\rangle = (-1)^n |n\rangle \quad \Rightarrow U_{CNOT} = e^{-iHT}$$

In pple. one can realize any U given an appropriate encoding.

But...

1) In gral we do not know the eigenvalues of an arbitrary U

2) N qubits need energy levels: $\{|0\rangle, |1\rangle, \ldots |2^N\rangle\}$

with highest energy $2^N \hbar\omega$!!!

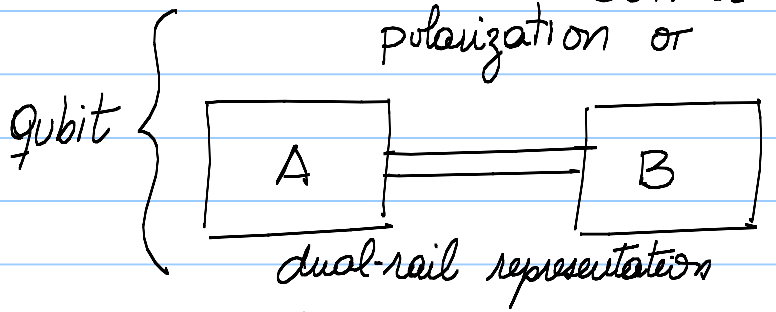
N 2-level systems → $N\hbar\omega$

Photon QC:

Optical photon QC

Qubit: Photon (quanta of the em field)

Features: - Can be transported along long distances with low losses
- Can be combined using beam splitters



$|01\rangle \rightarrow |0\rangle_L$

A B

$|10\rangle \rightarrow |1\rangle_L$

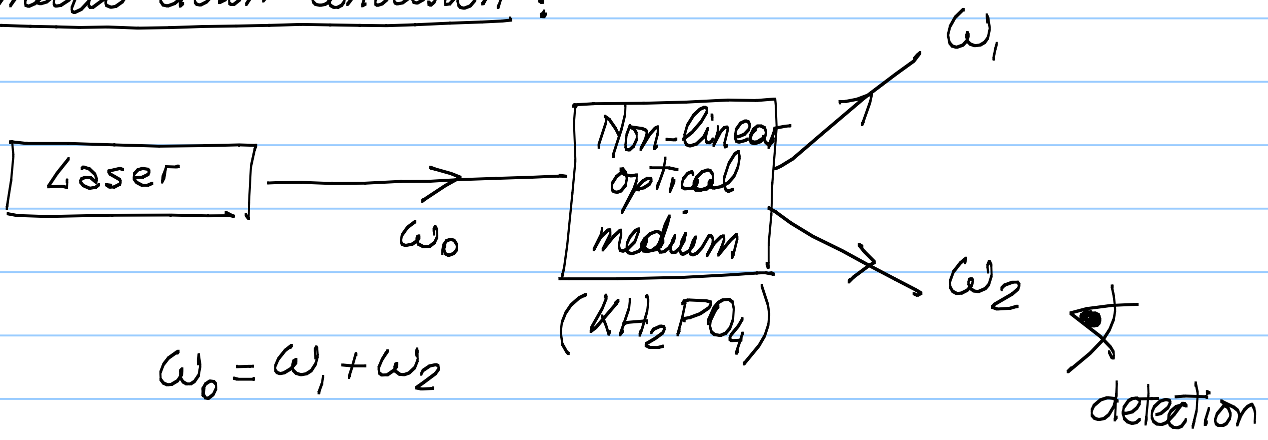
Source: Laser (single photons by laser attenuation)

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$ (coherent state)

$$\langle\alpha|a^\dagger a|\alpha\rangle = |\alpha|^2$$

Attenuation, small α

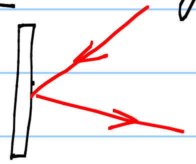
Parametric down-conversion:



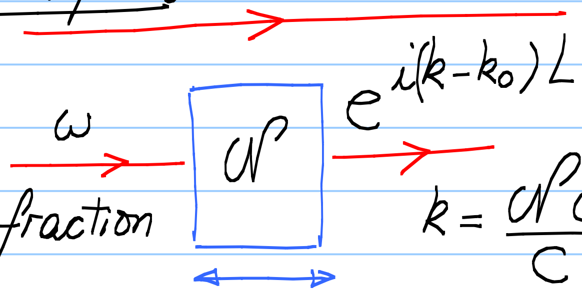
$$\omega_0 = \omega_1 + \omega_2$$

Detection : Quantum efficiency of a photodetector
(use a photomultiplier tube) (probability that a single γ generates a photocarrier pair)

Manipulation : – **mirrors** (high reflectivity, low losses):



– **phase shifters** :

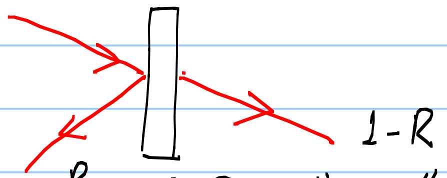


$\mathcal{N}$ = index of refraction

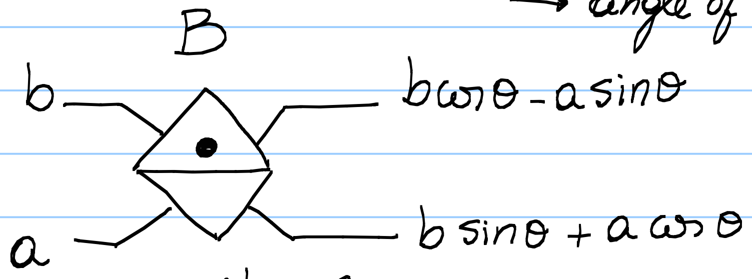
$$k = \frac{\mathcal{N}\omega}{c} \; ; \; k_0 = \frac{\mathcal{N}_0\omega}{c}$$

(borosilicate glass)

– **beamsplitters**:



