Jumpstarting Quantum Computing as Early as High-School and Middle School: A Guide for Both Teachers and Learners

Authors' names withheld for the purpose of the review associated with the initial anonymous submission.

In this column we answer the following questions: Why study Quantum Computing (QC)? What does QC give us that the standard, classical model of computation doesn't already have? Dogs and CSCI sophomores are beloved groups of highly intelligent beings: what do they have in common as far as QC and Quantum Information Science (QIS) are concerned? And, finally, how can we plan and organize our teaching so the concepts we present hit the learners at their most receptive point in the learning process? Answers to these and more (including some questions) in what follows.

Introduction

At the foundation of our field we have two rewriting systems: Turing machines and lambda calculus. Turing machines are important for many reasons, but especially because of two long-held beliefs regarding computation: the Church-Turing thesis says that everything that is computable can be computed with a Turing machine, although it could take a long time (e.g., exponential time in the size of the input). This correctly suggests that there are problems that cannot be computed—they are called undecidable problems, the most famous of which is the halting problem. Aside from such uncomputable problems, everything else can be computed, and it can be computed using a Turing machine.

The extended Church-Turing thesis is a(nother) foundational principle of computer science that says that the performance of all computers is only polynomially faster than a probabilistic Turing machine. In other words, any model of computation, be it the circuit model or something else, can be simulated by a probabilistic Turing machine with at most polynomial overhead. A probabilistic Turing machine is a Turing machine where the state of the system can be set probabilistically, such as by the flip of a coin. The strong (or extended) Church-Turing thesis says that a probabilistic Turing machine can perform the same computations as any other kind of computer, and it only needs at most polynomially more steps than the other computer.

In 1993, Bernstein and Vazirani showed that quantum computers could violate the extended Church-Turing thesis. Their quantum algorithm offered an exponential speedup over any classical algorithm for a certain computational task called recursive Fourier sampling. Another example of a quantum algorithm demonstrating exponential speedup for a different computational problem was provided in 1994 by Dan Simon. Quantum computation is the only model of computation to date to violate the extended Church-Turing thesis, and therefore only quantum computers are capable of exponential speedups over classical computers. It's equally important to understand that quantum computers would not violate the *regular* Church-Turing thesis. That is, what is impossible to compute will remain impossible. The hope, however, is that quantum computers will efficiently solve problems that are inefficient on classical computers. It is one of the main purposes of this paper to show how one can demonstrate in detail (at pre-algebra level) an elementary version of the Bernstein-Vazirani algorithm and, from that, the rest of all traditional intro topics in QIS (see e.g. [22].)

Key Concepts

In May 2020 on behalf of the Interagency Working Group on Workforce, Industry and Infrastructure, under the NSTC Subcommittee on Quantum Information Science (QIS), the National Science Foundation invited 25 researchers and educators to come together to deliberate on defining a core set of key concepts for future QIS learners that could provide a starting point for further curricular and educator development activities. The deliberative group included university and industry researchers, secondary school and college educators, and representatives from educational and professional organizations. The workshop participants focused on identifying concepts that could, with additional supporting resources, help prepare secondary school students to engage with QIS and provide possible pathways for broader public engagement. The workshop report [1] identified a set of nine key concepts: QIS, Quantum State, Quantum Measurement, Quantum Bit (Qubit), Quantum Entanglement, Quantum Coherence, Quantum Computers, Quantum Communication and Quantum Sensing. Some of these concepts can be refined further, and we do so below.

Misty States

The 12-year-olds of today may well have access to large quantum computers before they leave their teenage years. Yet a standard educational trajectory would see them still several years away from learning enough quantum theory to explore this technology's amazing potential meaningfully. In addition to barriers of convention ("This is the order in which things have always been taught") there are math-related barriers ("You can't understand quantum theory until you have mastered linear algebra in a complex vector space"). But, as has been shown, it is possible to replace linear algebra with some string-rewriting rules [19] which are no more complicated than the basic rules of arithmetic.

Furthermore, learning quantum computing is much easier [5] than learning quantum mechanics because QC deals with a very simple subset of quantum mechanics as follows: (a) a qubit—the foundation of quantum computing—is the simplest non-trivial quantum system; (b) you never have to solve the Schrödinger equation, or even learn what it is, because the quantum systems that carry out quantum computations evolve in a controlled manner based on the quantum gates applied to them; and (c) there's a model of quantum computation, so the most difficult aspect of quantum mechanics—the art of applying it to real systems—is absent.

