A Quantum View of the Origin of Randomness, Classicality and Complexity

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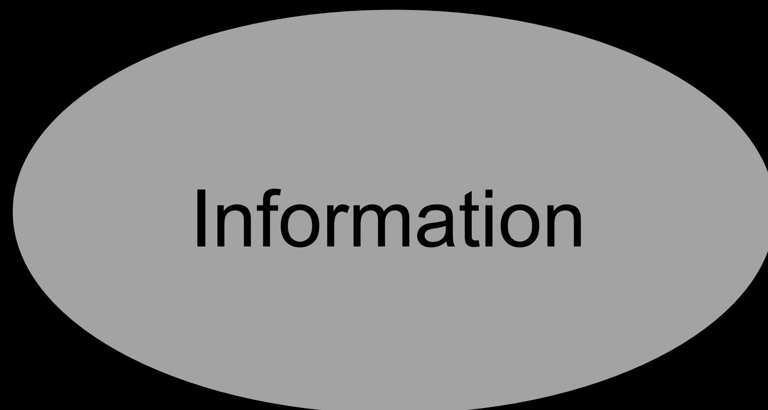
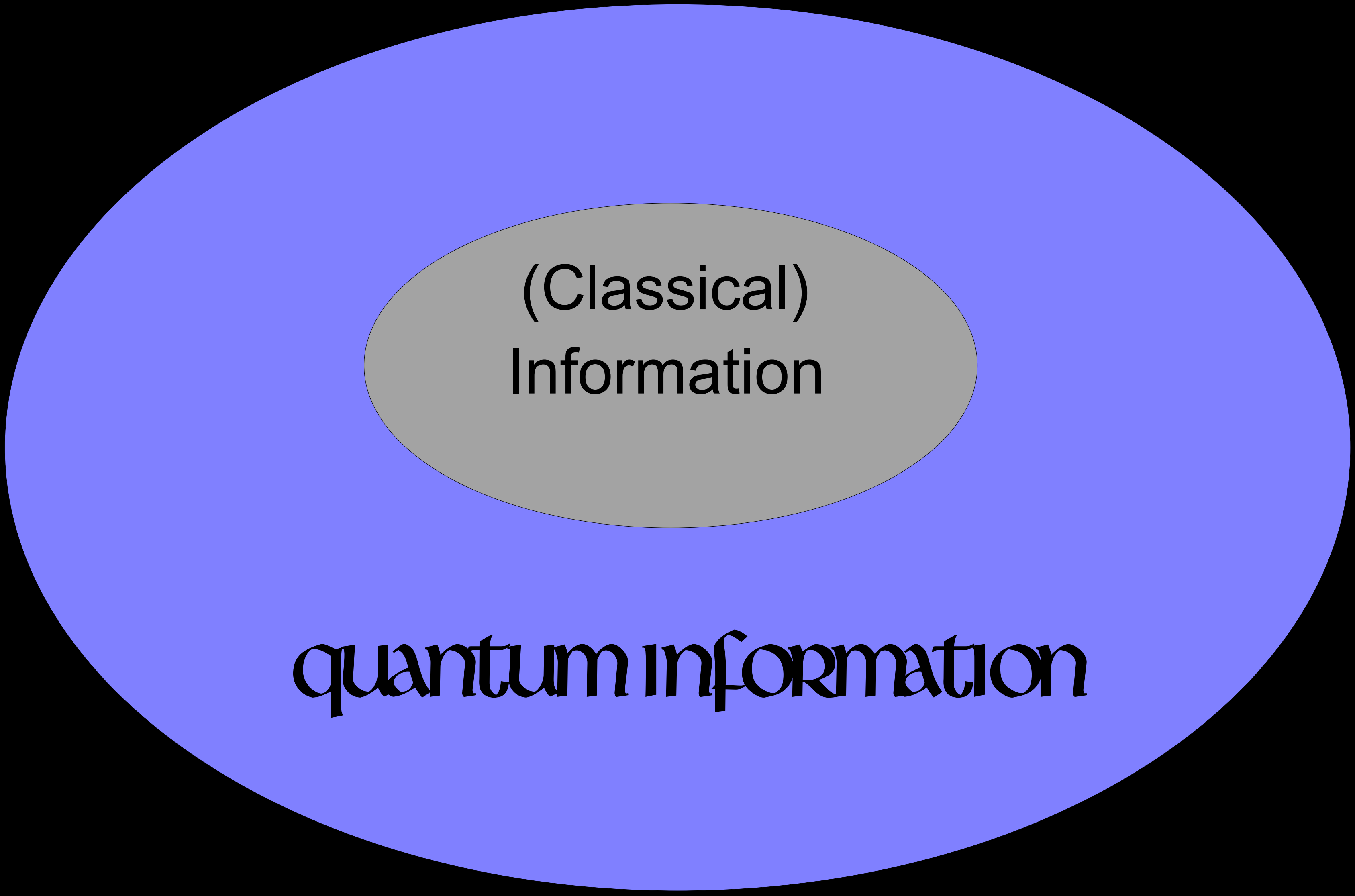
The Bloomington campus is not new to basic questions of epistemology and scientific method.

"There is an honesty in science which demands that the best means be used for the determination of truth. Certainly there are many sorts of truth in the universe, and many aspects of truth must be taken into account if man is to live most effectively in the social organization to which he belongs. But in regard to matter—the stuff of which both non-living materials and living organisms are made—scientists believe that there is no better way of obtaining information than that provided by the human sense organs. No theory, no philosophy, no body of theology, no political expediency, no wishful thinking, can provide a satisfactory substitute for the observation of material objects and of the way in which they behave…"

Kinsey, Pomeroy, Martin, and Gebhard, *Sexual Behavior in the Human Female* (1953)

- How quantum information is different from ordinary classical information
- Entanglement and the origin of randomness
- What quantum information and the Second Law can tell us about the permanence or impermanence of information in the world, the prerequisites for nontrivial computation and self-organization


Information


(Classical)
Information
quantum information

Information = Distinguishability, considered abstractly, separate from any physical medium.

(Using a pencil, a piece of paper can be put into various states distinguishable by eye. Using one's voice, the air can be put into various states distinguishable by ear, etc...)

- Information is reducible to bits (**0** ,**1**)

- Information processing, to reveal implicit truths, can be reduced to logic gates (**NOT**, **AND**)

- bits and gates are *fungible*, independent of their physical embodiment, making possible Moore's law (As bits become ever smaller and cheaper they become ever more useful, unlike shoes or cars)

Even though we have tried to abstract all the physics out of the notion of information, some physics is left. We take for granted that information

- cannot travel faster than light (relativity)

- can be erased when no longer wanted (thermodynamics)

- can be copied without disturbing it (quantum mechanics)

Indeed chemists and physicists have long known that

Information in microsopic bodies such as photons or nuclear spins obeys quantum laws. Such information

- cannot be read or copied without disturbance.

- can connect two spacelike separated observers by a correlation too strong to be explained by classical communication. However, this "entanglement" cannot be used to send a message faster than light or backward in time.

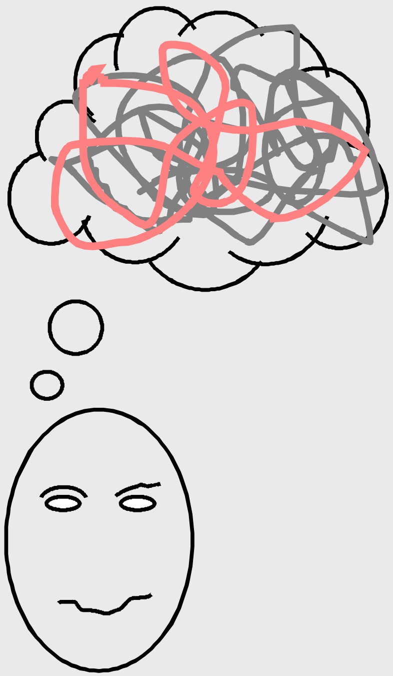
Quantum information is reducible to **qubits** i.e. two-state quantum systems such as a photon's polarization or a spin-1/2 atom.

Quantum information processing is reducible to one- and two-qubit gate operations.

Qubits and quantum gates are fungible among different quantum systems

Ordinary classical information, such as one finds in a book, can be copied at will and is not disturbed by reading it.

Quantum information is more like the information in a dream



• Trying to describe your dream changes your memory of it, so eventually you forget the dream and remember only what you've said about it.

• You cannot prove to someone else what you dreamed.

• You can lie about your dream and not get caught.

But unlike dreams, quantum information obeys well-known laws.

quantum laws

I. To each physical system there corresponds a Hilbert space [1] of dimensionality equal to the system's maximum number of reliably distinguishable states. [2]

2. Each direction (ray) in the Hilbert space corresponds to a possible state of the system. [3]

3. Spontaneous evolution of an unobserved system is a unitary transformation on its Hilbert space. [4]

-- more --

1. A linear vector space with complex coefficients and inner product

$$\langle \phi \mid \psi \rangle = \Sigma\, \phi_i^* \psi_i$$

2. For polarized photons two, e.g. vertical and horizonal

$$\leftrightarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \updownarrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

3. E.g. for photons, other polarizations

$$\nearrow\!\!\!\swarrow = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \nwarrow\!\!\!\searrow = \begin{pmatrix} +1 \\ -1 \end{pmatrix}$$

$$\circlearrowright = \begin{pmatrix} i \\ 1 \end{pmatrix} \quad \circlearrowleft = \begin{pmatrix} i \\ -1 \end{pmatrix}$$

4. Unitary = Linear and inner-product preserving.

4. The Hilbert space of a composite sysem is the tensor product of the Hilbert spaces of its parts. 1

5. Each possible measurement 2 on a system corresponds to a resolution of its Hilbert space into orthogonal subspaces $\{\mathbf{P}_j\}$, where $\sum \mathbf{P}_j = 1$. On state ψ the result j occurs with probability $|\mathbf{P}_j \psi|^2$ and the state after measurement is

$$\frac{\mathbf{P}_j |\psi>}{|\mathbf{P}_j |\psi>|}$$

1. Thus a two-photon system can exist in "product states" such as

$$\leftrightarrow\leftrightarrow \text{ and } \leftrightarrow\nearrow$$

but also in "entangled" states such as

$$\frac{\leftrightarrow\leftrightarrow - \updownarrow\updownarrow}{\sqrt{2}}$$

in which neither photon has a definite state even though the pair together does

2 Believers in the "many worlds interpretation" reject this axiom as ugly and unnecessary. For them measurement is just a unitary evolution producing an entangled state of the system and measuring apparatus. For others, measurement causes the system to behave probabilistically and forget its pre-measurement state, unless that state happens to lie entirely within one of the subspaces $\mathbf{P}_j$.

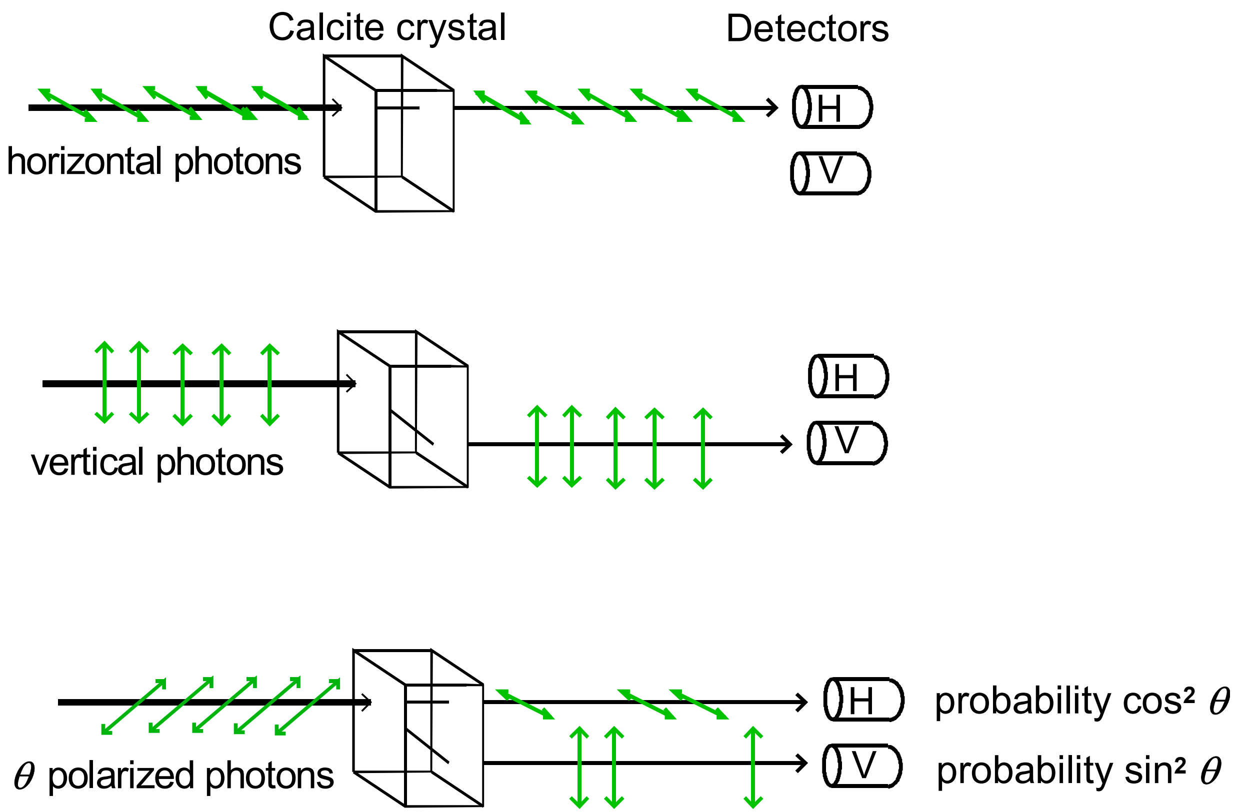
superposition principle

Between any two reliably distinguishable states of a quantum system

(for example horizontally and vertically polarized single photons)

there exists a continuum of intermediate states (representable as complex linear combinations of the original states) that in principle cannot be reliably distinguished from either original state.

(for example diagonal polarizations)



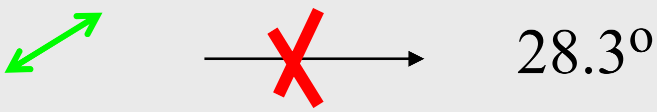
(Mathematically, a superposition is a weighted sum or difference, and can be pictured as an intermediate *direction* in space)

$$\nearrow = \frac{\leftrightarrow + \updownarrow}{\sqrt{2}}$$

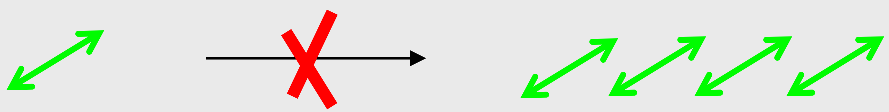
$$\nwarrow = \frac{\leftrightarrow - \updownarrow}{\sqrt{2}}$$

Non-orthogonal states like $\leftrightarrow$ and $\nearrow$ are in principle imperfectly distinguishable. $\leftrightarrow$ always behaves somewhat like $\nearrow$ and vice versa.

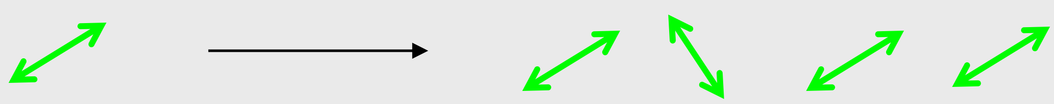
Measuring an unknown photon's polarization exactly is impossible (no measurement can yield more than 1 bit about it).

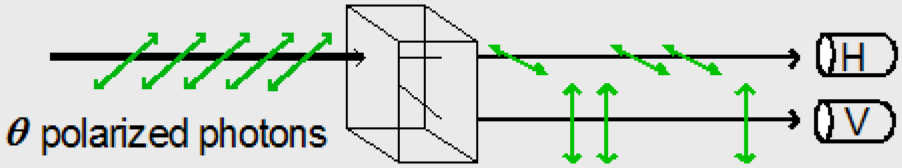


Cloning an unknown photon is impossible. (If either cloning or measuring were possible the other would be also).