$R = \cos\theta \rightarrow$ "angle" of the beamsplitter



– **Non-linear optical Kerr media** :

$$\mathcal{N}(I) = \mathcal{N} + \underbrace{\mathcal{N}_2}_{\text{effective interaction between } \gamma \text{ mediated by atoms}} I \xrightarrow{} \text{intensity}$$

Free space evolution : $H = \hbar\omega a^\dagger a$

$\downarrow$ cavity mode of em radiation

$|n\rangle$ = n- photon state

In the representation : $|\varphi\rangle = C_0\,|01\rangle + C_1\,|10\rangle$
(of a qubit)

$$e^{-iHt}\,|\varphi\rangle = \underset{\hookrightarrow}{e^{-i\omega t}}\,|\varphi\rangle$$

$\hookrightarrow$ overall phase (undetectable)

Phase shifter : $P\,|0\rangle = |0\rangle$; $P\,|1\rangle = e^{i\Delta}|1\rangle$

$$\Delta = \frac{(\mathcal{N} - \mathcal{N}_0)\,L}{c}$$

$$P = e^{-i\frac{\Delta}{2}\sigma^z}$$ over the logical states $|0\rangle_L |1\rangle_L$

$$P|\varphi\rangle = P\left(C_0|0\rangle_L + C_1|1\rangle_L\right)$$

$$= C_0\,e^{-i\frac{\Delta}{2}}|0\rangle_L + C_1\,e^{i\frac{\Delta}{2}}|1\rangle_L$$

Beamsplitter : $B = e^{\theta(a^\dagger b - a b^\dagger)} \in$ unitary

$$\begin{cases} B a B^\dagger = a\cos\theta + b\sin\theta \\ B b B^\dagger = -a\sin\theta + b\cos\theta \end{cases}$$

$a^\dagger a - b^\dagger b = \sigma^z$; $a^\dagger b = \sigma^+$; $b^\dagger a = \sigma^-$ $(a^\dagger a + b^\dagger b = 1)$

$\sigma^+ - \sigma^- = i\sigma^y = a^\dagger b - b^\dagger a$

In the logical basis $B = e^{i\theta\sigma^y}$

Non-linear Kerr media :

$$K = e^{i\chi L\, a^\dagger a\, b^\dagger b}$$

A CNOT can be constructed : For single γ states :

$$\begin{cases} K\,|00\rangle = |00\rangle \\ K\,|01\rangle = |01\rangle \\ K\,|10\rangle = |10\rangle \\ K\,|11\rangle = e^{i\chi L}\,|11\rangle \end{cases} \qquad \left(K\,|11\rangle = -|11\rangle \text{ if } \chi L = \pi\right)$$

Consider 2 dual-rail states (4 light modes) :

4 basis states : $|e_{00}\rangle = |1$ 🟥 $1\rangle$; $|e_{01}\rangle = |1$ 🟥 $0\rangle$

$|e_{10}\rangle = |0$ 🟥 $1\rangle$; $|e_{11}\rangle = |0$ 🟥 $0\rangle$

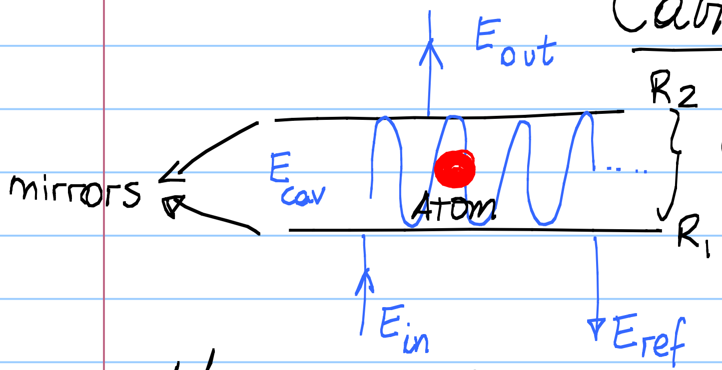
🟥 : apply Kerr $K = \begin{pmatrix} 1 & & & \\ & 1 & & \mathbb{0} \\ \mathbb{0} & & 1 & \\ & & & -1 \end{pmatrix}$

$$U_{CNOT} = (\mathbb{1} \otimes \text{Hadamard})\; K \; (\mathbb{1} \otimes \text{Hadamard})$$

But ... — Interaction is difficult : $\chi L \ll \pi$

+ absorption loss

Cavity QED QC



} Optical cavity with high Q (1 or 2 em modes with high $\vec{E}$) interacting thru dipolar interaction with the atom

Main physical interaction is the dipolar $\vec{d}\cdot\vec{E}$

The Fabry-Perot cavity realizes large $\vec{E}$ in a narrow band of frequencies and in a small volume.

A monochromatic single mode field:

$$\vec{E}(r) = i\,\vec{\epsilon}\,E_0\left(a e^{ikr} - a^\dagger e^{-ikr}\right)$$

polarization field strength $k = \omega/c$

$$H_{field} = \hbar\omega\, a^\dagger a$$

Two-level atom: $\hbar\omega_0 \updownarrow$ E_2, E_1 $\hbar\omega_0 = E_2 - E_1$

$$\langle\Psi_1|\hat{r}|\Psi_2\rangle \sim r_0\,\sigma^y$$

$$H_{atom} = \frac{\hbar\omega_0}{2}\sigma^z$$

(Change convention: Now $\sigma^z|0\rangle = -|0\rangle = -\begin{pmatrix}0\\1\end{pmatrix}$
$\sigma^z|1\rangle = +|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$)

Atom - cavity QED Interaction: $\vec{d}\cdot\vec{E} = H_I$

Atom at $r=0$ $(\hat{r}\cdot\vec{\epsilon}=1)$ $\longrightarrow$ $H_I = -ig\,\sigma^y(a - a^\dagger)$

$$H_I = g\,(\sigma^+ - \sigma^-)(a - a^\dagger)$$

$\sigma^+ a^\dagger$ and $\sigma^- a$ rotate at twice the frequency so we drop them (rotating wave approximation)

