Details of the 4D Rolling Ball

The 4D Rolling Ball matrix is a generalization of the 3D Rolling Ball matrix, and it takes the form:

$$\begin{vmatrix} 1+(c-1)\,n_x^2 & (c-1)\,n_x n_y & (c-1)\,n_x n_z & n_x\,s \\ (c-1)\,n_x n_y & 1+(c-1)\,n_y^2 & (c-1)\,n_y n_z & n_y\,s \\ (c-1)\,n_x n_z & (c-1)\,n_y n_z & 1+(c-1)\,n_z^2 & n_z\,s \\ -n_x\,s & -n_y\,s & -n_z\,s & c \end{vmatrix} .$$

Here we take $\overrightarrow{\mathbf{dx}} = (dx, dy, dz)$, with $r = |\overrightarrow{\mathbf{dx}}|$, as the fundamental control vector, and $c = \cos\theta$, $s = \sin\theta$ with $\theta = \arctan(r/R)$ for some scale R. From these we define the normalized control vector $\hat{n} = (n_x, n_y, n_z) = \overrightarrow{\mathbf{dx}}/r$. An exercise for the reader is to verify that this formula, with $\hat{n} = (\cos a, \sin a \cos b, \sin a \sin b)$, can be derived from a sequence of 4D-plane rotations of the form

$$R_{\mathbf{4Droll}} = R(b, yz) \cdot R(a, xy) \cdot R(\theta, wx) \cdot R^{-1}(a, xy) \cdot R^{-1}(b, yz) .$$