If you try to amplify an unknown photon by sending it into an ideal laser, the output will be polluted by just enough noise (due to spontaneous emission) to be no more useful than the input in figuring out what the original photon's polarization was.





Like a pupil confronting a strict teacher, a quantum system being measured is forced to choose among a set of distinguishable states (here 2) characteristic of the measuring apparatus.

Teacher: Is your polarization vertical or horizontal?

Pupil: Uh, I am polarized at about a 55 degree angle from horizontal.

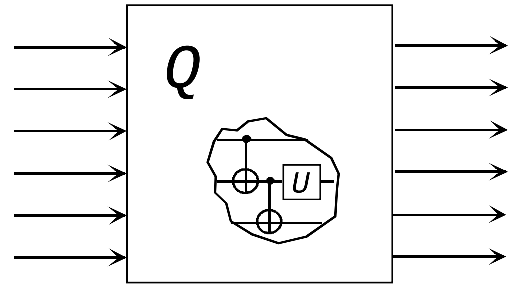
Teacher: **I believe I asked you a question.** Are you vertical or horizontal?

Pupil: Horizontal, sir.

Teacher: Have you ever had any other polarization?

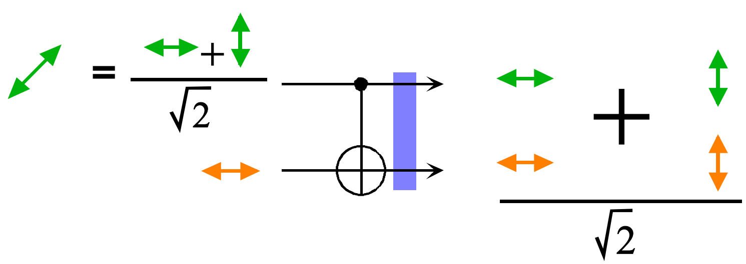
Pupil: No, sir. I was always horizontal.

Any quantum data processing can be done by 1- and 2-qubit gates acting on qubits.



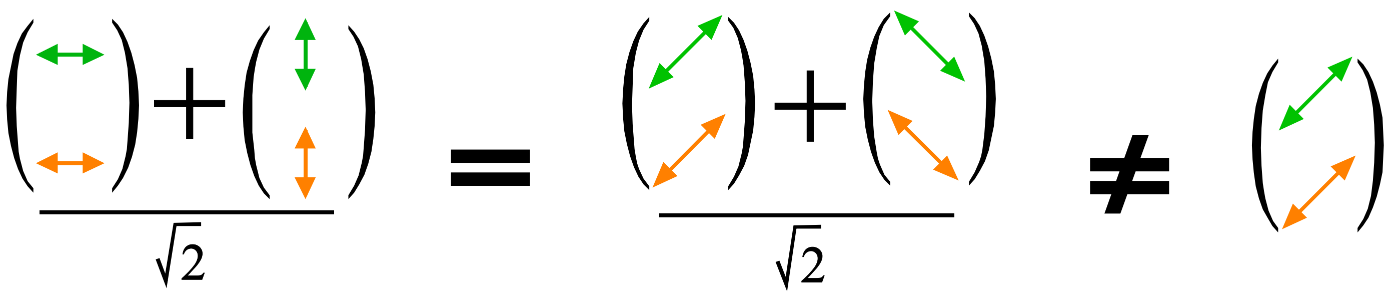
The 2-qubit XOR or "controlled-NOT" gate flips its 2nd input if its first input is 1, otherwise does nothing.

A superposition of inputs gives a superposition of outputs.



An entangled state

An entangled state is a state of the whole system that cannot be described by attributing states to its parts.

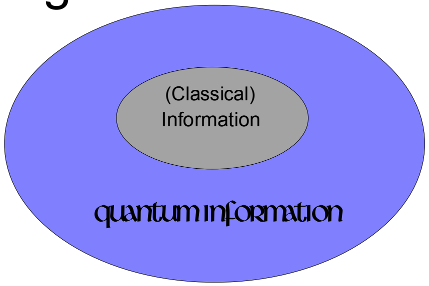


The two entangled photons can be said to be in a definite state of ***sameness*** even though neither has a state of its own.

Like two hippies in Woodstock or Haight-Ashbury, who sense that their spirits are in perfect agreement, even though neither has an opinion about anything.

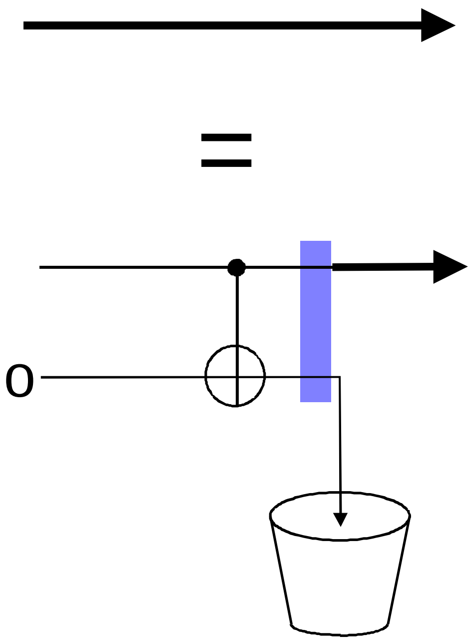
Re-expressing classical information in quantum language

A classical bit is just a qubit with one of the Boolean values **0** or **1**.



A classical wire is a quantum channel that conducts **0** and **1** faithfully, but randomizes superpositions of **0** and **1**.

(This occurs because the data passing through the wire interacts with its environment, causing the environment to learn the value of the data, if it was **0** or **1**, and otherwise become entangled with it.)



A classical channel is a quantum channel with an eavesdropper.

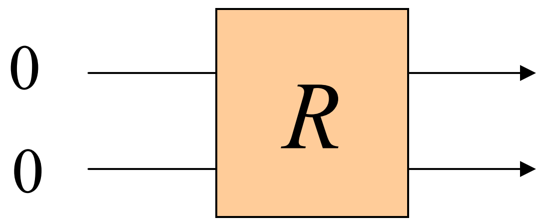
A classical computer is a quantum computer handicapped by having eavesdroppers on all its wires.

Entanglement:

A uniquely quantum phenomenon, allows entropy to increase in a subsystem even while the whole system remains in a state of zero entropy

Classical Dynamics

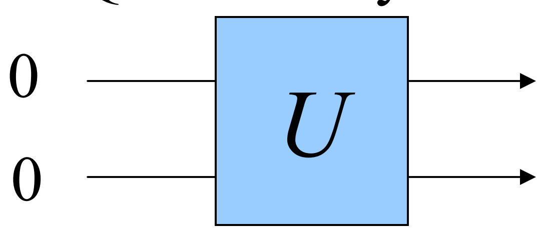
Zero entropy inputs



Each output subsystem has zero entropy

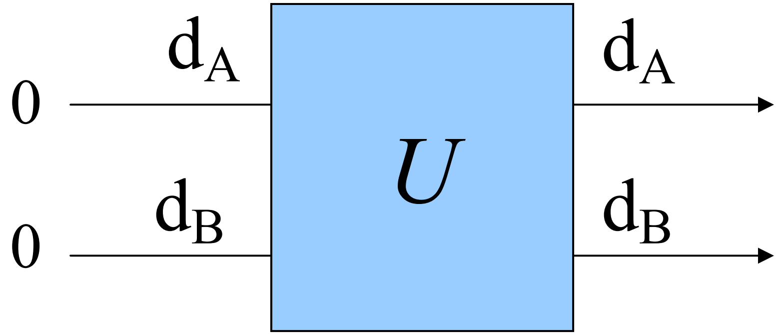
Quantum Dynamics

Zero entropy inputs



Whole output has zero entropy, but entangled subsystems can (and indeed typically do) have high entropy.

Generic Unitary Transformation

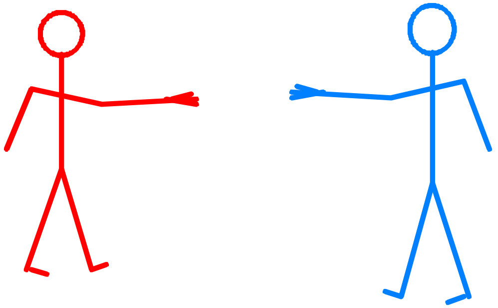


$$S_A = S_B \approx \log \min\{d_A, d_B\}$$

Entangled states are more numerous than unentangled ones.

In a large Hilbert space, most pure states are nearly maximally entangled (Seth Lloyd dissertation; Hayden, Leung, &Winter quant-ph/0407049) .

Personification of Entanglement



Some physical interaction is needed
to create entanglement. You can't become
entangled simply by talking on the telephone

Entanglement is monogamous —
the more entangled Bob is with Alice,
the less entangled he can be with anyone else.

Monogamy of Entanglement

In a perfectly entangled state of two parties, any measurement by one party yields complete information about the residual state of the other party (projects it into a definite local pure state).

But this does not scale up to three or more parties in the way some would hope. In a perfectly entangled 3-party state (e.g. the so-called GHZ state), measuring one party does not in general project each of other parties into a definite state. Instead, typically, it projects them into a definite entangled 2-party state.

In human terms this is like being Alice being perfectly in tune with the *relationship* between Bob and Charlie, but not with either gentleman individually.

What can Quantum Information and the Second Law tell us about the Big Questions.

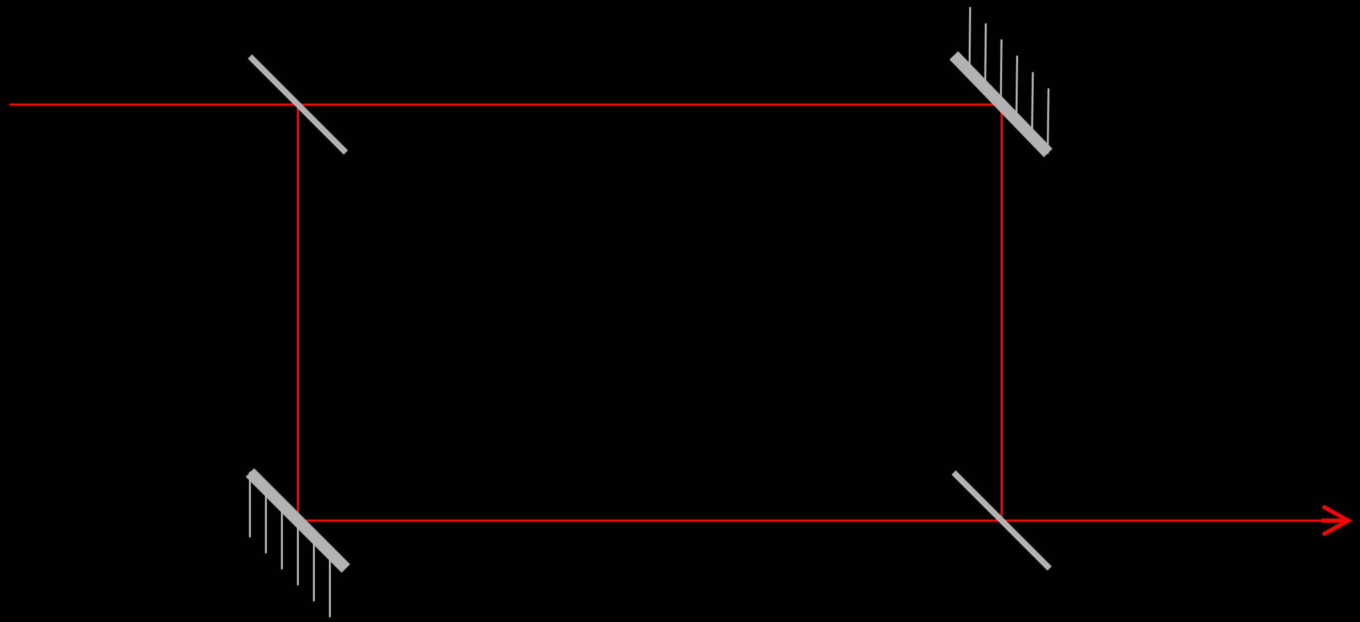
Why does the world seem so classical?

Why is the world complicated?

How real is the classical past?

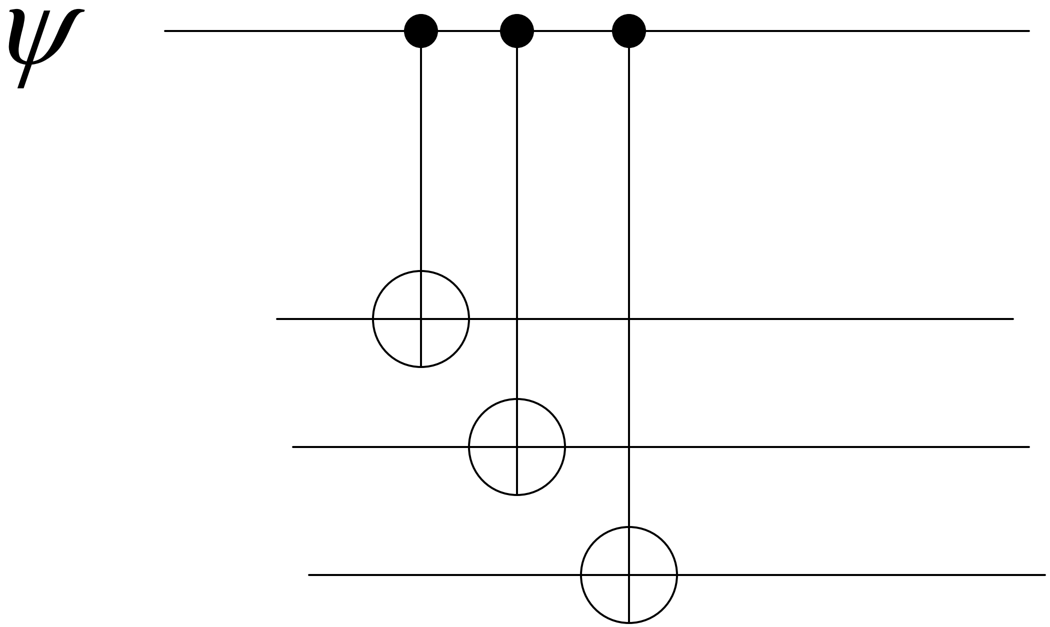
Could it be that the question, “What happened to Jimmy Hoffa?” is as meaningless as “Which path did the photon take through the interferometer?”

Quantum information is famously evanescent



If no record is made of which path a photon follows through an interferometer, or if a record is made but then unmade, the photon will have followed a superposition of both paths. After the experiment is over, even God doesn't remember.

Why does our quantum world seem so classical?



System

Environment:

Kinds of interactions present in our world typically determine a preferred basis for subsystems. In this basis, the environment contains many redundant copies of the system's information. In other bases it does not.

Information becomes classical when it is replicated redundantly throughout the environment.