Total H:

$$\boxed{H = \frac{\hbar\omega_0}{2}\sigma^z + \hbar\omega a^\dagger a + g\,(a^\dagger\sigma^- + a\sigma^+)}$$

(Jaynes - Cummings Hamiltonian)

$$\hat{N} = a^\dagger a + \frac{\sigma^z}{2} \quad / \quad [H, \hat{N}] = 0 \quad \Rightarrow$$

$$H = \hbar\omega\hat{N} + \underbrace{\delta}_{\text{detuning}}\sigma^z + g\,(a^\dagger\sigma^- + a\sigma^+)$$

$$\delta = \left(\frac{\omega_0 - \omega}{2}\right)$$

Single-γ, single-atom:

We would like to use a single atom to obtain interaction between photons

Basis states: $|\underset{\text{photon}}{0}\overset{\text{Atom}}{0}\rangle$, $|01\rangle$, $|10\rangle$

$$H = \begin{pmatrix} -\delta & 0 & 0 \\ 0 & \delta & g \\ 0 & g & -\delta \end{pmatrix}$$

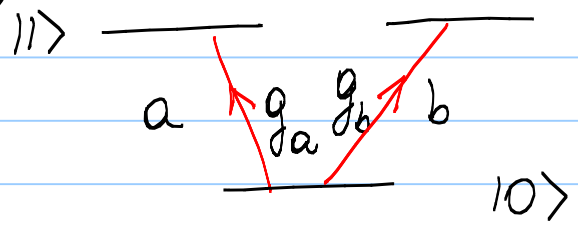
$$\left(H = \delta\sigma^z + g\,(a^\dagger\sigma^- + a\sigma^+)\right)$$

$$U = e^{-iHt} \Rightarrow \qquad \Omega = \sqrt{\delta^2+g^2}$$

$$U = e^{i\delta t}|00\rangle\langle 00| + \left(\cos\Omega t - i\frac{\delta}{\Omega}\sin\Omega t\right)|01\rangle\langle 01|$$
$$\left(\cos\Omega t + i\frac{\delta}{\Omega}\sin\Omega t\right)|10\rangle\langle 10|$$
$$\underbrace{-i\frac{g}{\Omega}\sin\Omega t\left(|01\rangle\langle 10| + |10\rangle\langle 01|\right)}_{\text{Rabi oscillations}}$$

Two photon modes (at most 1 γ each) and single atom:

(Kerr effect)



3 level atom

2 modes a and b

$$H = \delta\begin{pmatrix}-1&0&0\\0&1&0\\0&0&1\end{pmatrix} + g_a\left(a\begin{pmatrix}0&0&0\\1&0&0\\0&0&0\end{pmatrix} + a^{\dagger}\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix}\right) +$$
$$+ g_b\left(b\begin{pmatrix}0&0&0\\0&0&0\\1&0&0\end{pmatrix} + b^{\dagger}\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix}\right)$$

Basis: $|a, b, \text{atom}\rangle$:

$$\{|000\rangle, |100\rangle, |001\rangle, |010\rangle, |002\rangle, |110\rangle,$$
$$|011\rangle, |102\rangle\}$$

One can use these techniques to perform computation in $\neq$ ways:

(1) Quantum info carried by γ

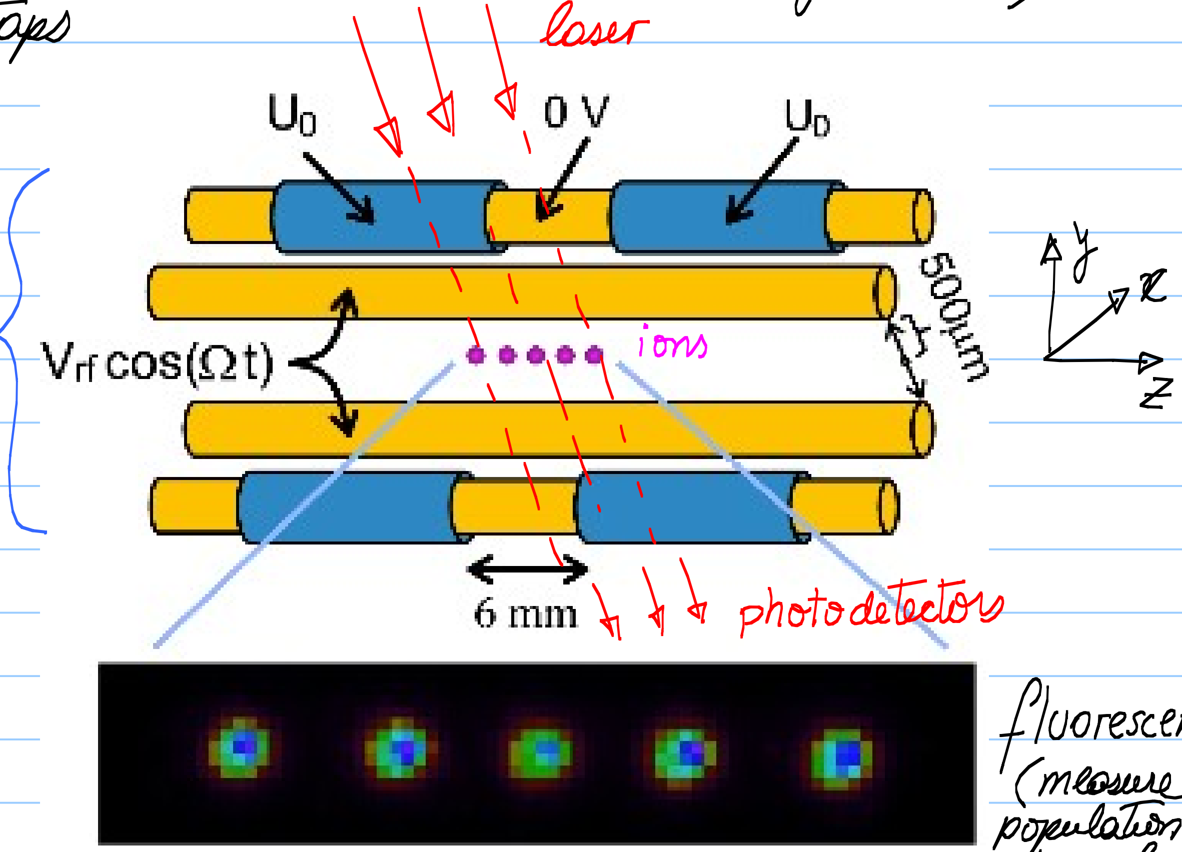
(2) " " " " atoms

But ...