In 2017 Terry Rudolph proposed a method of teaching quantum mechanics and quantum computing using only the simple rules of arithmetic to students as early as sixth grade. The method is incredibly effective and in a sequence of conference workshops and tutorials it has already been shown [7] how one can use it to introduce superposition, phase, interference and entanglement with virtually no mathematical overhead. The CS2023 project [8, 9, 11] then showed how a complete eight week introductory course (for computer science sophomores) can be built around this approach with these milestones: quantum gates and circuits, phase kickback, the Deutsch-Josza algorithm, Grover quantum search algorithm, superdense coding, Bernstein-Vazirani and the extended Church-Turing thesis, the GHZ game, quantum teleportation and entanglement swapping.

Mathematics

If you have tried teaching yourself (or others) the basics of Quantum Computing (or Quantum Information Science) using one of the many already available wonderful textbooks only to feel overwhelmed by the mathematical apparatus (or just its syntax) know that there is an alternative pedagogical approach that acts as a bridge to the standard quantum computation curriculum but in which the mathematics starts to feel supportive, organic and helpful, instead of oppressive. We should stress that, in our view, there is nothing wrong with mathematics; mathematics by itself is not oppressive. We use mathematics to help describe things going on in the physical world around us. To a physicist, the math is an inextricable part of our understanding. Unfortunately, not everyone is good at math, and most have little, if any, training in physics. A system devised by Terry Rudolph based on a rewriting system (effectively working with what the author calls Misty States, i.e. superpositions of quantum states) can accurately be used to guide our students to the place where we would all like them to be, no less, but going through a stage where they feel that they "really understand" what our mathematics "means" in terms of stuff that goes on in the physical world.

No Compromises

There is undisputed, general consensus that the actual mathematics behind quantum computation is an inevitable and desirable destination for our students. But for those students lacking an adequate mathematical background (HS and younger students) the Terry Rudolph method (i.e., quantum computing with misty states [19]) can be used to reliably communicate a visual and entirely operational understanding of key quantum computing concepts without resorting to complex numbers or matrix multiplication. It's been shown [10] that this approach can create a genuine bridge to the actual mathematics behind quantum computation: and as soon as we exhaust the standard, CS2023 recommended curriculum in QC [11] we can also identify an elementary break-even point when creating a three-qubit W-entangled state. Terry Rudolph's formalism is based on a paper by Shih that Toffoli plus Hadamard gates are universal. When trying to create the W-entangled state we need to accommodate rotations and we must use controlled-Hadamard gates. And this is what allows for a break-even point: a Hadamard gate controlled by the output of another Hadamard gate breaks the ubiquitous symmetry in Terry's original system, and from then on one has to carry around (i.e., specify) the actual probability amplitudes in misty states. This means that students can proceed to developing, in parallel, with (extended) misty states and Dirac notation. And after crossing that bridge we have an entirely conventional Quantum Computation course, but the intuition we acquired while computing with misty states faithfully stays with us.

Disc Dog

This is a game played by both dogs and CSCI sophomores. Anyone watching a dog catching a frisbee in the air can't deny that the dog does know Physics (or at least a bit of it). The dog has acquired that knowledge through sheer, direct and unmitigated (but supervised) practice and interaction. It's debatable if the dog can communicate or otherwise teach that knowledge through any other means. Experience teaches us to understand the classical world. This approach is also accesible to humans (e.g., CSCI sophomores) although we tend to also use heavily language-based methods (e.g., books or the article that you are currently reading). The classical world is accessible to us all, humans and dogs, through direct interaction and experience. Things are completely different, however, when it comes to the quantum physical phenomena that surround us [13]. They ultimately come from the fact that size is absolute, and absolutely small particles just don't behave the way classical objects, that is absolutely big objects, behave. When is a particle absolutely small and subject to the new world of quantum physics? Dirac taught us that there is a minimum disturbance that accompanies a measurement, a disturbance that is inherent in the nature of things and can never be overcome by improved experimental technique. If the disturbance is negligible, then the object is large in an absolute sense, and it can be described by classical physics. However, if the minimum disturbance accompanying a measurement is nonnegligible, then the object is absolutely small, and its properties fall in the realm of quantum mechanics [6]. The quantum properties of absolutely small particles are not strange; they are just unfamiliar and not subject to our classical intuition (we can't interact directly with them as we do with frisbees). They remain entirely accessible to computer science sophomores through classes, videos, interactive sites, and even books with humorous, or clever titles [17].