"Quantum Darwinism"

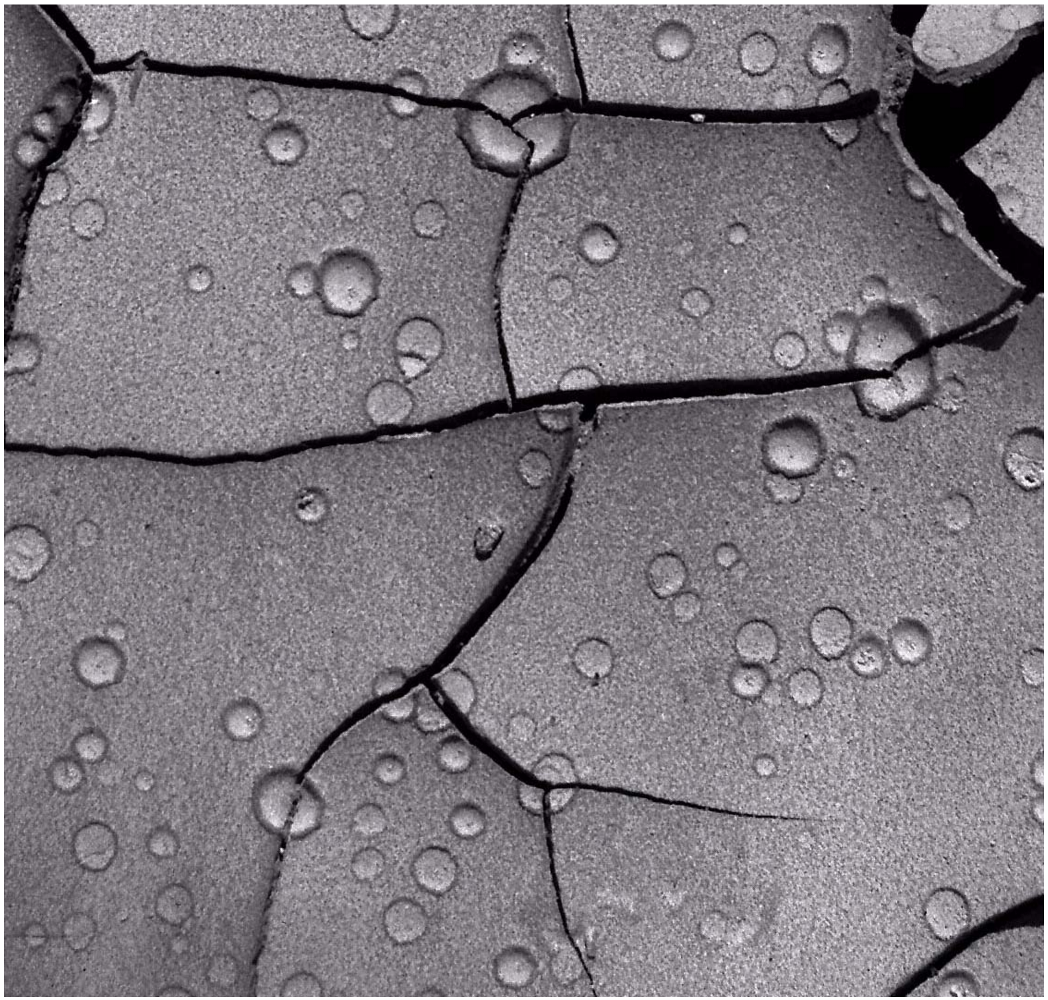
Blume-Kohout, Zurek quant-ph/0505031 etc.

Nowadays, it is tempting to believe that once information has become public, and someone has written about it in their blog or put it on You-tube, it can never be destroyed.

The modern world appears very different in this regard from the ancient pre-Gutenberg era, when major literary works were written down, performed, and widely known, but then then lost.

Sappho, ca 620-525 BC, by Gustav Klimmt

But even in today's world, much macroscopic, publicly accessible information is lost because no person, nor any natural process, happens to record it in a durable medium.



Raindrop marks in dried mud in a river bed in Las Vegas, USA in 1965.
A few days later these cracks and craters were washed away by a subsequent rain.

If no one had photographed them, would all record of them have been lost?

It is tempting to believe that such macroscopic information is not lost, just that it becomes so diffusely and complexly spread out as to be irrecoverable in practice while being stored somewhere in the universe (just as when a book is burned its contents are in principle recoverable from the exact state of the smoke, ashes, and heat it generated).

To believe otherwise is venturing dangerously close to the postmodernist view, abhorred by most scientists as arrogantly anthropocentric, that the past (or maybe even the present) has no objective reality independent of our beliefs about it, and therefore that it is pointless to inquire what "actually" happened.

But I will argue that much formerly classical information about the past eventually acquires nearly the same ontologically ambiguous status as which-path information in an interferometer.

I think some information is really lost, not from the universe but from the world (i.e. the planet Earth). Why? –because most information we might care about is washed away by much larger entropy flows into and out of the Earth.

The Earth has finite information storage capacity, but it exports a lot of randomness (generates a lot of entanglement with its environment, in the quantum way of speaking) in the form of thermal radiation into the sky.

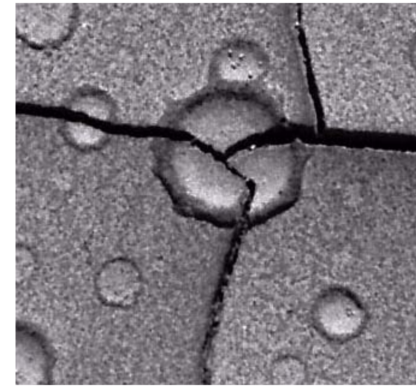
Thermal entropy export rate $\approx$ 200 watts/sq meter at 300K
$\approx 10^{30}$ bits per square meter per year.

Geological information capture rate in "hard" degrees of freedom, stable for geological times against thermal motion (e.g. atomic substitutional disorder and crystal lattice defects in solid rock of earth's crust) = crust thickness ($\approx$10 km) × rock information density ($\approx$1 bit/cubic nm) / rock lifetime ($\approx 10^8$ yr)
$\approx 10^{22}$ bits / per square meter per year.

Human digital information capture rate 100GB/person x 10^9 people who are heavy information users $\approx 10^{21}$ bits per year

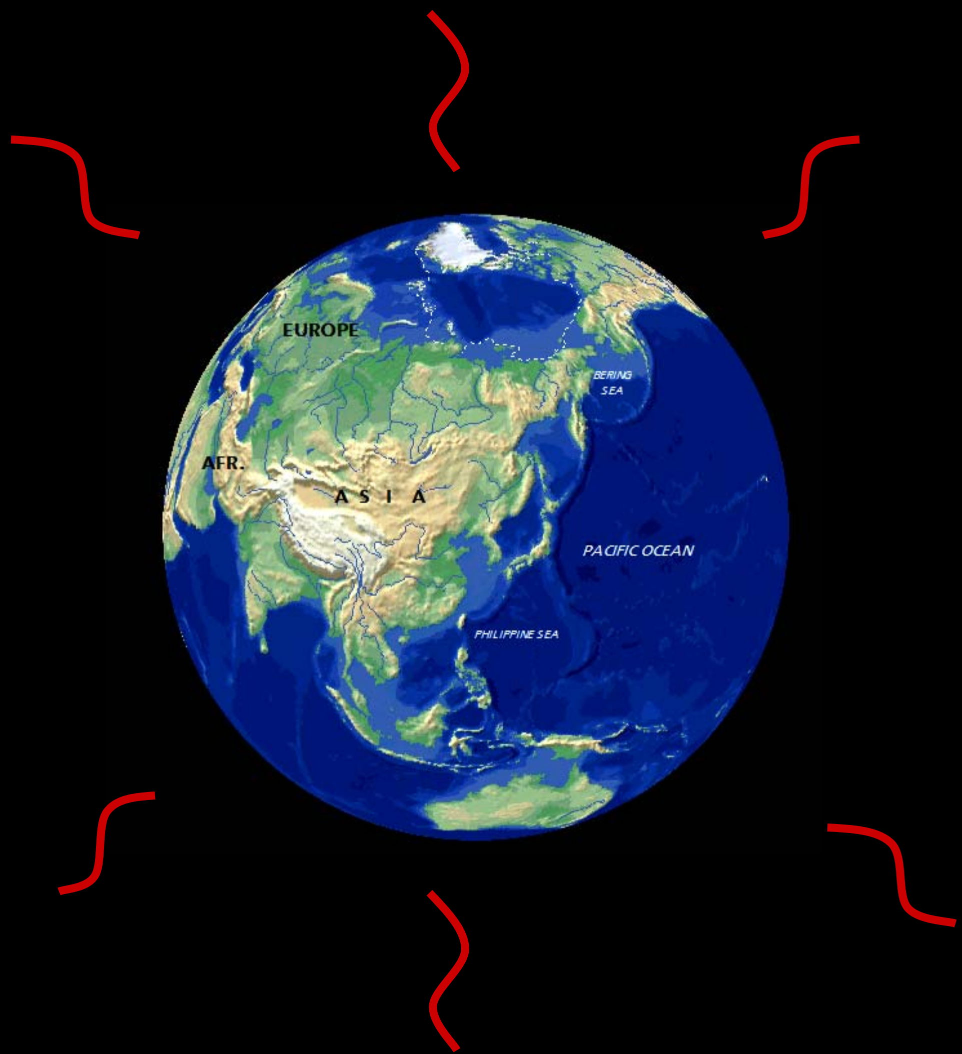
(that's for the whole world, not per sq meter)

Randomizing dynamics in a representative case.



Though the raindrop originates in quantum and thermal fluctuations, it does not fall in a superposition of places. Independent observers would agree where it fell, and as long as the drop or its crater exists, reflected light will generate a torrent of replicas of the information, fulfilling the classicality criterion of quantum Darwinism.

However, unless the crater is lucky enough to get fossilized, it will be washed away, and its former location will then lose any stable earthly embodiment. The torrent of optical replicas will cease, and the old optical replicas will escape into space. So the classical information of where it was remains in the universe, but not the Earth. This still sounds more real than which-path information about a particle in an interferometer, but I will argue that it is not.



To catch up with the thermal radiation leaving Earth, one would need to travel faster than light. So the information is still in the universe, but not recoverable by us.

Mysteries of the Past:

Still recorded on earth, though unknown to any human and inaccessible with current technology:

- Locations of gold rings, dropped in an annual ceremony into the Venice Lagoon over a period of several centuries, to symbolize Venice's marriage to the Sea.

Maybe still recorded on earth, maybe escaped: Fates of mysteriously disappeared persons such as

- Physicist Ettore Majorana disappeared 1938
- Labor leader Jimmy Hoffa disappeared 1975
- Computer Scientist Jim Gray disappeared 2007

Escaped:

- Unrecorded raindrops from past rain storms.
- Pattern of foam on my yesterday morning's cappuccino.

What can we do to make a particular chosen body of information long-lasting (say until the sun turns into a red giant)?

Why would we want to?
- To preserve important works of literature
- To preserve evidence of a crime until it is safe to publicize, thereby discouraging crime even in times of despotism and corruption
- Because we hate postmodernism and want to make even unimportant details of the past uncontestable.

Record the information in a durable digital medium, and bury many copies in geologically stable rock formations in various parts of the world, as if it were nuclear waste.

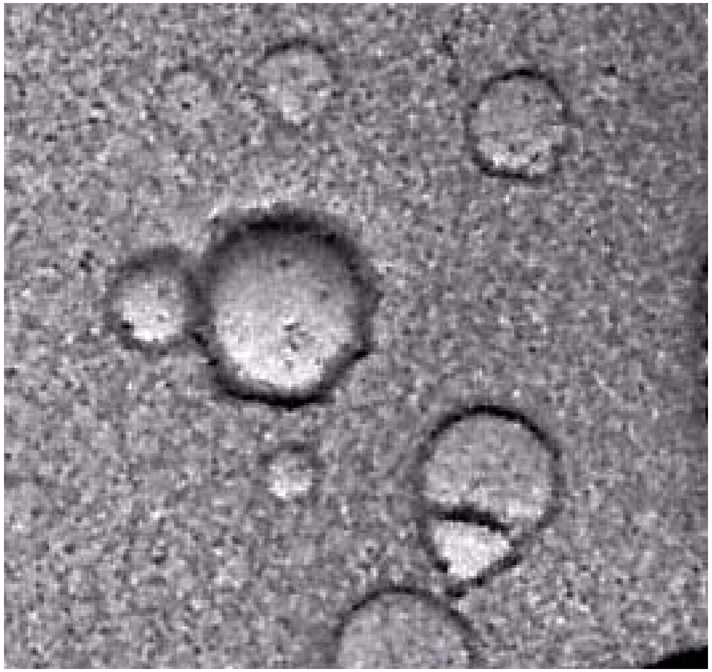
But suppose we wanted to store not all or most, but a lot of information, say a real-time video surveillance of entire earth surface at millimeter-millisecond resolution.

This works out to about 10^{16} bits/sq m year, well within geological capture rate.

Is this scary thing perhaps happening already, automatically, without deliberate human effort, just because frozen accidents in newly formed rock in a sense provide a hash of the current state of the earth?

Probably not, due to randomizing effect of dynamics.

Randomizing dynamics in a representative case.



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How to obliterate earthly evidence of Jimmy Hoffa’s demise? (Former US labor leader disappeared in 1975, presumed murdered by the New York City Mafia, but body was never found. Police are still searching.)

- Cremate his body and let the smoke and heat escape

- Dissolve the ashes to make a clear liquid, with no solid fragments, then pour the liquid into the ocean

- Don’t tell anyone you did it, even on your deathbed

- For good measure, have yourself cremated and your ashes dissolved to make sure physical traces of your memory are thoroughly gone.

Some further Questions

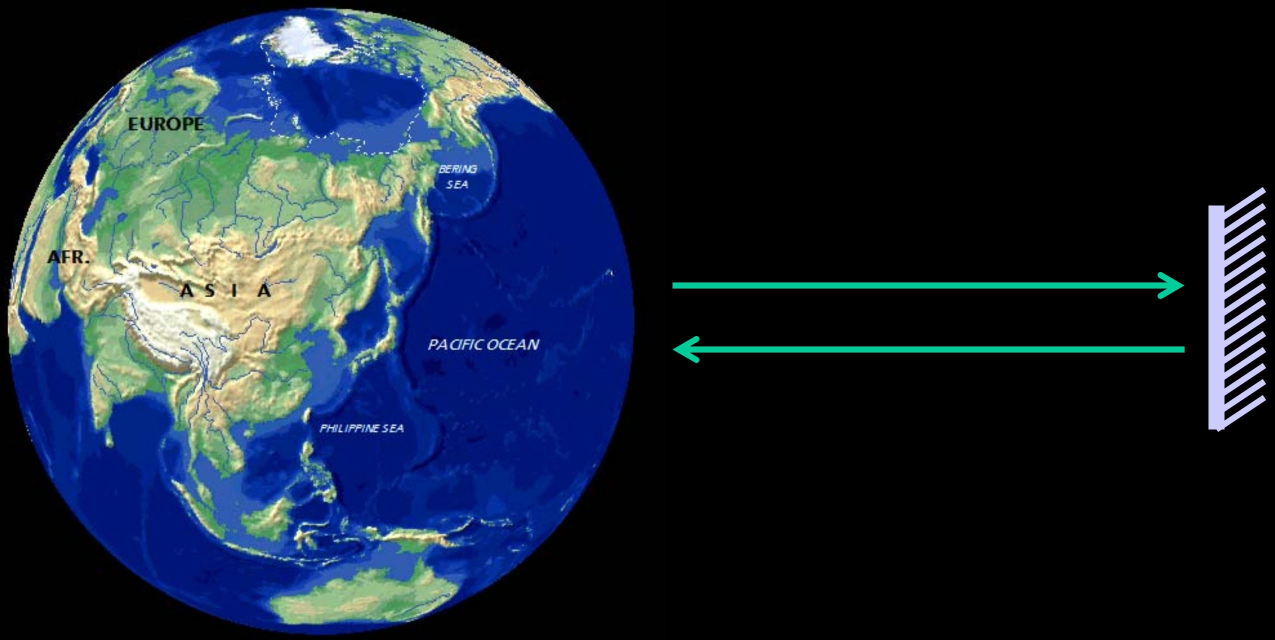
- Is it really impossible to recapture the escaped radiation?

- What is the ontological status of escaped information? Does God remember where the raindrops fell, even after all terrestrial evidence is gone?