It is desirable to increase g, i.e. the atom-field coupling

Ion traps QC (up to 4 qubits)

Here we represent qubits with ions (charged atoms) in em traps



em trap (linear)

fluorescence (measure population of hyperfine states)

Physical Hamiltonian : $(\omega_x, \omega_y \gg \omega_z)$

$$H = \sum_{i=1}^{N} \frac{1}{2} \underset{\downarrow \text{ mass of the ion}}{M} \left(\omega_x^2 x_i^2 + \omega_y^2 y_i^2 + \omega_z^2 z_i^2\right) + \frac{p_i^2}{2M} + \sum_{i<j} \frac{\overset{\text{charge} \uparrow}{e}^2}{4\pi\epsilon_0 r_{ij}}$$

The quantum of vibrational energy is called phonon

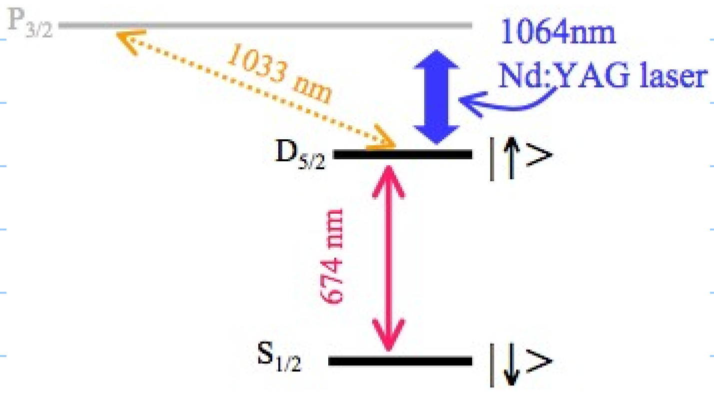
We work in a regime where :

- the CM phonons are the only relevant ones
- the ions are cooled such that $kT \ll \hbar\omega_z$ (close to 0 phonons)

(How? → Doppler cooling)

– the width of the ion oscillation < λ_{laser}

<u>Qubit</u> = Internal states of the ion : hyperfine states (nuclear + electron spin)
+ lowest vibrational modes

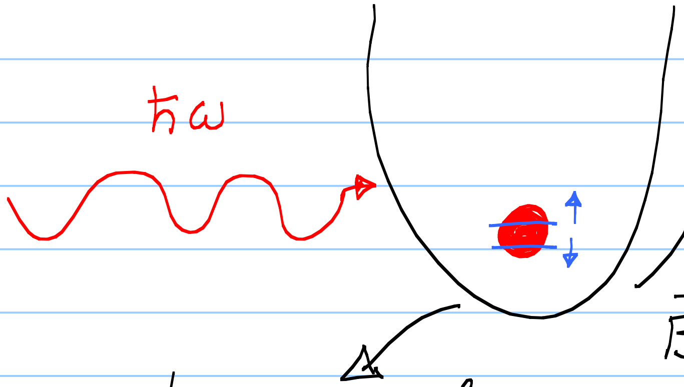


$^{88}Sr^{+}$

<u>Atom-em field coupling</u>: $H_I = g(a^{\dagger}\sigma^{-} + a\sigma^{+})$

Produces spontaneous emission but it has remarkably long coherence times (10^{2}-10^{3} seconds) for hyperfine states.

<u>Simplified ion trap QIPD</u>:



$\vec{\mu} = \mu_m \vec{S}$

$\vec{B} = B_1 \hat{x} \cos(kz - \omega t + \varphi)$

harmonic confinement $\hbar\omega_z$

position of the confined ion : $z = z_0 \overset{\overset{\text{phonons}}{\uparrow}}{(a+a^\dagger)}$

The interaction Hamiltonian :

$$H_I = -\vec{\mu}\cdot\vec{B} = -\mu_m \vec{S}\cdot\hat{x}\, B_1 \left[\frac{e^{i(kz-\omega t+\varphi)} + e^{-i(kz-\omega t+\varphi)}}{2}\right]$$

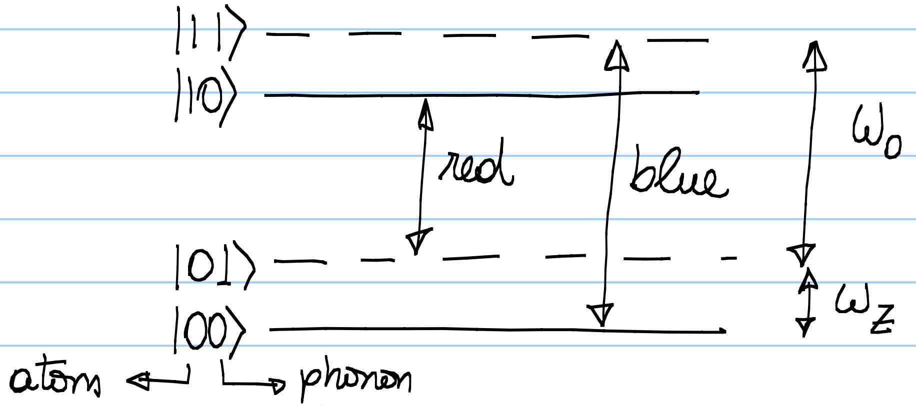
$$H_I = -\hbar\Omega\,(S_+ + S_-)\left[\frac{e^{i(kz-\omega t+\varphi)} + e^{-i(kz-\omega t+\varphi)}}{2}\right]$$