The net result is that students (as opposed to dogs) need and can be exposed to at least some general knowledge of quantum physics. As others noticed [15] for many years computer science students have been led to believe that they can get by with some knowledge of discrete mathematics and little understanding of physics at all. In the field of quantum computing, and throughout this report, computers that process information according to classical laws of physics are referred to as "classical computers," in order to distinguish them from "quantum computers," which rely upon quantum effects in the processing of information. Computer and communication systems using quantum effects have remarkable properties but a successful transition to them presents challenges. Students and professionals interested in quantum information sciences will need to adopt a different kind of thinking than the one used to construct today's algorithms. It's important to understand that as a discipline, with respect to quantum computing, computer science is of necessity going back to an age when a strong relationship between physics and computer science existed.

Sudent Agency

Student agency is the ability to manage and advance one's learning. Seymour Papert, a prominent figure in education and computer science, famously stated that "the trauma of going to school is that you must stop learning and you must now accept being taught". He believed that education should foster independent exploration and construction of knowledge, rather than passive acceptance of instruction. George Bernard Shaw also famously stated, "What we want to see is the child in pursuit of knowledge, not knowledge in pursuit of the child," emphasizing the importance of fostering a student's intrinsic motivation and curiosity over rote memorization. Through active learning, teachers and students become equal partners in the learning process. Though we agree that a motivated student will always be in pursuit of knowledge, all too often in school we find that knowledge is in fact in heavy pursuit of the student. All students are intrinsically motivated to learn but learn to be unmotivated if they repeatedly fail. Every student has the basic needs to belong, to be competent and to influence what happens to them; motivation to learn only exists when these three conditions are satisfied. This is true at both the elementary level as it is in higher education.

The quotes above guide our classroom strategy which we aim to describe below. In what follows we present an operational approach to jumpstarting quantum computing education to learners as early as middle school and HS.

Quantum Gates

Here are some of the basic building blocks of quantum algorithms:

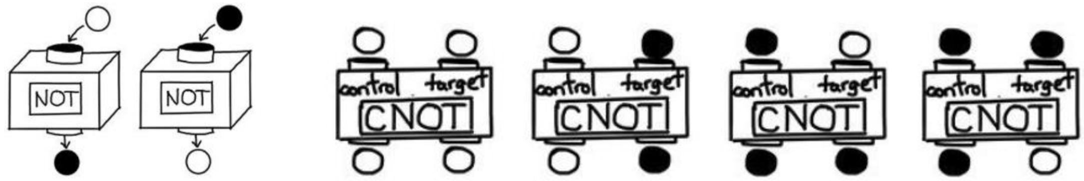
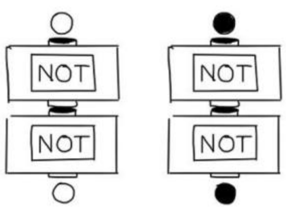
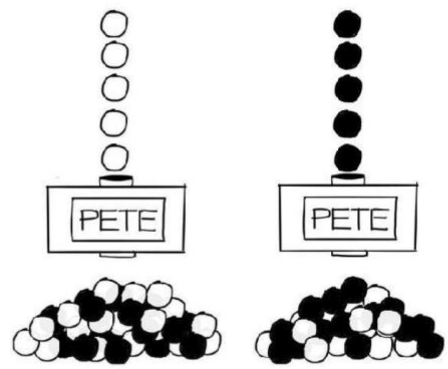


Figure 1. First is a one-qubit quantum gate. The second is a two qubit quantum gate. They both send a classical vibe.

Important property of quantum gates in general that can be very easily proved (for this gate) with what we already know.





We define the Hadamard gate and immediately state that this can be observed experimentally.

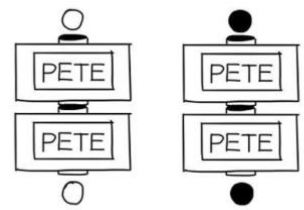


Figure 2. Reversible quantum gates. Introducing the Hadamard (PETE) gate that creates a misty state (superposition).

We state that if we look in between the first two NOT boxes the experiment is undisturbed. But if we were to try to peek in between the Hadamard (PETE) gates we'd completely ruin the property stated in the diagram. Thus, looking at things (sometimes) disturbs them and superpositions are represented by misty states (phase included).