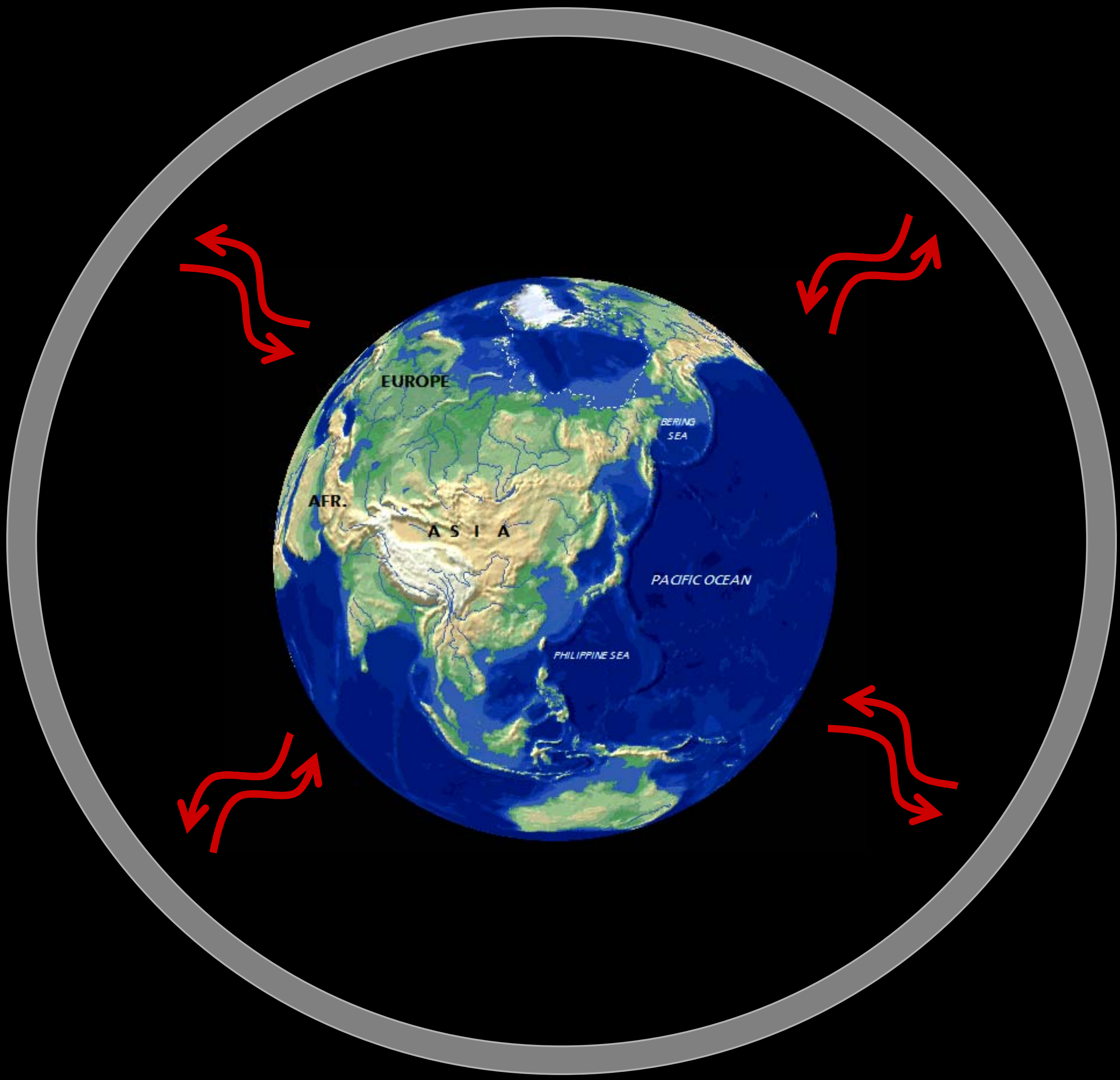
- Is random input (e.g. entropy of incoming solar radiation) necessary to make the earth forget things?

- Even after classical information is lost from "hard" geological storage, might it still be retained in the earth's far more numerous "soft" degrees of freedom such as phonons in the earth's core and mantle?

Can we arrange for escaped information to be reflected back to us later, making it again accessible?

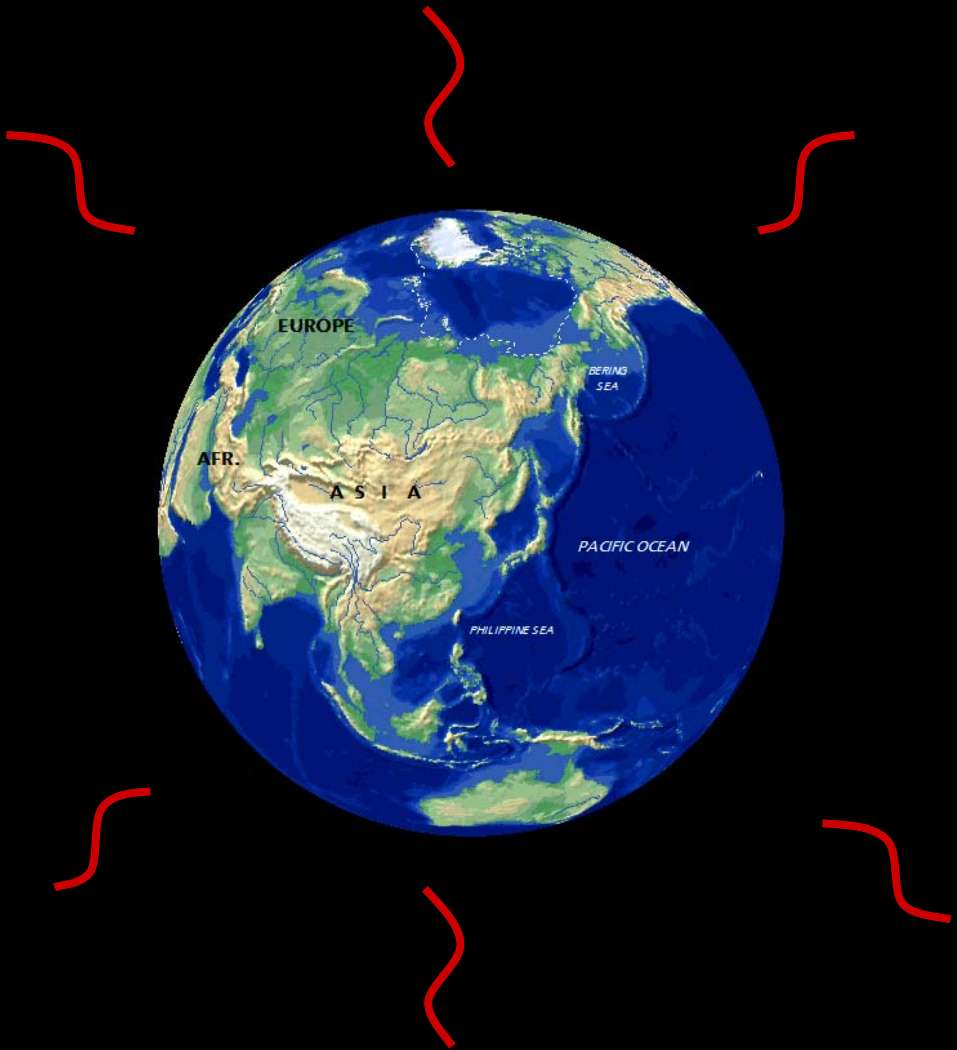


Yes. For specific items of non-thermalized outgoing radiation (e.g. optical earth views, old TV broadcasts) , this could be arranged, with advance planning, or it might happen accidentally. Such information could be called extraterrestrial fossils.



But for fully thermalized radiation we would have to catch and reflect back so much of it, to reconstruct any particular item of interest, that the earth would badly overheat.

- Can we outrun the radiation?



For example, one might hope to outrun the thermal radiation, because the refractive index of interstellar space is slightly >1.

But this hope is probably dashed by the accelerating expansion of the universe (a.k.a. cosmological constant, dark energy) which causes remote objects now visible (e.g. other galaxies) to eventually become inaccessible.

Ontological Status of Escaped Information

Consider a raindrop that may fall in one of 2 locations **L** or **R**. Suppose that it forms, falls, and finally evaporates, so that all earthly record of where it fell is lost as radiation into the sky.

(**LLLL**+**RRRR**) /√2 Drop forms, falls and begins to emit radiative replicas into space. All observers, terrestrial and celestial, will see the drop as having fallen in one of two places. God sees a cat state-like superposition in which both outcomes happen.

(**LLLLL**+**RRRRR**) /√2 Drop begins to evaporate, emitting further radiative replicas.

(**LLLLL**+**RRRRR**) /√2 Drop has entirely evaporated. No terrestrial information remains about where it fell.

- Conclusion: Escape of last classical information from earth restores terrestrial observers to God's Olympian viewpoint in which both outcomes happened. Escaped information not different from which-path information.

"The past exists only insofar as it is recorded in the present." —Wheeler

The Evolving Human Viewpoint

- Before raindrop forms: Humans know it's about to rain, but they don't know where the drops will fall.

- (**LLLL**+**RRRR**) /√2 Humans agree where drop has fallen. Depending on how it affects their plans, they may or may not care, but they all agree that one of the outcomes R or L is real while the other is merely an unrealized possibility.

- (**LLLLL**+**RRRRR**) /√2 Drop has entirely evaporated. People may remember that it fell but no one remembers where. No other terrestrial memory remains about which of the possibilities, L or R, "happened" and which did not.

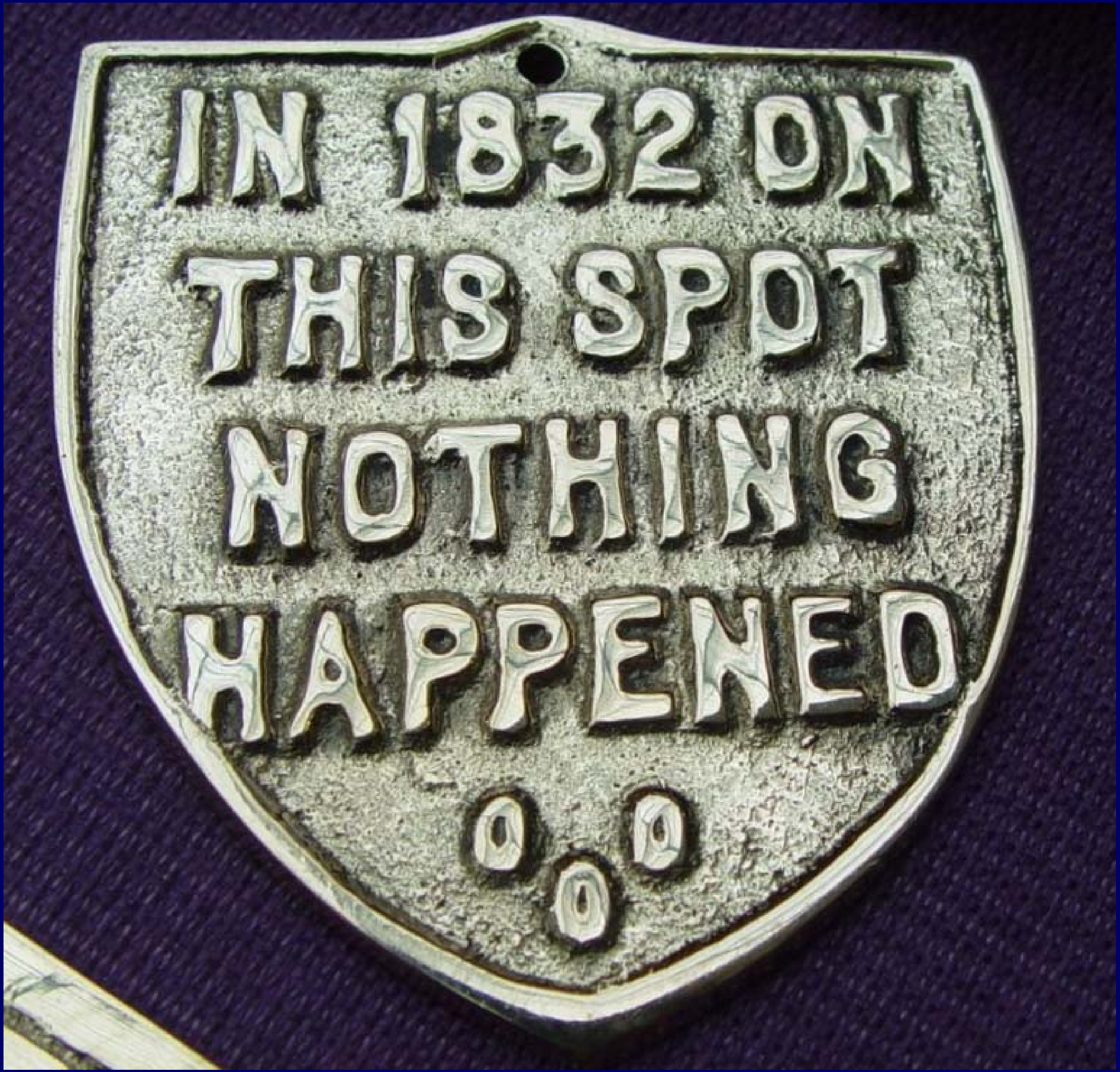
In the end, humans will come to accept what God knew all along, that the distinction between what happened and what might have happened is transient and illusory.

Most classical information, such as the pattern of snow flakes on the ground last winter, or rice grains on my dinner plate last night, is impermanent, eventually losing its durable embodiment and escaping from the earth in outgoing radiation.

Occasionally information is lucky enough to get fossilized by natural processes or recorded by humans in a durable medium. Such information can last billions of years.

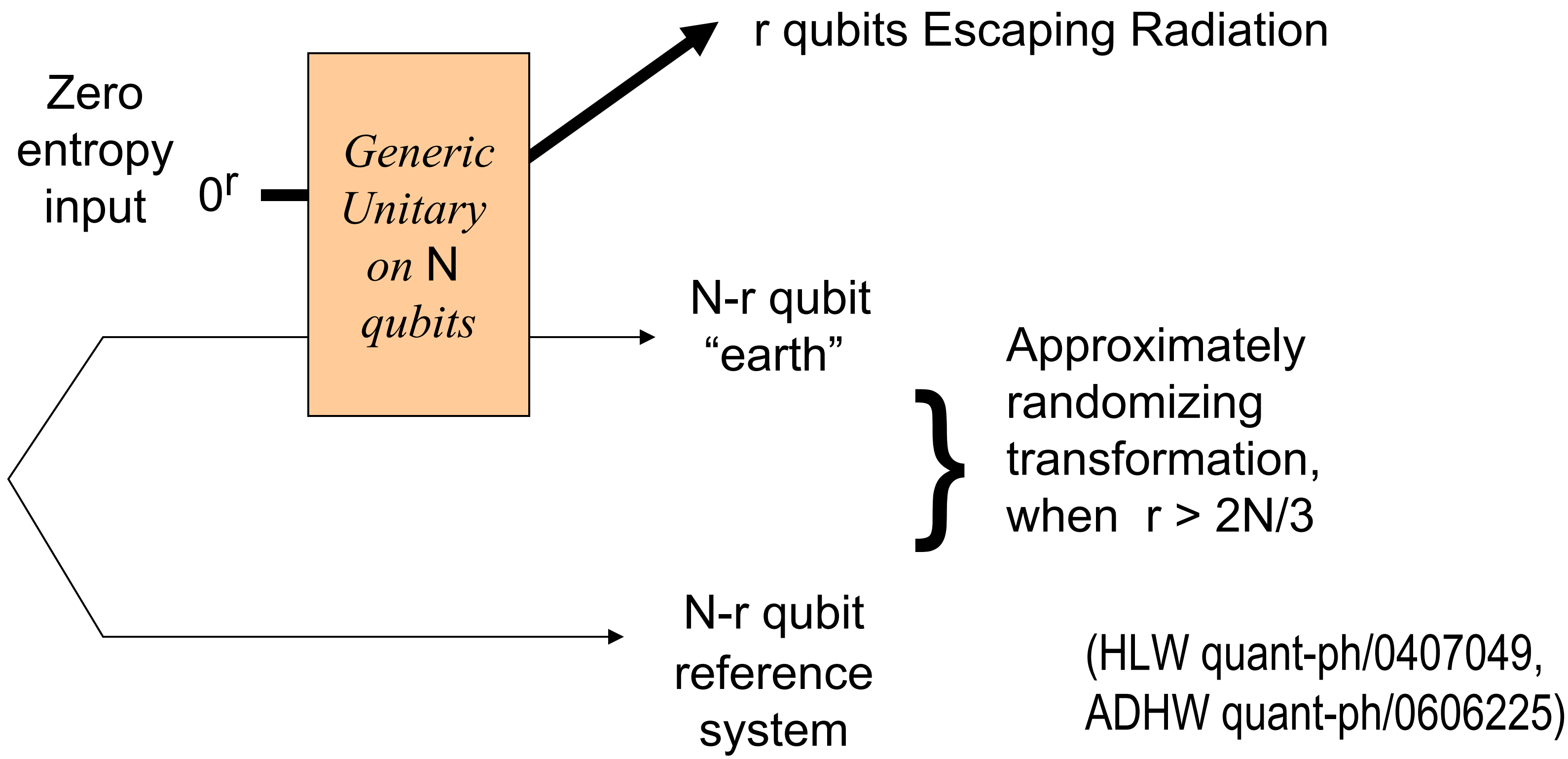
Escaped information still exists in the universe, but it is inaccessible on earth.