Assuming $kz_0 = \eta \ll 1$; $\Omega = \frac{\mu_m B_1}{2\hbar}$

$$H_I \simeq -\frac{\hbar\Omega}{2}(S_+ + S_-)\Big[(1+i\eta(a+a^\dagger))e^{i(\varphi-\omega t)} + (1-i\eta(a+a^\dagger))\,e^{-i(\varphi-\omega t)}\Big]$$

$$H_I \simeq -\left[\frac{\hbar\Omega}{2}(S_+ + S_-)\left(e^{i(\varphi-\omega t)} + e^{-i(\varphi-\omega t)}\right)\right] -$$

$$-\underbrace{\left[i\,\eta\frac{\hbar\Omega}{2}(S_+ + S_-)(a+a^\dagger)\left(e^{i(\varphi-\omega t)} - e^{-i(\varphi-\omega t)}\right)\right]}$$



$$H_0 = \hbar\omega_0 S_z + \hbar\omega_z a^\dagger a$$

$$H_I' = e^{iH_0t} H_I e^{-iH_0t} \qquad \text{(rotated frame)}$$

$$H_I' \simeq \begin{cases} i\eta\frac{\hbar\Omega}{2}\left(S_+ a^+ e^{i\varphi} - S_- a e^{-i\varphi}\right); & \omega = \omega_0 + \omega_z \\ i\eta\frac{\hbar\Omega}{2}\left(S_+ a e^{i\varphi} - S_- a^+ e^{-i\varphi}\right); & \omega = \omega_0 - \omega_z \end{cases}$$

Single Qubit ops.: laser tuned to ω_0 during appropriate t

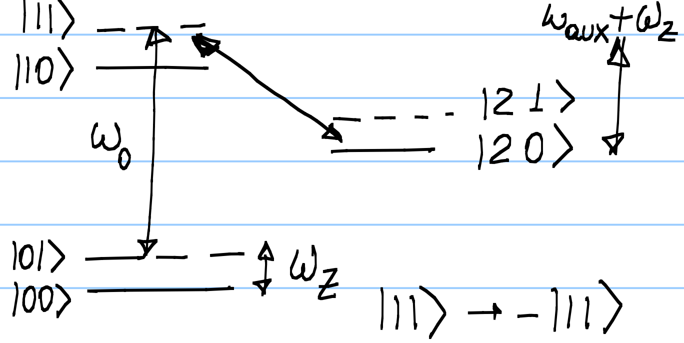
$$H_I^{\text{internal}} = \frac{\hbar\Omega}{2}\left(S_+ e^{i\varphi} + S_- e^{-i\varphi}\right)$$

$$\downarrow \quad R_x \text{ and } R_y$$

Controlled phase-flip gate:

1 qubit in the atom, another qubit in $|0\rangle$ and $|1\rangle$ ph

$$\begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & -1 \end{pmatrix}$$



$|11\rangle \rightarrow -|11\rangle$

Swap gate:

Swap atom and ph qubits (laser $\omega_0 - \omega_z$)

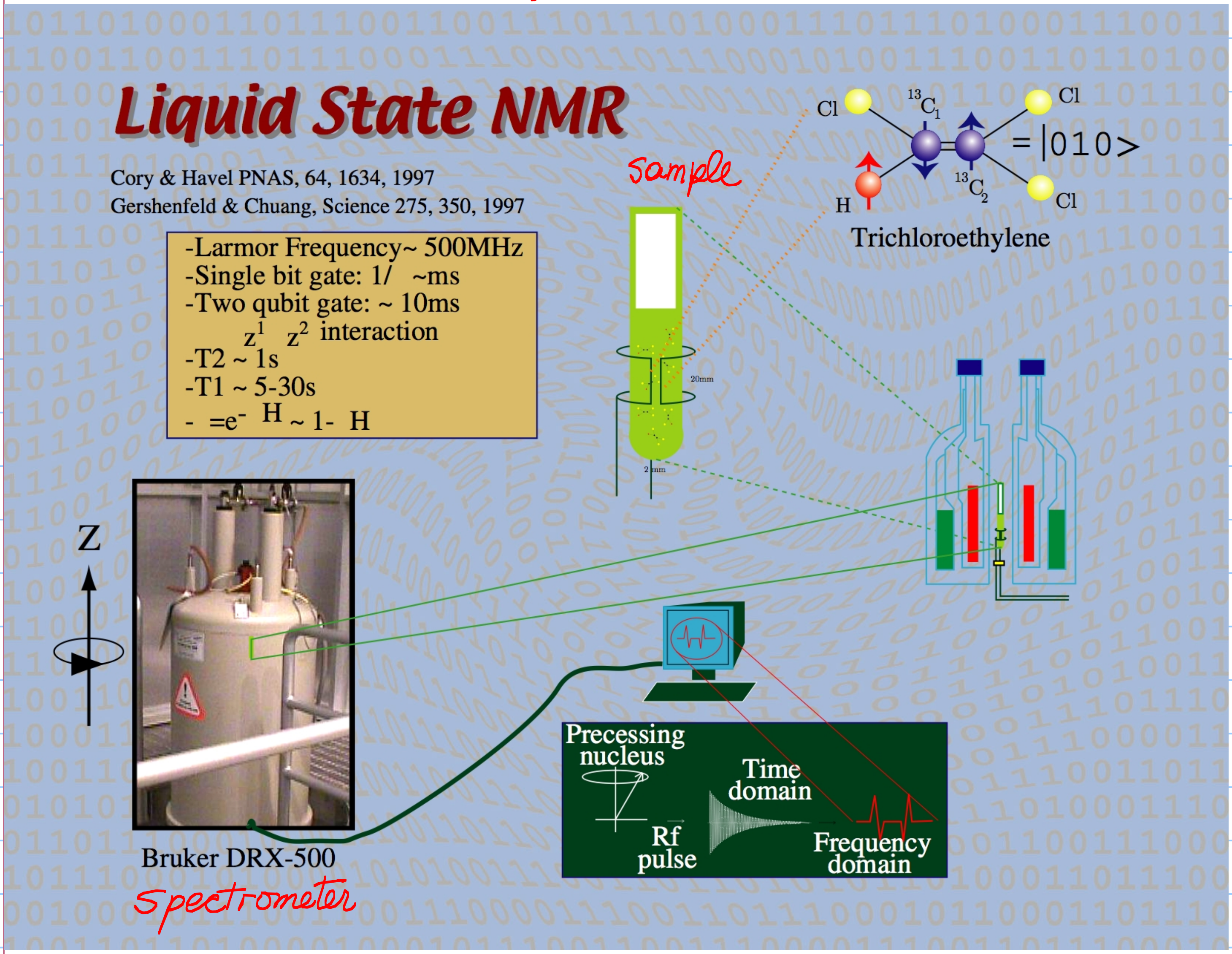
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