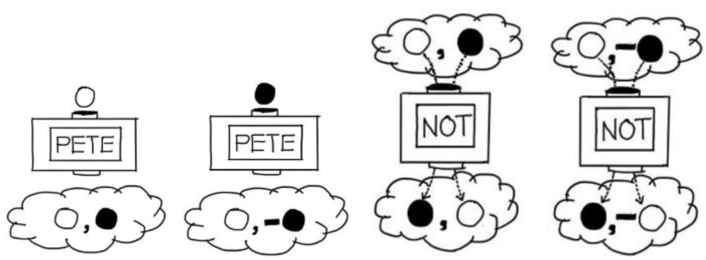


Figure 3. Quantum gates act linearly on (superpositions of) quantum states.

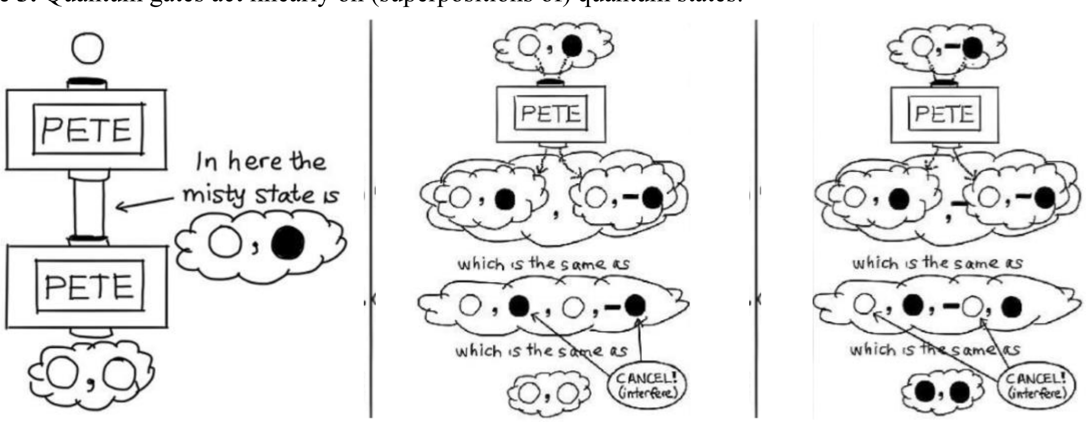
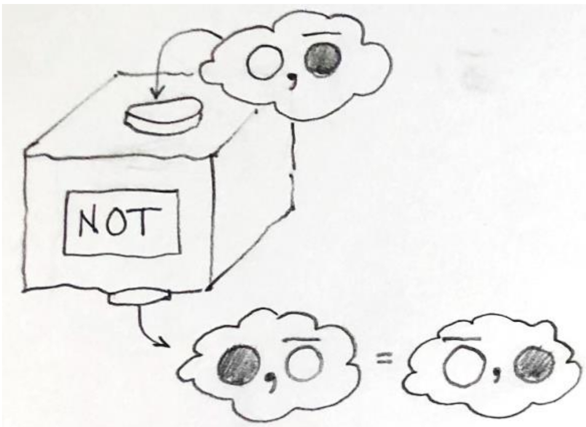


Figure 4. The mistery of stacked Hadamard gates explained.

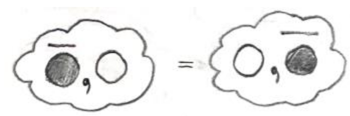
Misty States for Phase Kickback

In this section we prove the phase kickback phenomenon entirely visually using the (pure) misty state formalism introduced in [19]. It is assumed that the reader is familiar with the diagrams in Part 1 (Q-Computing). At the outset we need to establish some facts which we will need later during the actual proof. We start with the effect of a NOT gate on a superposition of states (with phase) as shown on the left (this diagram also appears on the previous page):



This is $\mathrm{NOT}\left(\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)\right) = -\left(\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)\right)$

or, simpler: $\mathrm{NOT}\left(|-\rangle\right) = -|-\rangle$



The order of states in a mist is not important although some (canonical) order is usually preferred.