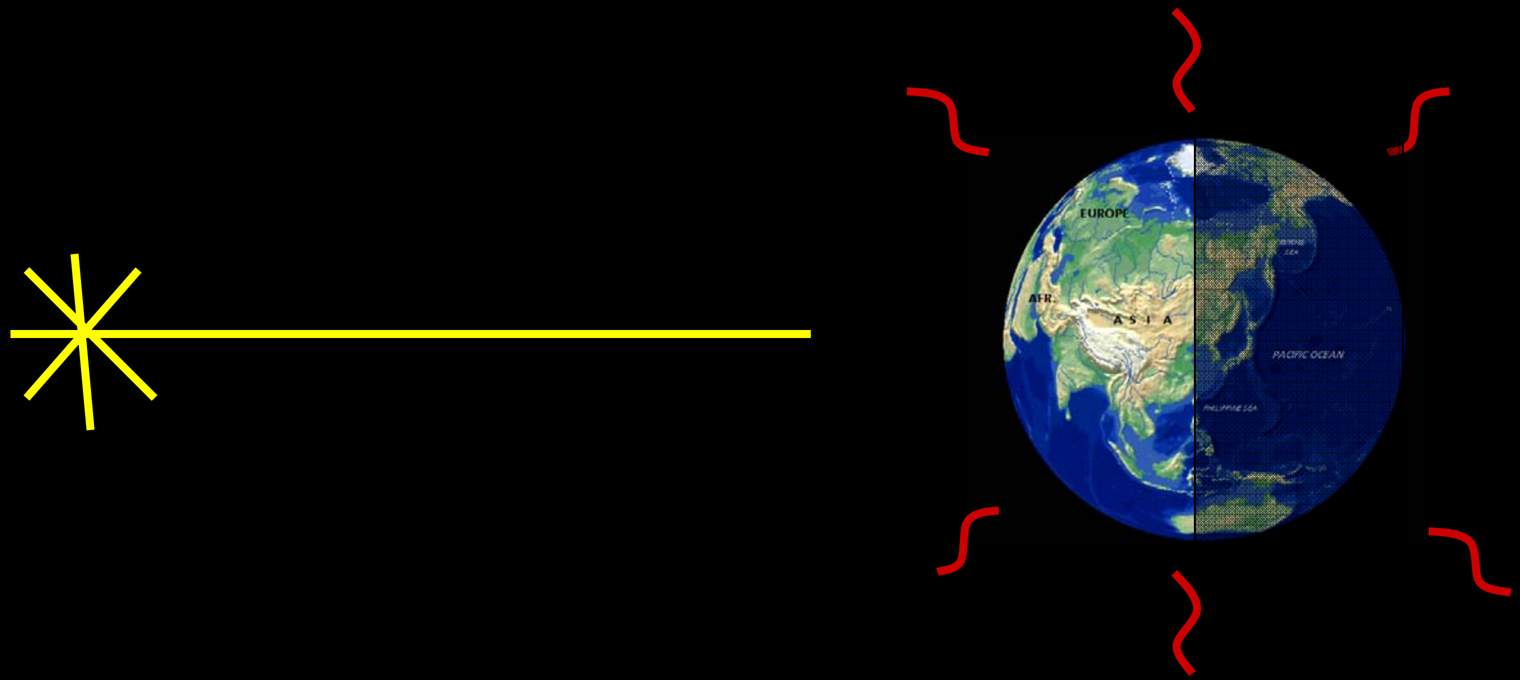
Humans have little justification for continuing to think that one alternative really happened and the others didn’t.


IN 1832 ON
THIS SPOT
NOTHING
HAPPENED

• Is random input (eg radiation from the sun) necessary to achieve randomization?

Not always. Unlike a classical system, a deterministically evolving quantum system can be randomized simply by allowing enough information to escape from it.





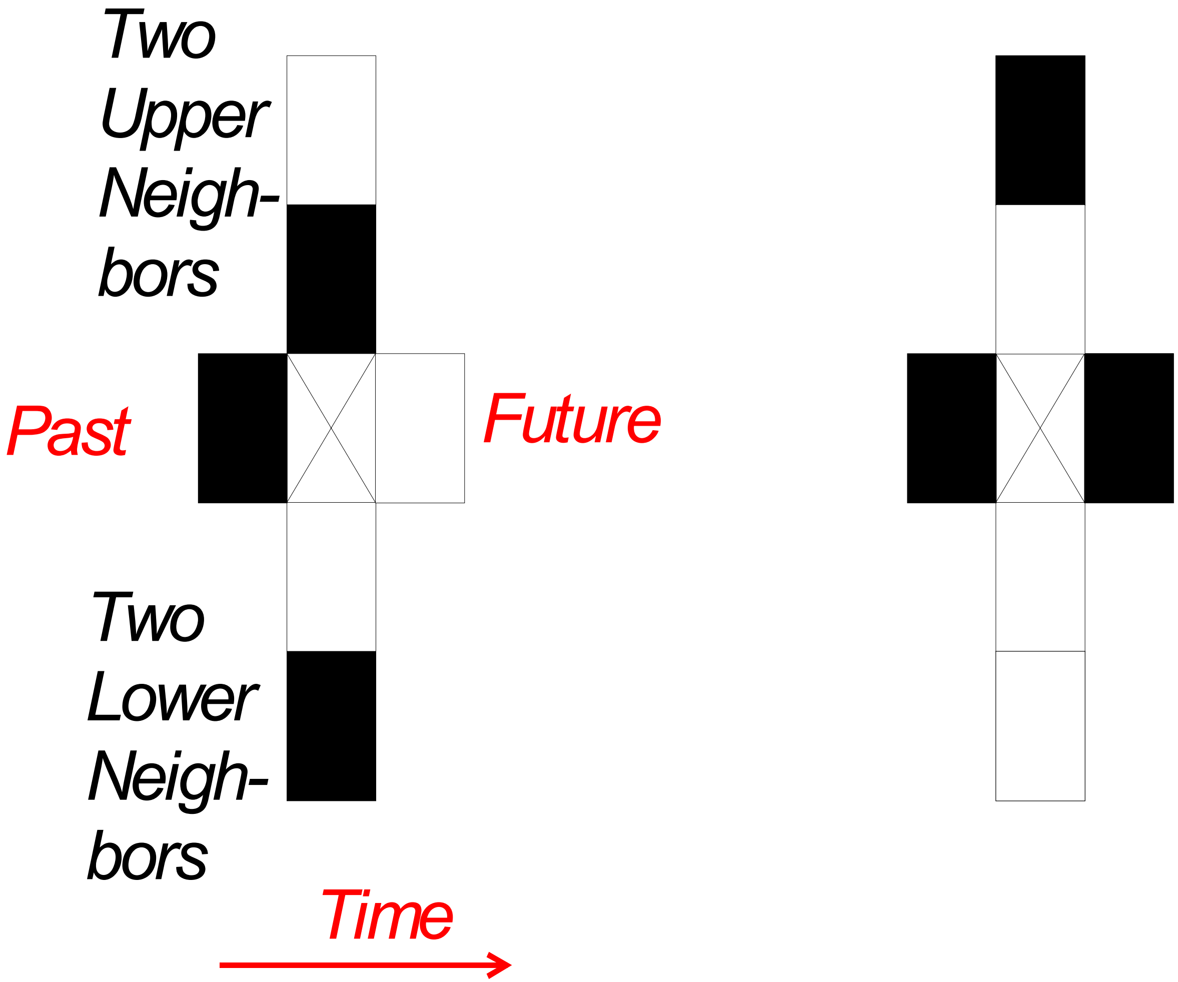
If the earth's solar input were replaced by a laser beam of equal power, the input entropy would be zero while its apparent output entropy rate would be about the same. Thus at a steady state the output entropy rate would also be zero, because of entanglement among the output modes. The earth would be functioning as a giant down-converter. Unlike an ordinary down-converter, the correlations would be exceedingly computationally complex and unobservable in practice.

Radiation from a deterministically evolving system with zero input entropy

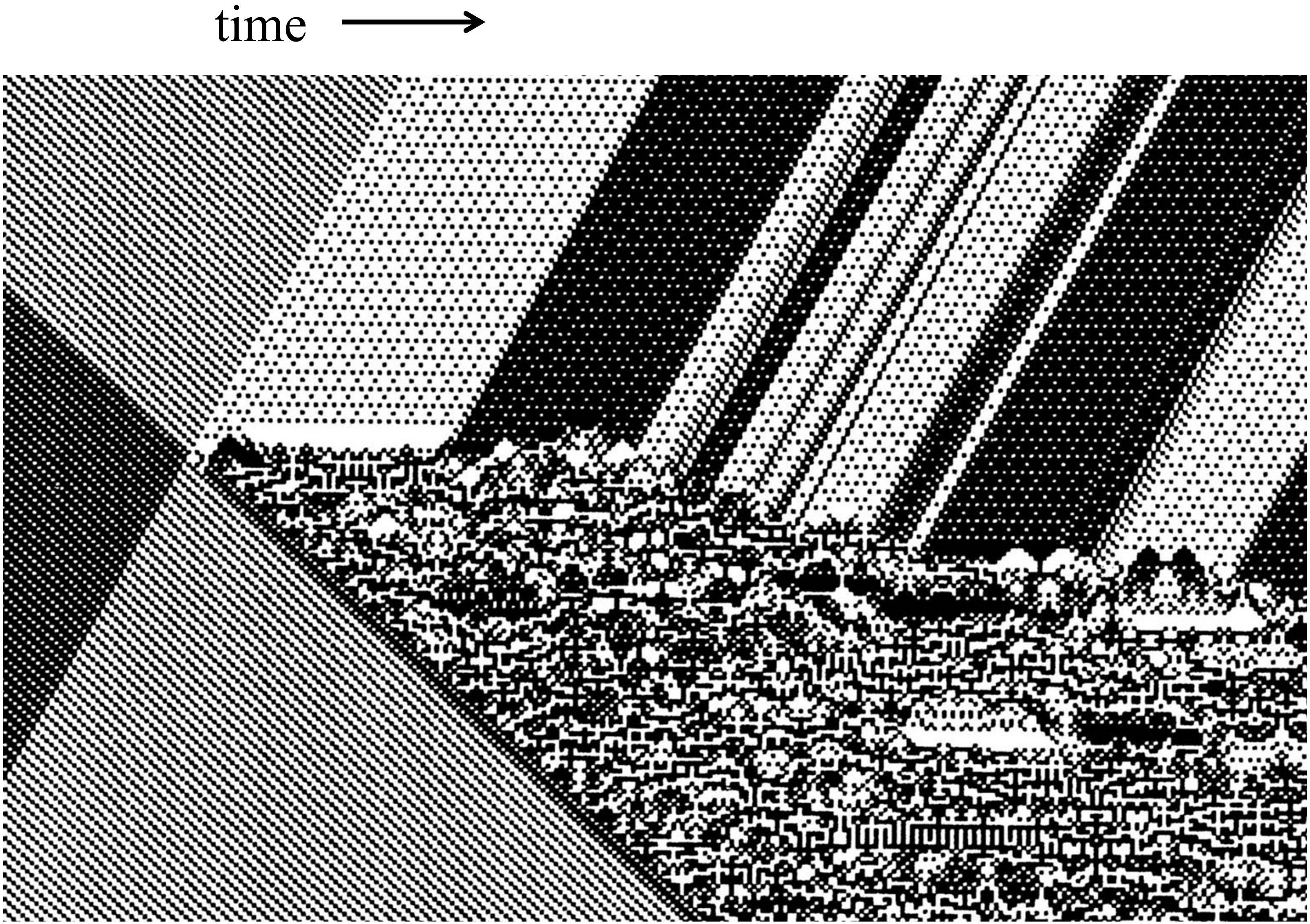
Discrete Classical (e.g. reversible cellular automaton):
    Pseudorandom radiation, pseudorandom residual system

Continuous Classical (chaotic)
    Random radiation, random residual system (both from mining the infinite-precision generic initial condition. If initial condition were special, both residual system and radiation would be pseudorandom)

Discrete Quantum:
    Random residual system, random-looking radiation, entangled with itself and with residual system



Range-2, deterministic, 1-dimensional Ising rule. Future differs from past if exactly two of the four nearest upper and lower neighbors are black and two are white at the present time.

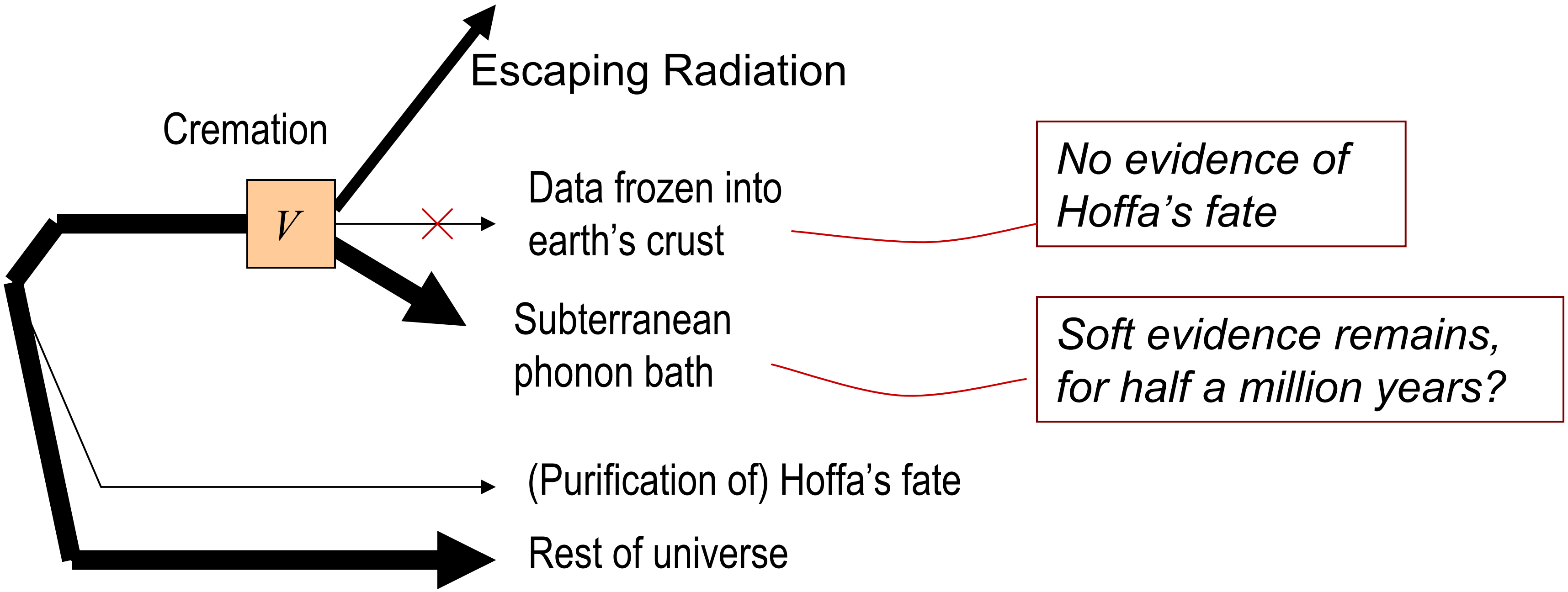


“Radiation” from a hot pseudorandom state formed by collision of two domain edges in simple initial condition in range-2 reversible deterministic 1d Ising cellular automaton