But...

- Phonons lifetimes are short
- difficult to cool

Nuclear Magnetic Resonance (NMR) (up to 12 qubits)

- Uses Nuclear spins of molecules
- Nuclear magnetic moment is small ⇒ large # of molecules needed to have a measurable signal
  ⇓
  ensemble QC
- $kT \gg \hbar\omega \rightarrow$ highly mixed state
- Best developed technology so far



Liquid State NMR

Cory & Havel PNAS, 64, 1634, 1997
Gershenfeld & Chuang, Science 275, 350, 1997

-Larmor Frequency~ 500MHz
-Single bit gate: 1/   ~ms
-Two qubit gate: ~ 10ms
z^1 z^2 interaction
-T2 ~ 1s
-T1 ~ 5-30s
- =e$^-$ H ~ 1- H

sample

Bruker DRX-500
Spectrometer

Physical Hamiltonian: (single spin)

$$H = -\vec{\mu}\cdot\vec{B} \quad ; \quad \vec{B} = B_0\hat{z} + B_1(\hat{x}\cos\omega t + \hat{y}\sin\omega t)$$

$$B_1 \ll B_0$$

$$H = \frac{\omega_0}{2}\sigma^z + g(\sigma^x\cos\omega t + \sigma^y\sin\omega t)$$

$$i\partial_t|\chi\rangle = H|\chi\rangle$$

In the "rotating frame" $|\varphi(t)\rangle = e^{i\frac{\omega t}{2}\sigma^z}|\chi\rangle$

$$\Rightarrow \quad i\partial_t|\varphi\rangle = \left[\frac{(\omega_0-\omega)}{2}\sigma^z + g\sigma^x\right]|\varphi\rangle$$

$$\Downarrow$$

$$|\varphi(t)\rangle = \underbrace{e^{-i\left[\frac{(\omega_0-\omega)}{2}\sigma^z + g\sigma^x\right]t}}_{R_{\hat{n}}(\theta)}|\varphi(0)\rangle$$

$$R_{\hat{n}}(\theta) = e^{-i\frac{\theta\hat{n}\cdot\vec{\sigma}}{2}} \quad ; \quad \hat{n} = \frac{(\omega_0-\omega)\hat{z} + 2g\,\hat{x}}{\sqrt{(\omega_0-\omega)^2 + 4g^2}}$$

$$\theta = t\sqrt{(\omega_0-\omega)^2 + 4g^2}$$

Resonance: $\omega_0 \simeq \omega$

Spin-spin Coupling: - dipolar couplings average to zero

- J- couplings : $H_{12} = \frac{\hbar J}{4}\vec{\sigma}_1\cdot\vec{\sigma}_2$

When the nuclei have very ≠ precession frequencies:

$$H_{1,2} \simeq \frac{\hbar J}{4} \sigma_1^z \sigma_2^z$$

Initial State (equilibrium):

$$\rho = \frac{e^{-\beta H}}{Z} \sim 2^{-N} [\mathbb{1} - \beta H]$$

(pseudopure states)

Readout: Magnetization (NMR spectrum)

$$M_{\hat{x}-\hat{y}}(t) = M_0 \, T_r \left[\rho(t) \; (i\sigma_k^x + \sigma_k^y) \right]$$

$\downarrow$

exponential decay: decoherence

T_1, T_2

(cause: - inhomogeneity of B_0
- thermalization)

Summarizing:

$$H = \sum_i \omega_i \sigma_i^z + \sum_{i,j} J \sigma_i^z \sigma_j^z + H^{RF} + H^{Dipole} + H_{e-S}$$

or

$$H = \sum_i \omega_i \sigma_i^z + \sum_{i,j} J \sigma_i^z \sigma_j^z + \sum_i \left(g_i^x(t) \sigma_i^x + g_i^y(t) \sigma_i^y \right)$$

Refocusing : Needed to perform arbitrary unitaries

2 spins $\quad H = H_s + H^{RF}$

$$H_s = a\sigma_1^z + b\sigma_2^z + c\,\sigma_1^z\sigma_2^z$$

At resonance $\longrightarrow$ $e^{-iHt} \sim e^{-iH^{RF}t}$

$$R_{x1} = e^{-i\frac{\pi}{4}\sigma^x} \quad \Rightarrow \quad \underbrace{R_{x1}^2\, e^{-ia\sigma_1^z t} R_{x1}^2 = e^{ia\sigma_1^z t}}_{\text{refocusing}}$$

$\Rightarrow$

$$e^{-iH_s t} R_{x1}^2\, e^{-iH_s t} R_{x1}^2 = e^{-i2b\sigma_2^z t}$$

$\Rightarrow$ useful technique for removing interactions

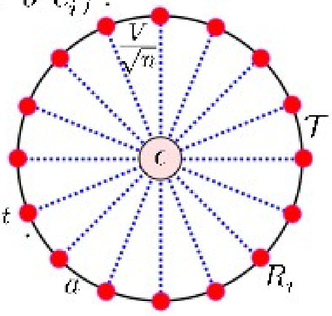
But ...

