B522: Assignment 3

1 A Small Functional Language

Syntax:

$$
\begin{array}{rcl}
o & ::= & + \mid \star \mid - \mid / \mid = \mid < \mid \\
\tau & ::= & \mathsf{int} \mid \mathsf{bool} \mid \tau \to \tau \\
e & ::= & x \mid n \mid o(e_1, \cdots, e_n) \mid \texttt{true} \mid \texttt{false} \mid \\
 & & \texttt{if } e \texttt{ then } e_1 \texttt{ else } e_2 \mid \lambda x.e \mid e_1 e_2 \\
 & & \texttt{fix } x.e \mid \texttt{error} \\
v & ::= & x \mid n \mid \texttt{true} \mid \texttt{false} \mid \lambda x.e
\end{array}
$$

Reduction rules:

$$
\begin{array}{rcl}
+(n_1, n_2) & \to & n \text{ where } n \text{ is the sum of } n_1 \text{ and } n_2 \\
\star(n_1, n_2) & \to & n \text{ where } n \text{ is the product of } n_1 \text{ and } n_2 \\
-(n_1, n_2) & \to & n \text{ where } n \text{ is the difference of } n_1 \text{ and } n_2 \\
/(n_1, n_2) & \to & n \text{ where } n \text{ is the quotient of } n_1 \text{ and } n_2 \\
/(n, 0) & \to & \texttt{error} \\
=(n, n) & \to & \texttt{true} \\
=(n_1, n_2) & \to & \texttt{false} \text{ if } n_1 \text{ and } n_2 \text{ are different} \\
<(n_1, n_2) & \to & \texttt{true} \text{ if } n_1 \text{ is less than } n_2 \\
<(n_1, n_2) & \to & \texttt{false} \text{ if } n_1 \text{ is not less than } n_2 \\
\texttt{if true then } e_1 \texttt{ else } e_2 & \to & e_1 \\
\texttt{if false then } e_1 \texttt{ else } e_2 & \to & e_2 \\
(\lambda x.e)v & \to & e[v/x] \\
\texttt{fix } x.e & \to & e[\lambda x.(\texttt{fix } x.e)x/x] \\
 & & \\
o(v_1, \cdots, \texttt{error}, e_i, \ldots) & \to & \texttt{error} \\
\texttt{if error then } e_1 \texttt{ else } e_2 & \to & \texttt{error} \\
\texttt{error } e & \to & \texttt{error} \\
v \texttt{ error} & \to & \texttt{error}
\end{array}
$$

2 To do

- Design a type system for the language
- Complete the formalization of the evaluation by defining evaluation contexts
- *State only* the type soundness property
- *Prove* the interesting cases needed to establish subject reduction.