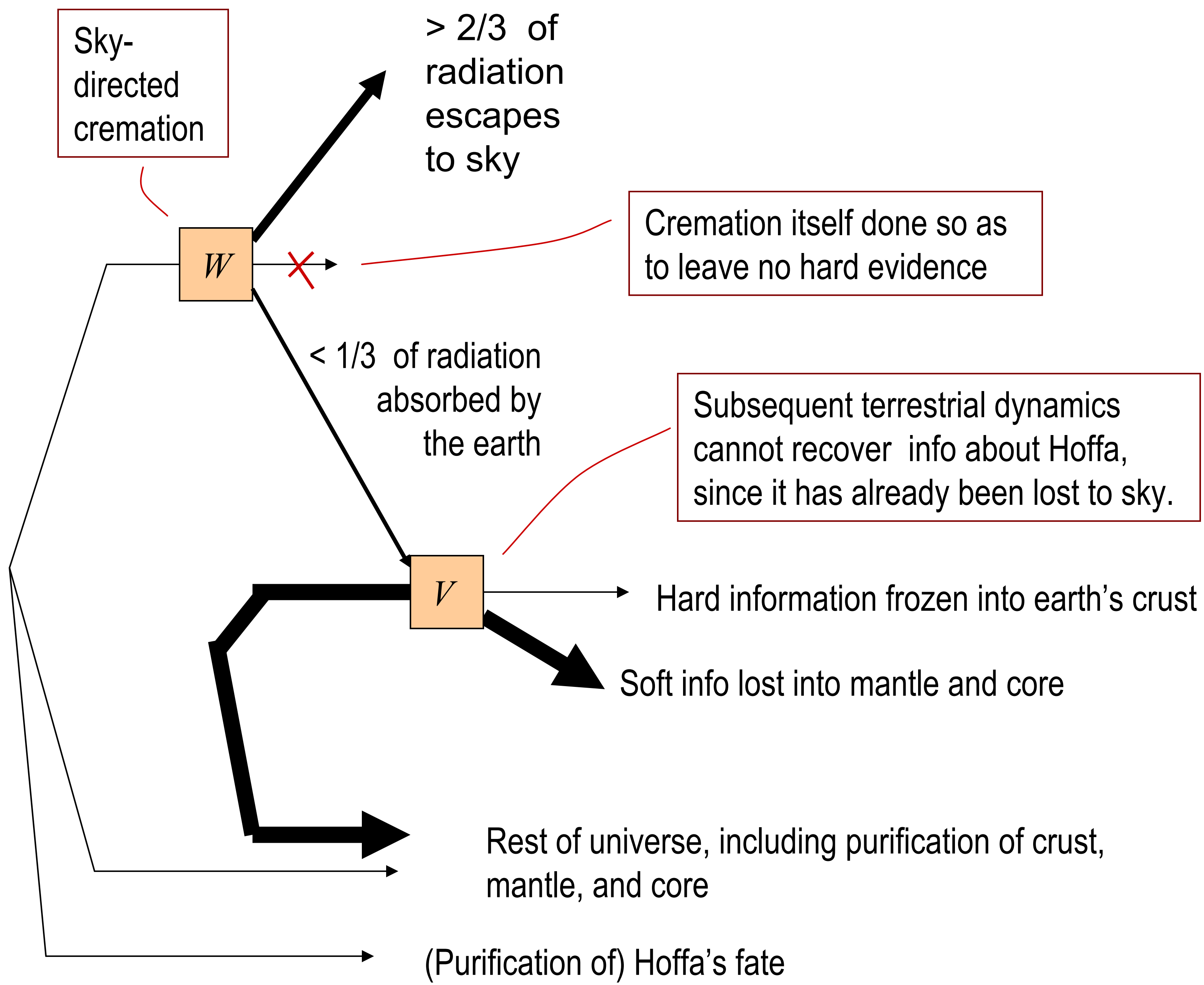
Up to now, I have considered only geologically stable "hard" degrees of freedom in the crust, and neglected the far more numerous "soft" degrees of freedom (e.g. phonons and photons) in not only the crust but the whole body of the earth. These degrees of freedom have entropy about a million times *greater* than the annual radiant entropy flux leaving the earth.

Therefore the condition $r > 2N/3$ for quantum randomization without random input is badly unfulfilled.

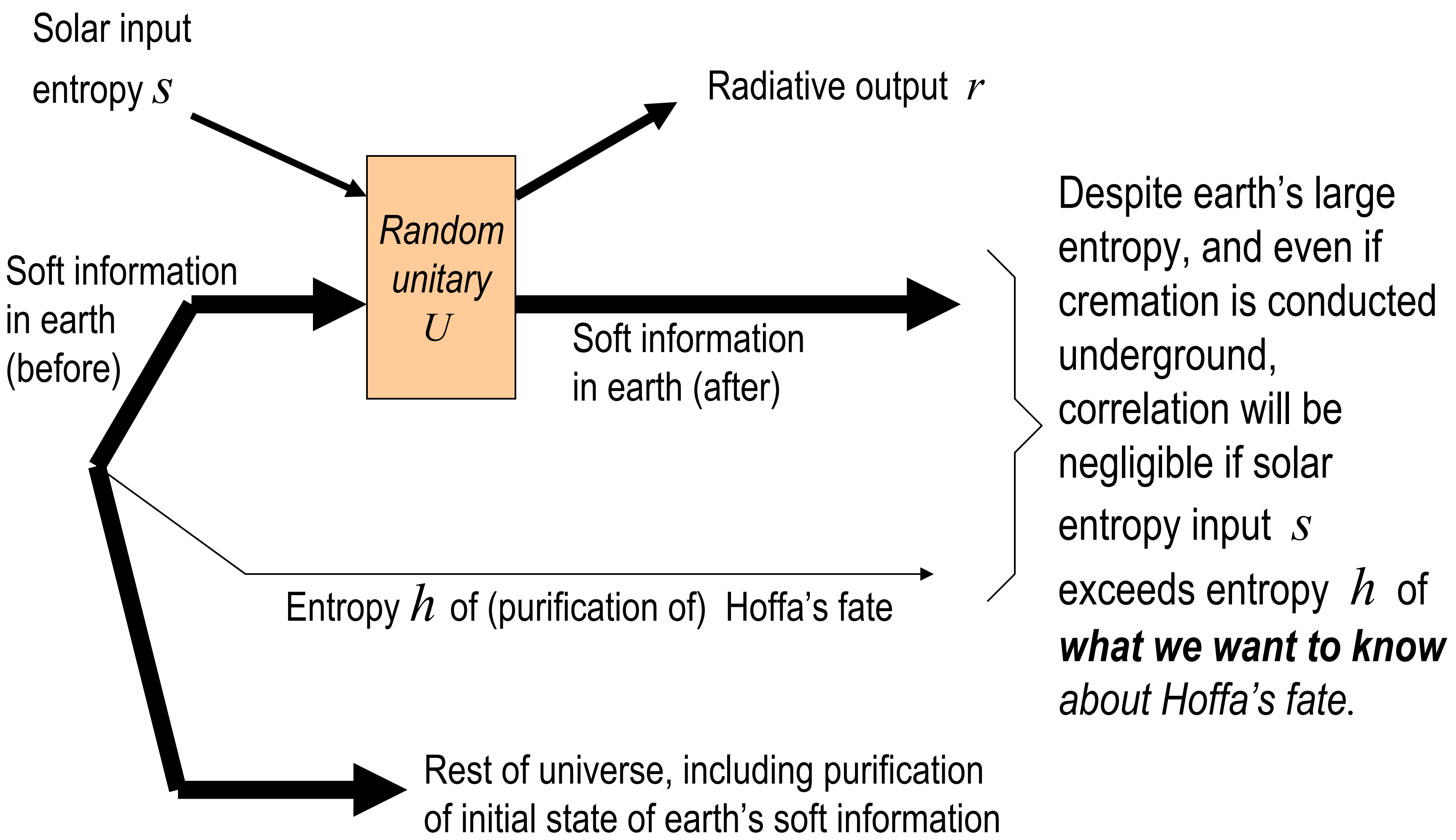
Could it be that, if Hoffa were cremated in a way that efficiently couples to soft degrees of freedom in the earth (deep in a mine, for example), this soft evidence will persist for about half a million years, even after the hard evidence is gone?

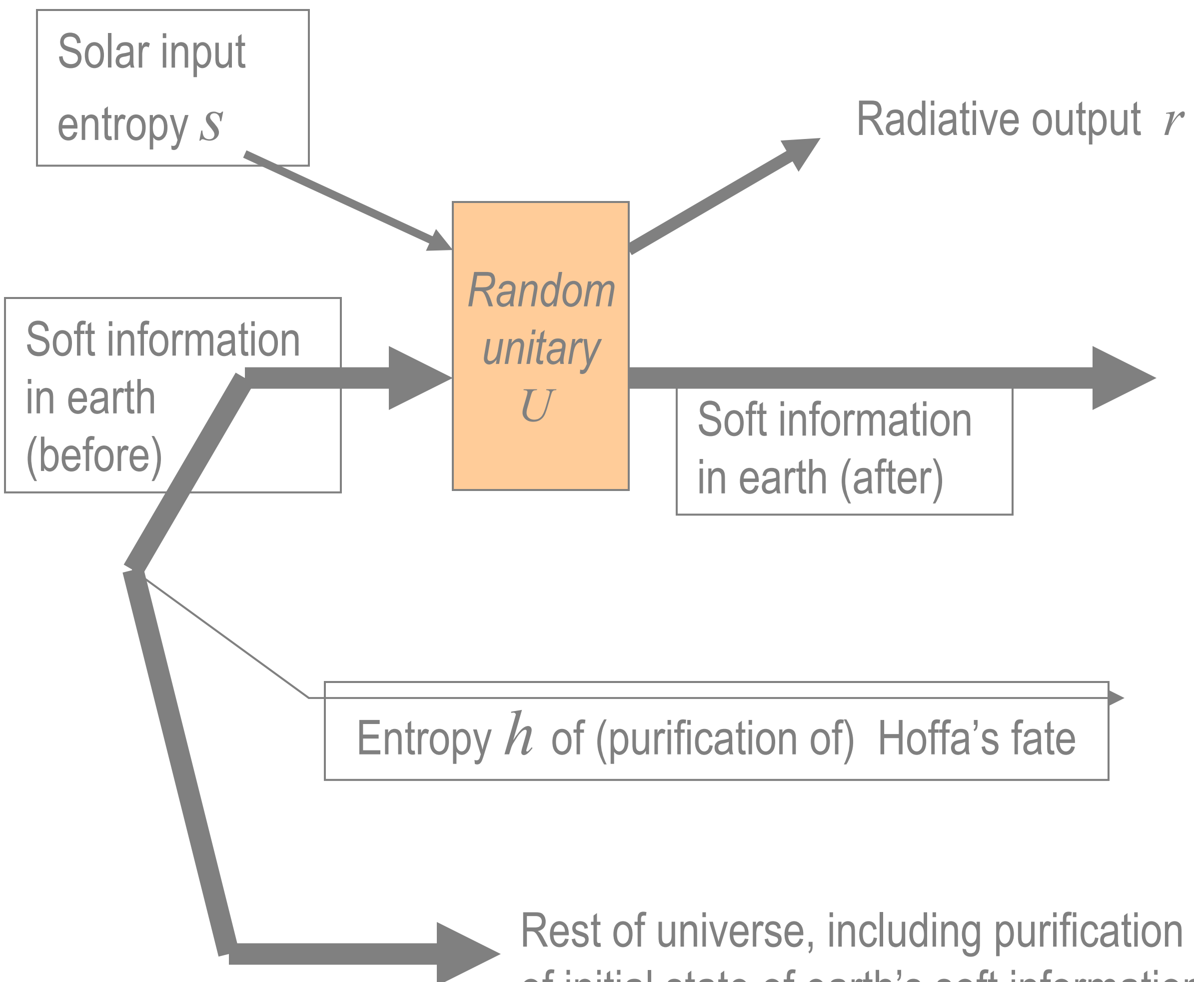


To keep the earth from storing soft evidence of Hoffa's death, cremate him outdoors in an insulated reflective pan that directs most of the radiation skyward and otherwise randomizes his state. Avoid doing it on a cloudy day.


Sky-directed cremation
> 2/3 of radiation escapes to sky
W
Cremation itself done so as to leave no hard evidence
< 1/3 of radiation absorbed by the earth
Subsequent terrestrial dynamics cannot recover info about Hoffa, since it has already been lost to sky.
V
Hard information frozen into earth's crust
Soft info lost into mantle and core
Rest of universe, including purification of crust, mantle, and core
(Purification of) Hoffa's fate

But such elaborate precautions are probably unnecessary, because of additional randomization from solar input radiation (the sun is not a laser).





An example of information locking.

Earth doesn't remember whether Hoffa was killed in New York or New Jersey, but does remember a useless and complicated function of that, Sappho's poems, last winter's snowfall, yesterday's cappuccino, etc.

Enough about information & remembering and forgetting.

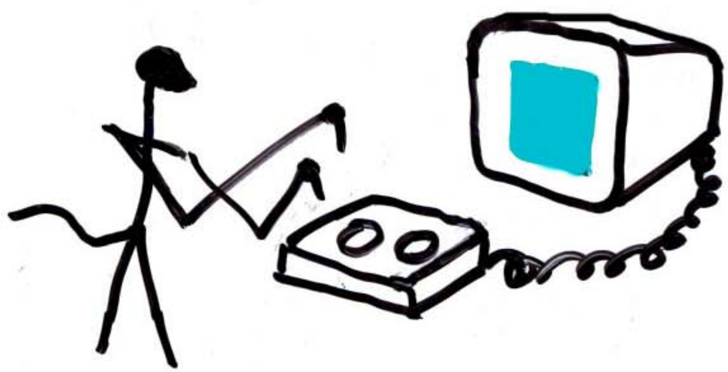
Can we find a non-anthropocentric definition of what kind of information is *worth* remembering?

How should *complexity* be defined?

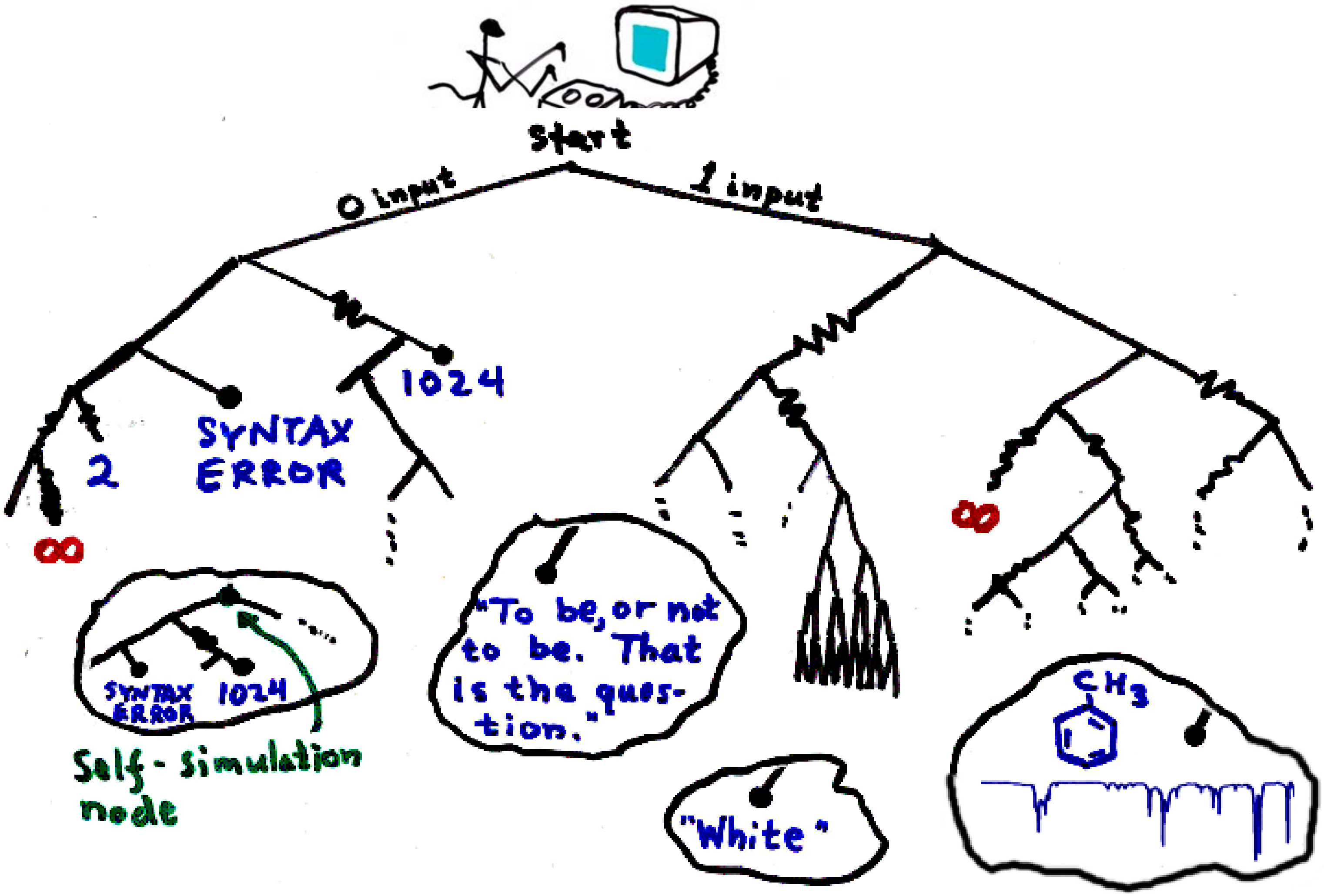
How does it originate?