Pseudopure state preparation reduce the signal exponentially in the # of qubits, unless the initial polarization is high

Example: Resonant Impurity Scattering

A) System to simulate: $L = na$, $R_i = ia$ (n := # of modes, $c^\dagger_{i+n} = c^\dagger_i$)

$$H = -\mathcal{T} \sum_{i=1}^{n} (c^\dagger_i c_{i+1} + c^\dagger_{i-1} c_i) + \epsilon\, b^\dagger b + \frac{V}{\sqrt{n}} \sum_{i=1}^{n} (c^\dagger_i b + b^\dagger c_i) \ .$$



B) Property to compute: Probability to stay in $|\Psi(0)\rangle$

$$G(t) = \langle\Psi(0)|b(t)b^\dagger(0)|\Psi(0)\rangle \ , \ b(t) = e^{iHt}\, b(0)\, e^{-iHt} \ .$$

C) Initial state: Fermi sea of $N_e \leq n$ fermions

$$|\Psi(0)\rangle = |FS\rangle = \prod_{i=0}^{N_e-1} c^\dagger_{k_i}|0\rangle \ , \ c^\dagger_{k_i} = \frac{1}{\sqrt{n}} \sum_{j=\mathbf{1}}^{n} e^{ik_i R_j}\, c^\dagger_j$$

$k_j = k_j = \frac{2\pi n_j}{L}$, with n_j an integer ($-\frac{\pi}{a} < k \leq \frac{\pi}{a}$), $|0\rangle$:= vacuum

Quantum Algorithm to compute $G(t)$

Spin-Fermion Mapping: First Fourier-transformed modes

$$\begin{aligned} b &= \sigma^1_- & b^\dagger &= \sigma^1_+ \\ c_{k_0} &= -\sigma^1_z \sigma^2_- & c^\dagger_{k_0} &= -\sigma^1_z \sigma^2_+ \\ &\vdots & &\vdots \\ c_{k_{n-1}} &= (-1)^n \sigma^{\mathbf{1}}_z \sigma^2_z \cdots \sigma^n_z \sigma^{n+1}_- & c^\dagger_{k_{n-1}} &= (-1)^n \sigma^1_z \sigma^2_z \cdots \sigma^n_z \sigma^{n+1}_+ \ . \end{aligned}$$

Standard Model Hamiltonian: (2-qubit problem) $\mathcal{E}_{k_i} = -2\mathcal{T} \cos k_i a$

$$2H = \left[\epsilon + \sum_{i=0}^{n-1} \mathcal{E}_{k_i}\right] \mathbb{1} + \epsilon\sigma^1_z + \sum_{i=0}^{n-1} \mathcal{E}_{k_i} \sigma^{i+2}_z + V(\sigma^1_x \sigma^2_x + \sigma^{\mathbf{1}}_y \sigma^2_y) \ .$$

Preparation Initial State: Same as before

Physical Quantity: $G(t) = \langle \mathcal{A}(t) \rangle$

$$\mathcal{A}(t) = b(t)b^{\dagger}(0) = e^{i\bar{H}t}\sigma_{-}^{1}e^{-i\bar{H}t}\sigma_{+}^{1} \, .$$

$$\bar{H} = \frac{\epsilon}{2}\sigma_z^1 + \frac{\mathcal{E}_{k_0}}{2}\sigma_z^2 + \frac{V}{2}(\sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2) \, .$$

Exact Unitary Mapping: $e^{-i\bar{H}t} = U e^{-iH_{P1}t} U^{\dagger}$

$$U = e^{i\frac{\pi}{4}\sigma_x^2} e^{-i\frac{\pi}{4}\sigma_y^1} e^{-i\frac{\theta}{2}\sigma_z^1\sigma_z^2} e^{i\frac{\pi}{4}\sigma_y^1} e^{i\frac{\pi}{4}\sigma_x^1} e^{-i\frac{\pi}{4}\sigma_x^2} e^{-i\frac{\pi}{4}\sigma_y^2} e^{i\frac{\theta}{2}\sigma_z^1\sigma_z^2} e^{-i\frac{\pi}{4}\sigma_x^1} e^{i\frac{\pi}{4}\sigma_y^2} \, ,$$

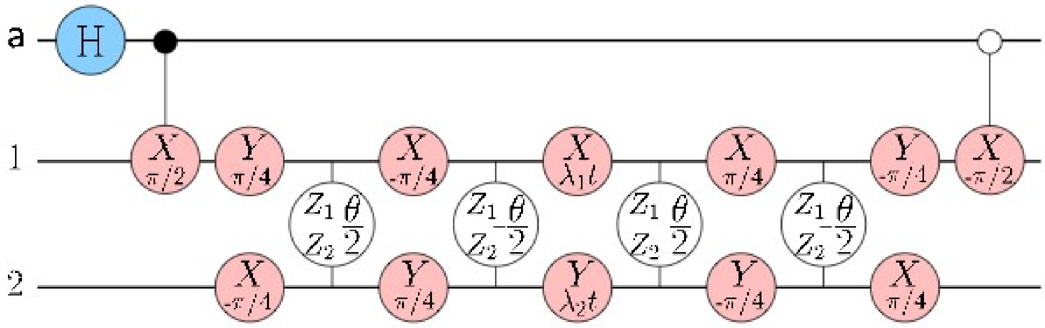
$$H_{P1} = \frac{1}{2}(E - \sqrt{\Delta^2 + V^2})\sigma_z^1 + \frac{1}{2}(E + \sqrt{\Delta^2 + V^2})\sigma_z^2 \, ,$$