Why is the universe complex?

Defining complexity: use a computerized version of the old idea of a monkey at a typewriter eventually typing the works of Shakespeare. Of course a modern monkey uses a computer instead of a typewriter.

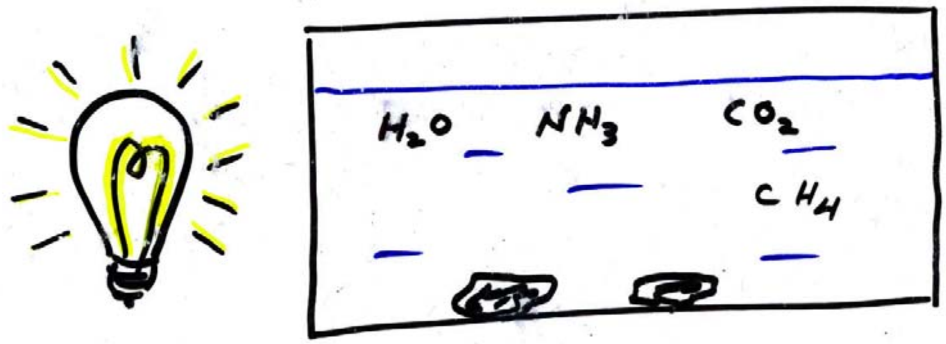


A monkey randomly typing 0s and 1s into a universal binary computer has some chance of getting it to do any computation, produce any output (Chaitin 1975)

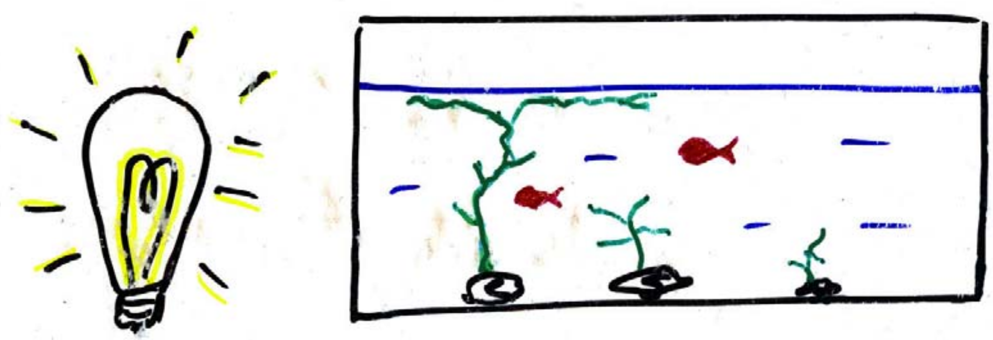


The input/output graph of this or any other universal computer is a microcosm of all cause/effect relations that can be demonstrated by deductive reasoning or numerical simulation.

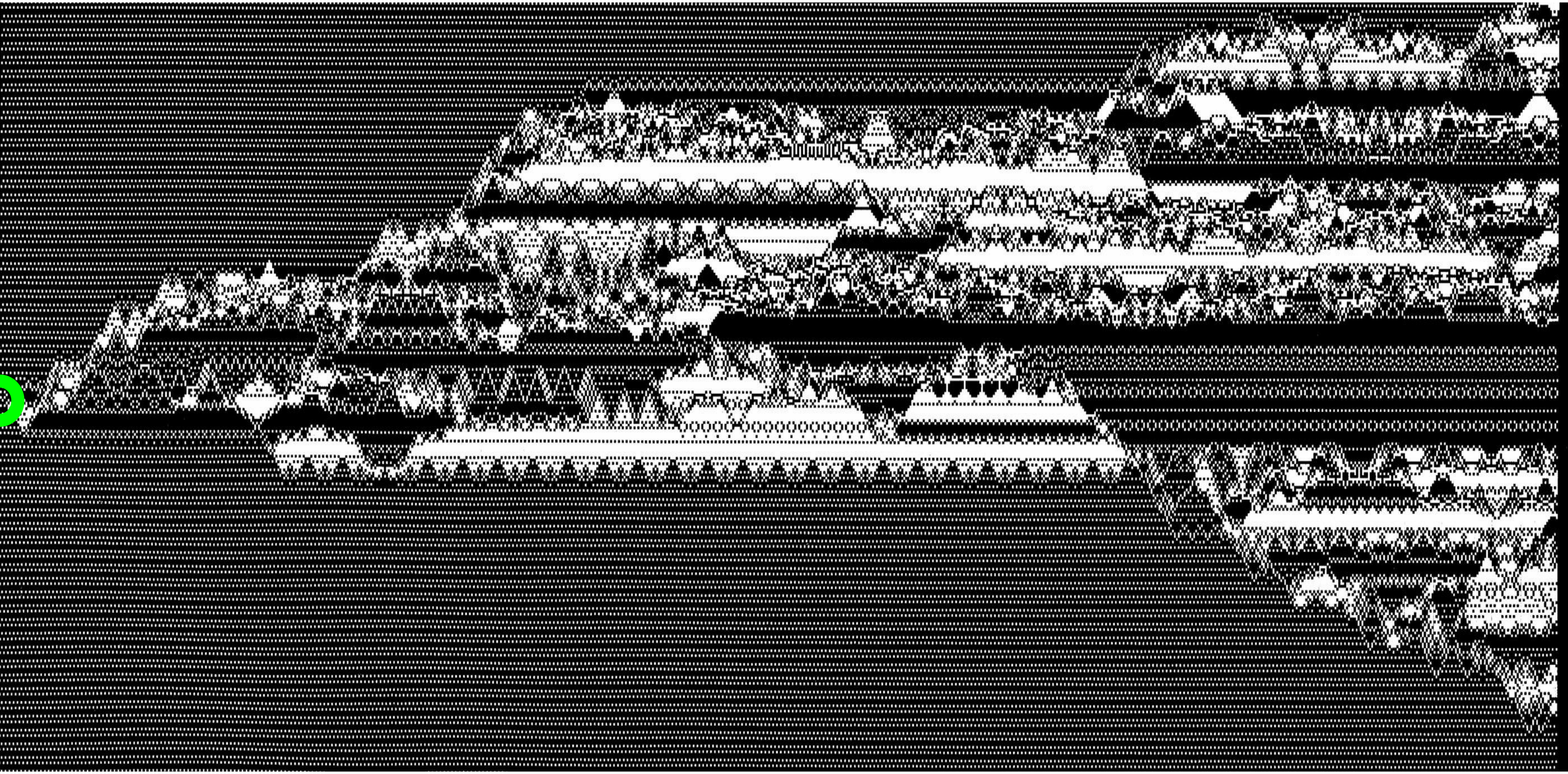
A simple cause can have a complicated effect, but not right away.



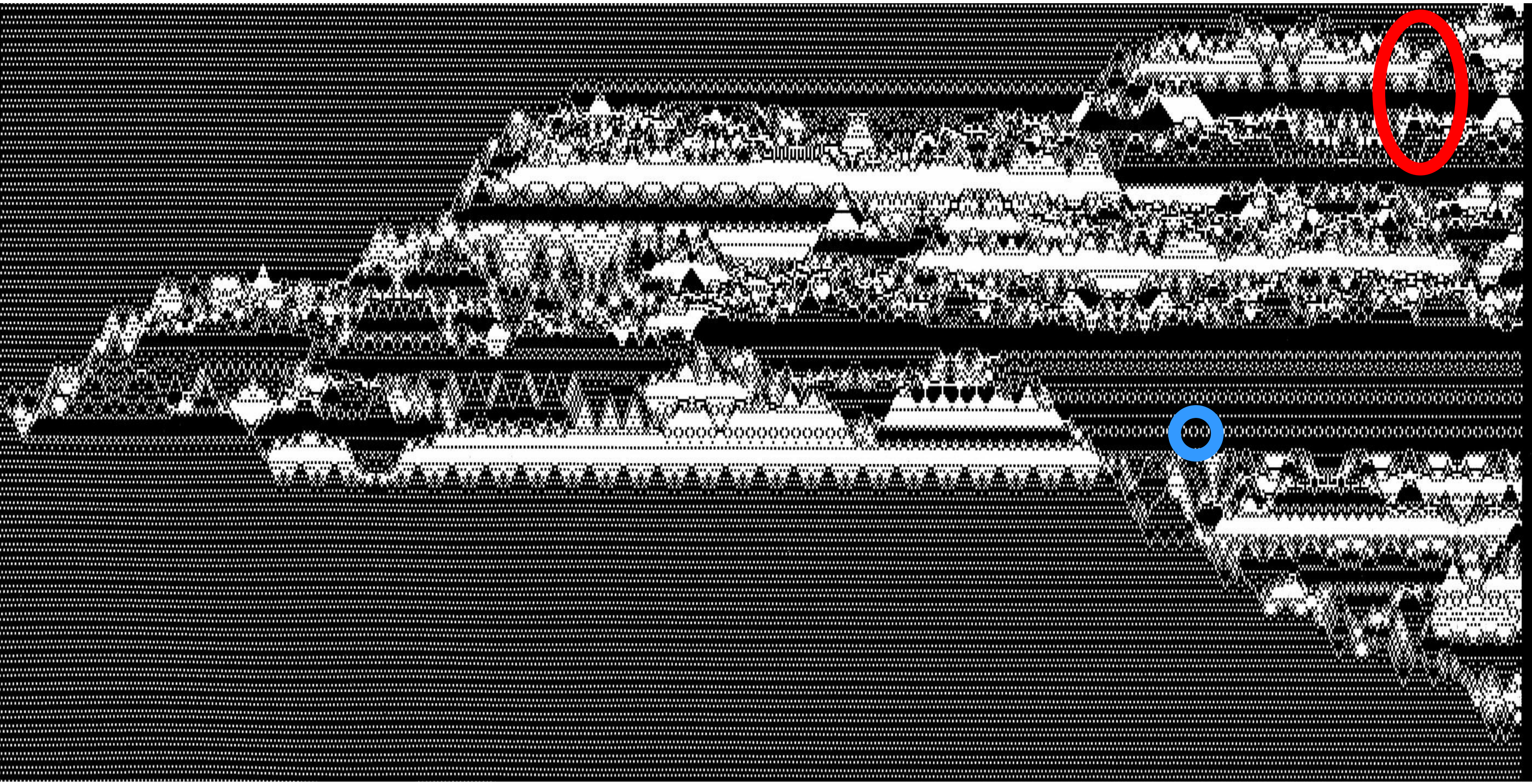
Much later



Self-organization, the spontaneous increase of complexity.
A simple dynamics (the same 1 dimensional reversible cellular automaton) can produce a complicated effect from a simple cause.

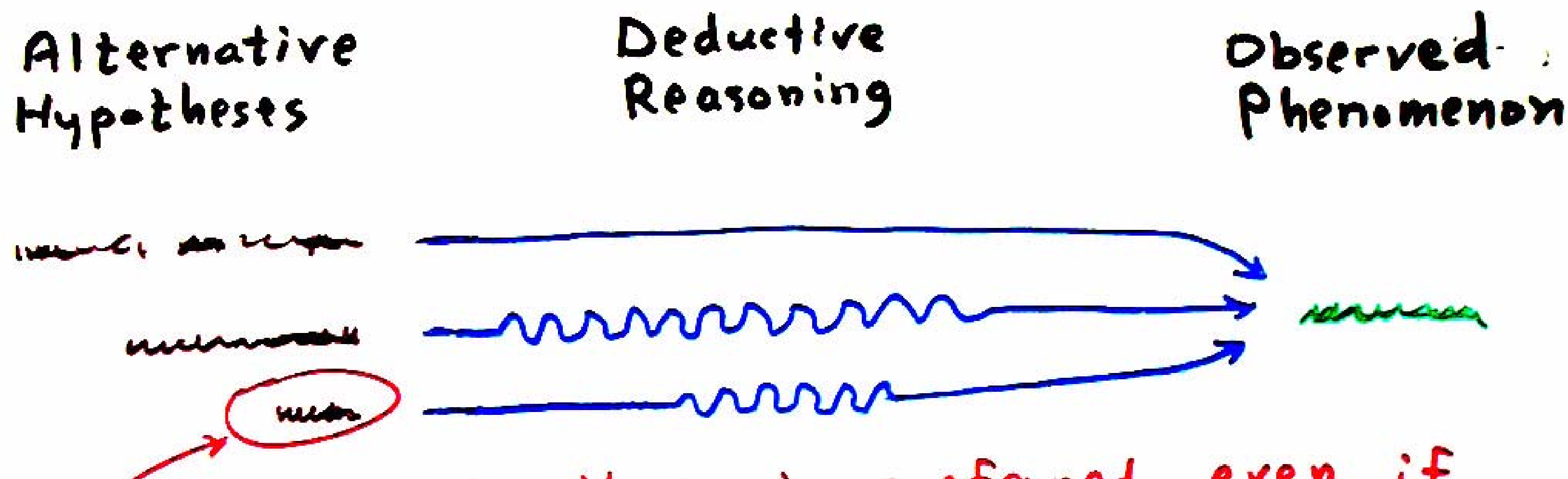


Small irregularity (green) in initial pattern produces a complex deterministic "wake" spreading out behind it.



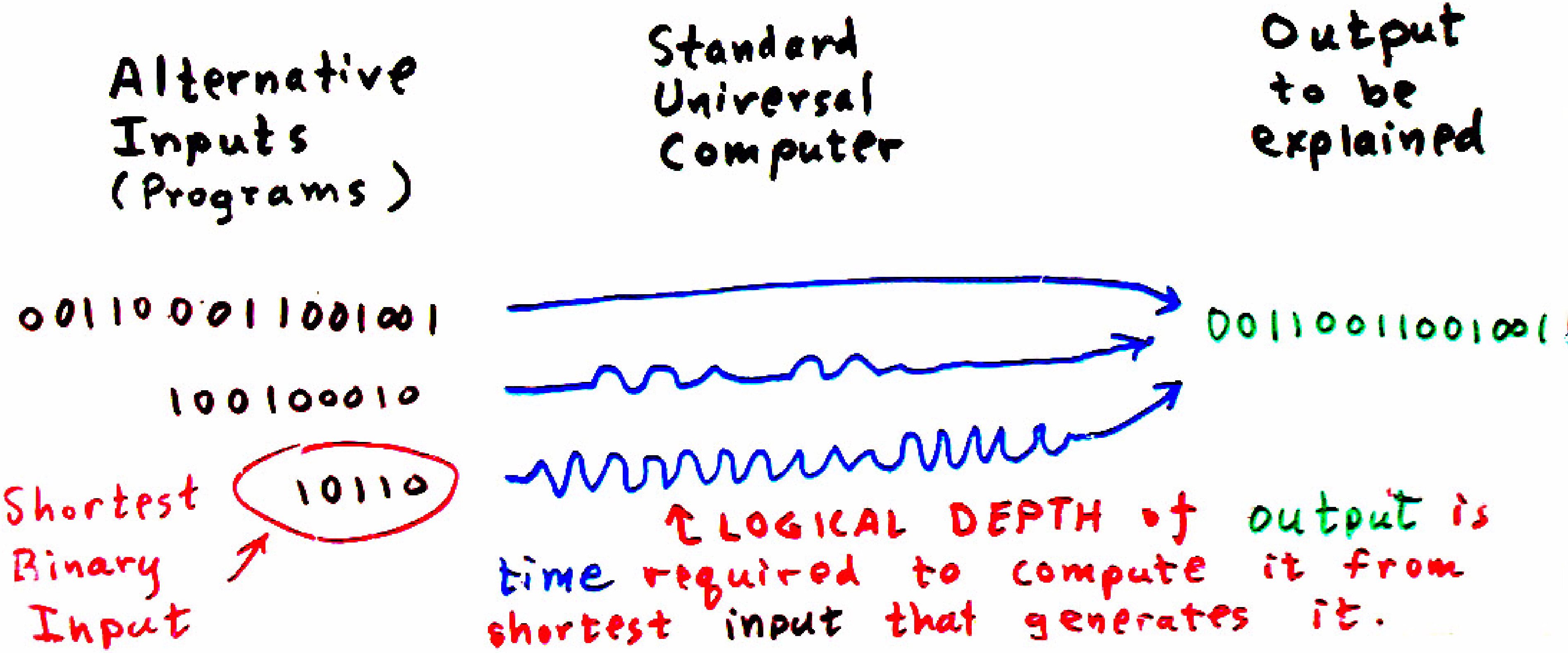
A sufficiently big piece of the wake (red) contains enough evidence to infer the whole structure. A smaller pieces (blue) does not.

In the philosophy of science, the principle of Occam's Razor directs us to choose the most economical hypothesis able to explain a given body of observed phenomena.



Most economical hypothesis is preferred, even if the deductive path connecting it to observation is long.

This idea is formalized using a universal computer (a device versatile enough in principle to follow any deductive path, or derive the consequences of any physical laws):



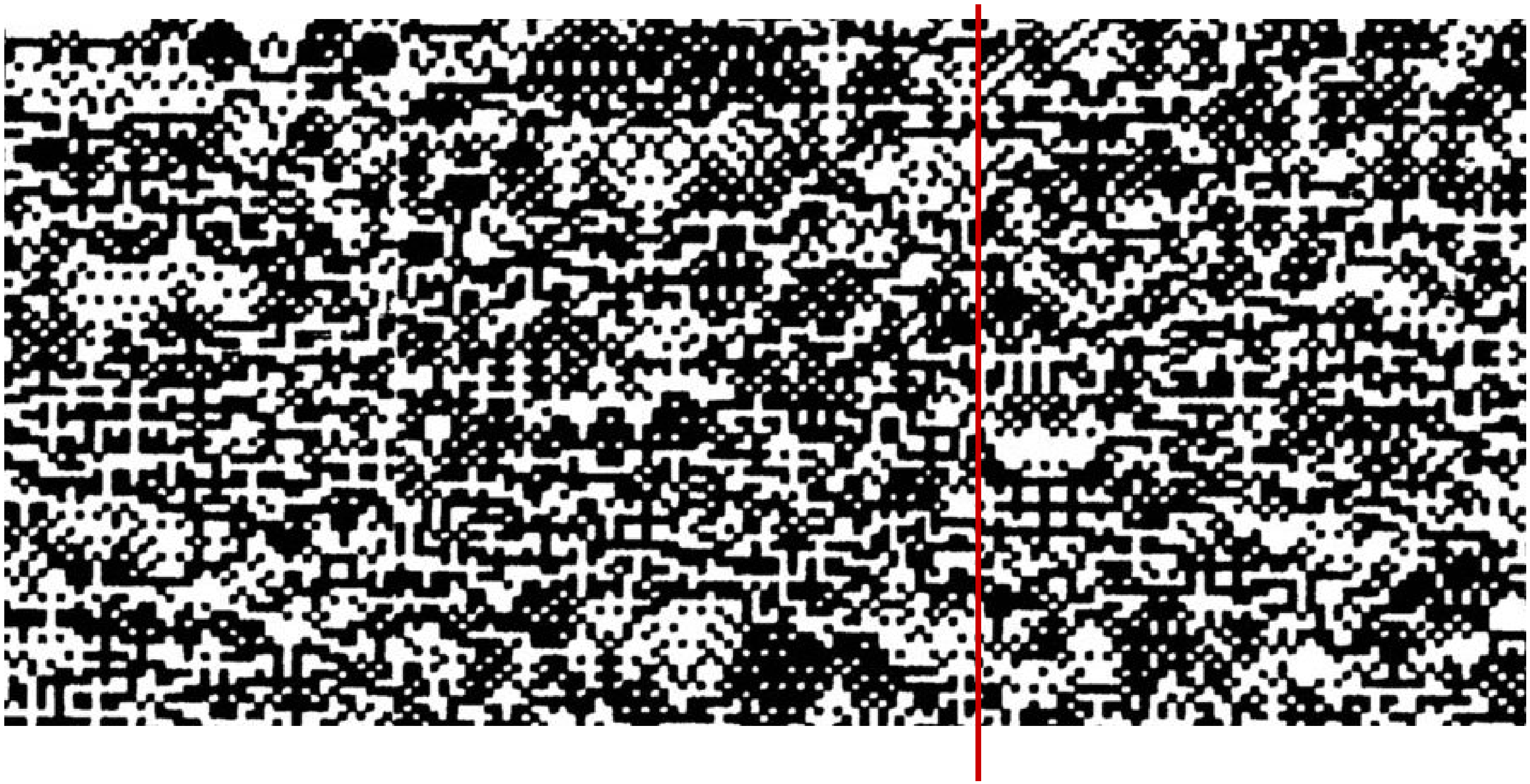
Logically deep objects contain internal evidence of having undergone a long and complicated evolution.

A trivially orderly sequence like 111111… is logically shallow because it can be computed rapidly from a short description.

A typical random sequence, produced by coin tossing, is also logically shallow, because it essentially its own shortest description, and is rapidly computable from that.
Depth thus differs from Kolmogorov complexity or algorithmic information, defined as the length of the shortest description, which is high for random sequences.

If a reversible local dynamics (e.g. the 1d system considered earlier) is allowed to run long enough in a closed system, comparable to the Poincaré recurrence time, the state becomes trivial and random.

Our world is complex because it is out of equilibrium.



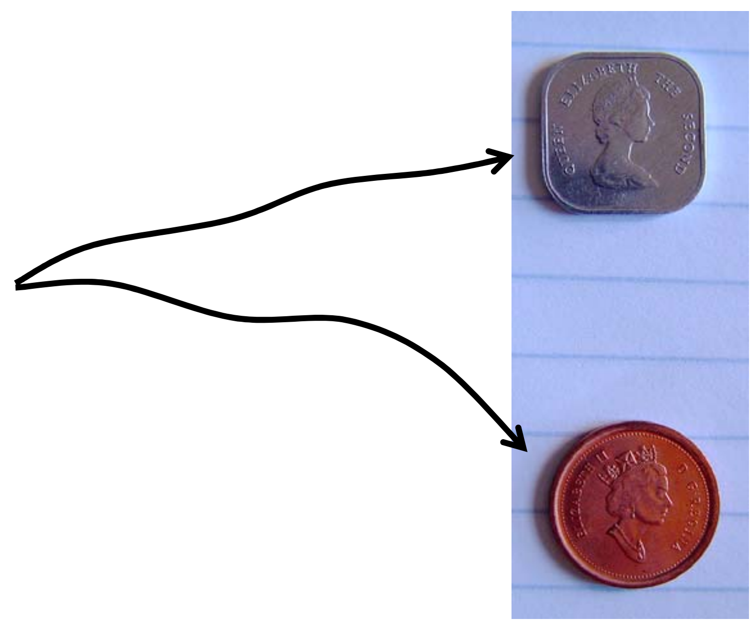
After equilibration, typical time slice is shallow, with only local correlations.

In an equilibrium world with local interactions (e.g. a thermal ensemble under a local Hamiltonian) correlations are generically local, mediated through the present.

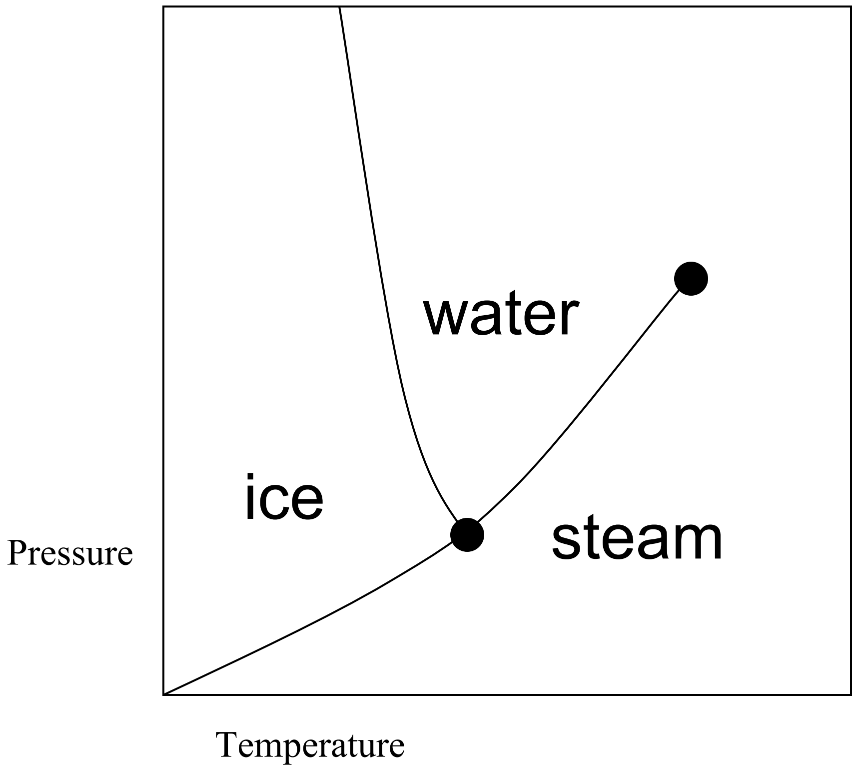
By contrast, in a non-equilibrium world, local dynamics can generically give rise to long range correlations, mediated not through the present but through a V-shaped path in space-time representing a common history.

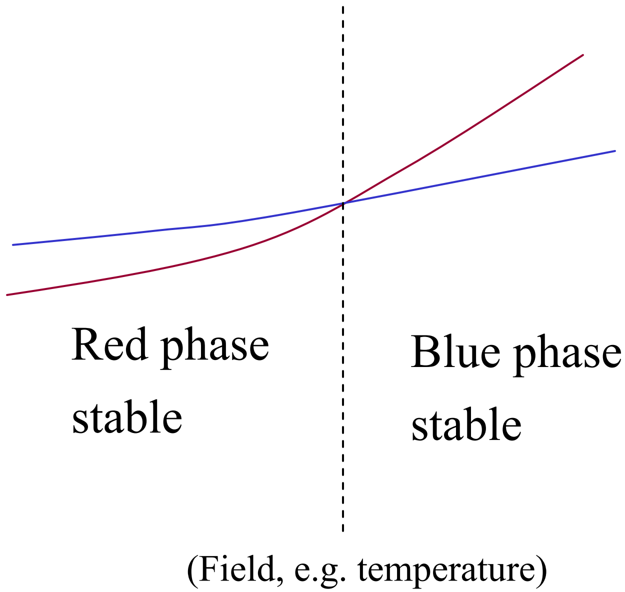
Equilibrium correlations mediated only through present

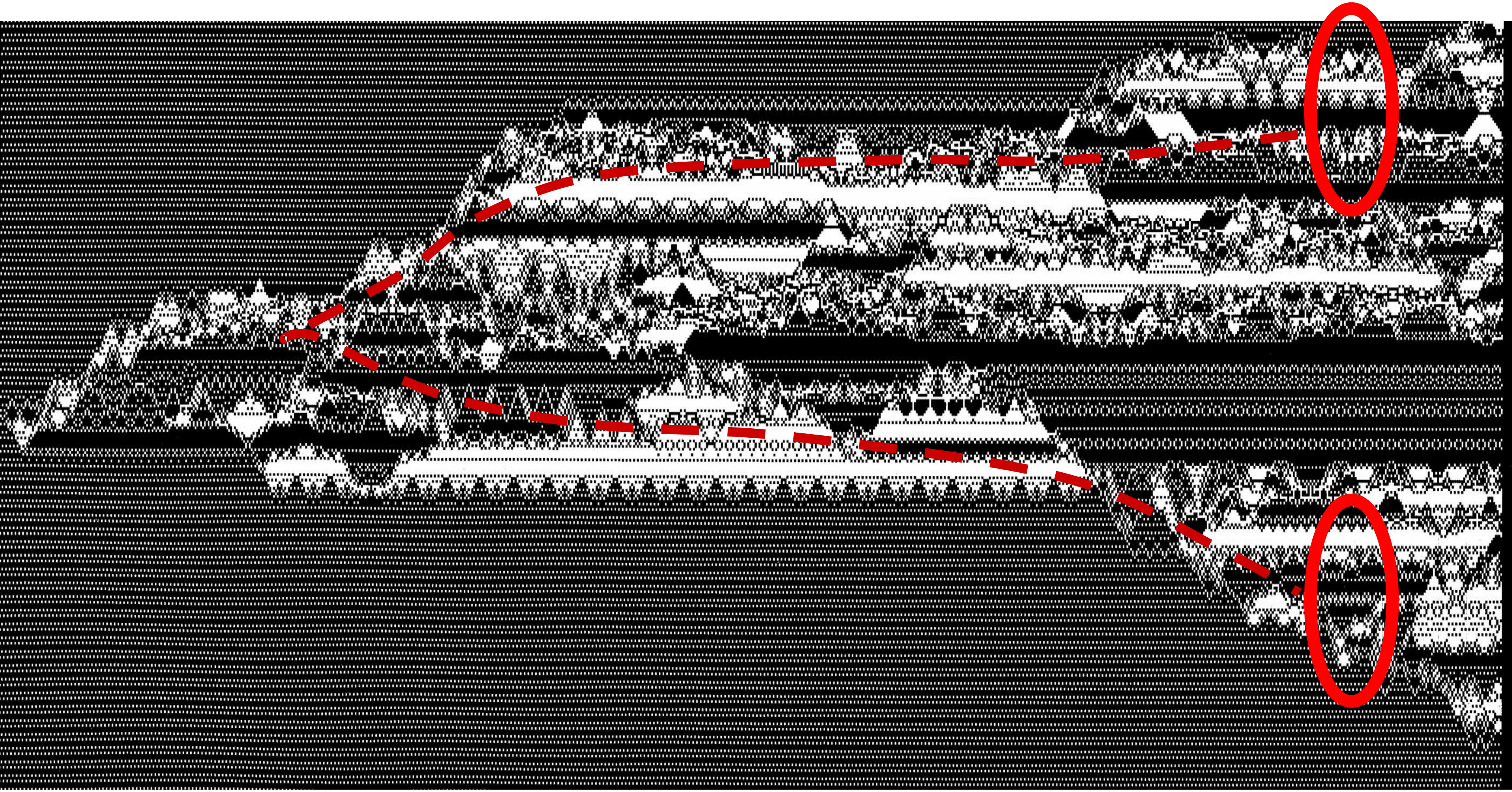
time →



Gibbs Phase Rule (cf also Pirogov-Sinai theory): At equilibrium, for systems with short-range interactions under generic conditions of external fields, there is a unique structurally simple thermodynamic phase of lowest free energy.







V-shaped path in spacetime, mediating the plausible correlation between two noncontiguous regions (red ovals) in the present, need not go back all the way to the beginning. Indeed, if P=PSPACE, it cannot back more than polynomially in the size (height) of the system.

Limits on attainable complexity (logical depth).

• For systems at equilibrium, with generic parameter values, Only one Gibbs phase will be stable, and it will have only short range correlations, being therefore logically shallow.

• For out-of-equilibrium or driven systems, with unbounded space (memory), arbitrarily long computations can be done, generating arbitrarily deep results and structures.

• For driven systems with bounded space n, self-organization can generate structures with depth exponential (or, if, alas, P=PSPACE) polynomial in n.

[We leave aside the important subject of fault-tolerance, whereby noisy processes contrive to simulate the computations of ideal noiseless ones.]