Approximate Unitary Mapping: Trotter breakup

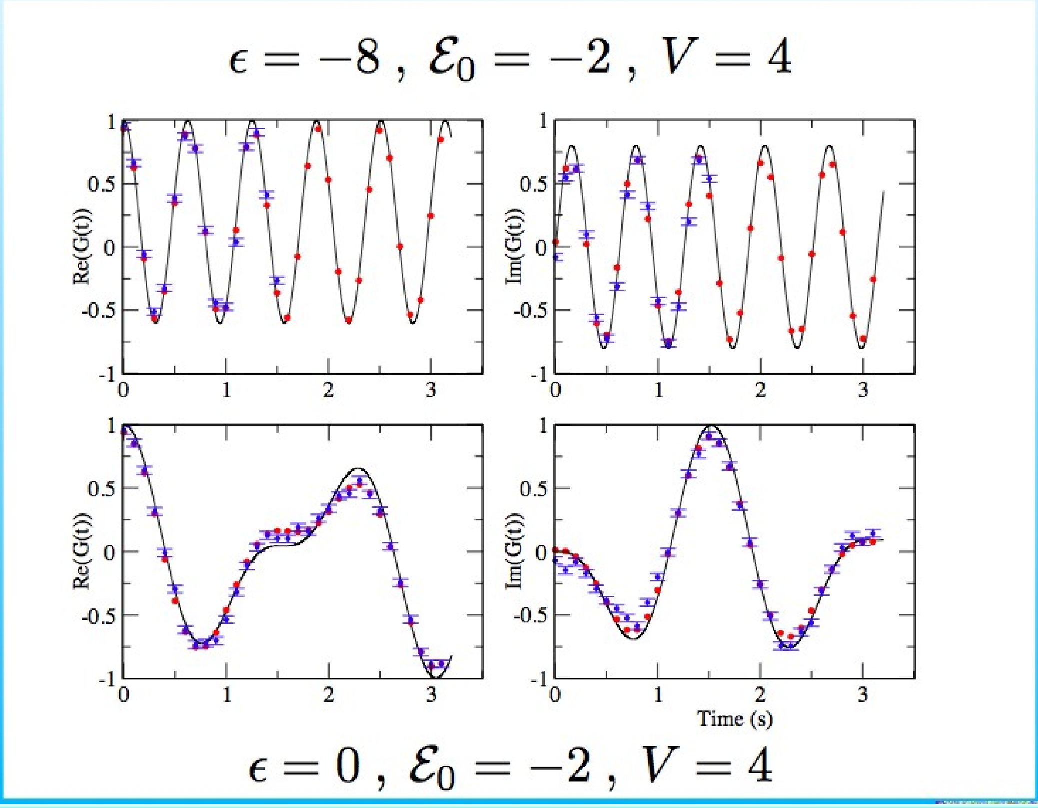
$$e^{i\bar{H}t} = \Big[e^{i\bar{H}s} \Big]^{M} = \Big[e^{i\bar{H}_z s} e^{i\bar{H}_{xy} s} + \mathcal{O}(s^2) \Big]^{M} \, , \; \bar{H} = \bar{H}_z + \bar{H}_{xy} \, .$$

Quantum Network for Resonant Scattering

$$|\Psi(0)\rangle = |0\rangle_{\mathsf{a}} \otimes |1\rangle_1 \otimes |0\rangle_2$$



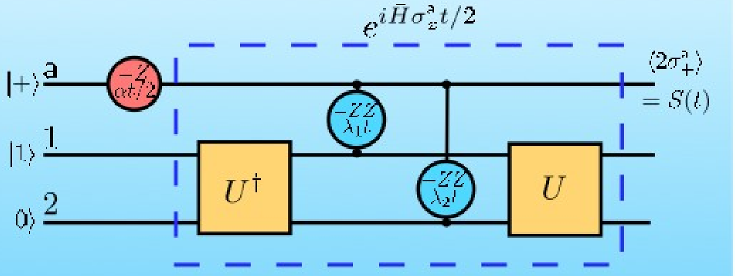
Green's function G(t)

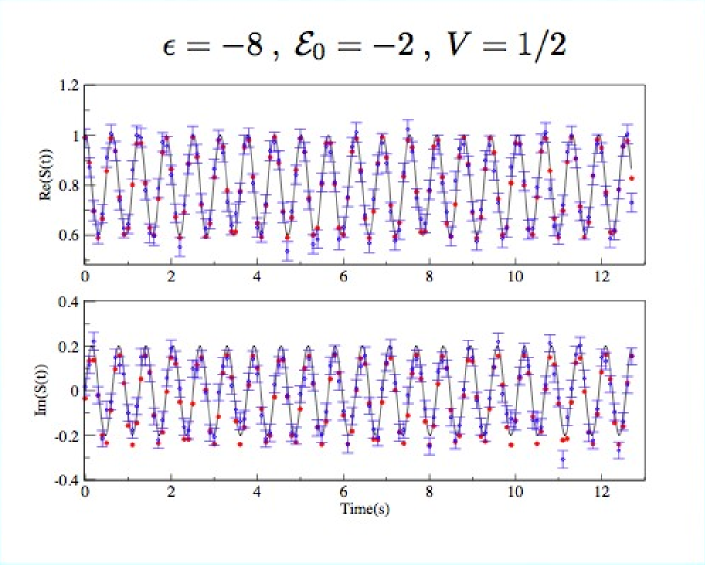


$\epsilon = -8\ ,\ \mathcal{E}_0 = -2\ ,\ V = 4$

$\epsilon = 0\ ,\ \mathcal{E}_0 = -2\ ,\ V = 4$

Energy Spectrum





$\epsilon = -8\ ,\ \mathcal{E}_0 = -2\ ,\ V = 1/2$