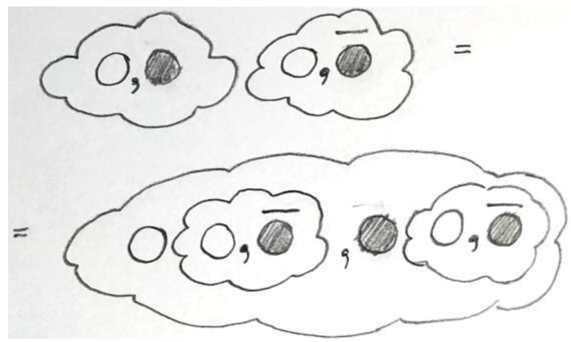


Figure 5. Left: same transformation as in the right half of Figure 3. For the lower diagram on the right: A two qubit state is expressed as a (tensorial) product. The operation is known as FOIL (acronym) in pre-algebra. A mist acts like a sum, while a (tensorial) product as a (non-commutative) multiplication. The phase acts as a sign (plus or minus). A negative phase is shown as an overbar. In the process we may decide to keep the second mist intact.

We remind the reader that the order is not important in a mist and that NOT is a linear gate, that is, when we apply the gate to a mist of states the result is a mist comprised of NOT applied to each of the states in the original mist. The phase acts as a minus and is also linear. For every diagram we draw there is a corresponding, unambiguous mathematical translation using Dirac notation but we won't use (or show) that here because it might distract the reader from our main message (which is the phase kickback phenomenon, followed by the Bernstein-Vazirani derivation).

Figure 6. Phase is a linear operator so a negative phase applied to a mist distributes over its constituent states.

The reader is encouraged now to make sure they understand beyond any doubt the derivation shown in Figure 5 for the transformation $\mathrm{NOT}\left(|-\rangle\right) = -|-\rangle$ as that result will now be used below.

So now let's remind ourselves what it is that we want to prove:

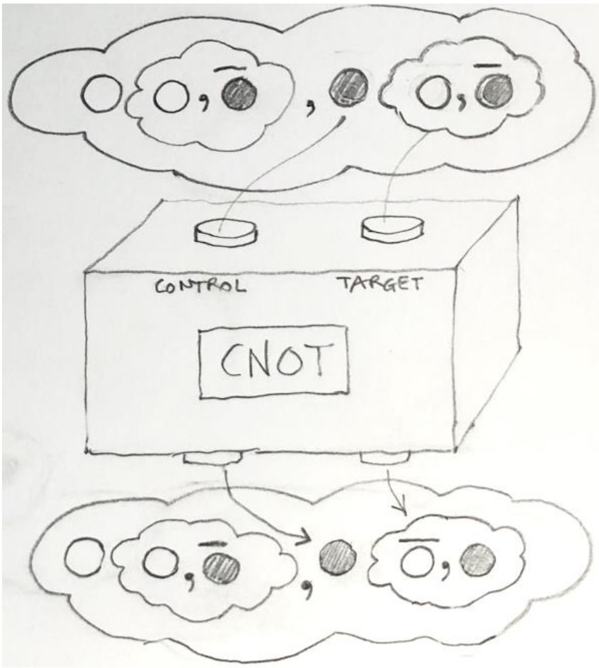
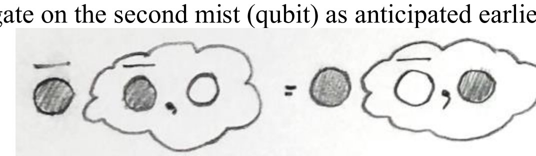
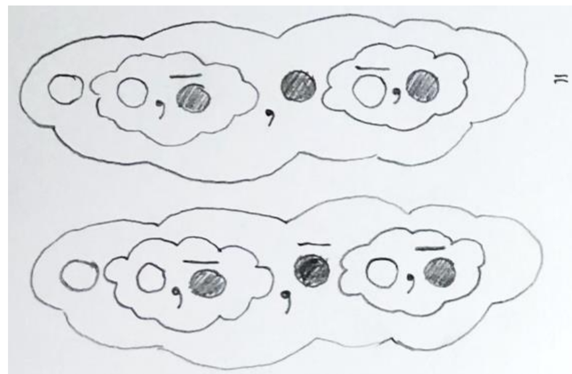


Figure 7. Action of the controlled NOT quantum gate on the state $|+\rangle|-\rangle$ as previously shown (in Figure 5).

Note that if you trace carefully the transformations in the figure on the left you can see the effect of the NOT gate on the second mist (qubit) as anticipated earlier.



Since a (tensorial) product is acting as a multiplication (though without being commutative) the phase operator acts as expected. Here we move the sign from the first qubit to the mist that represents the second qubit. We're ready to process the output from the figure on the left:



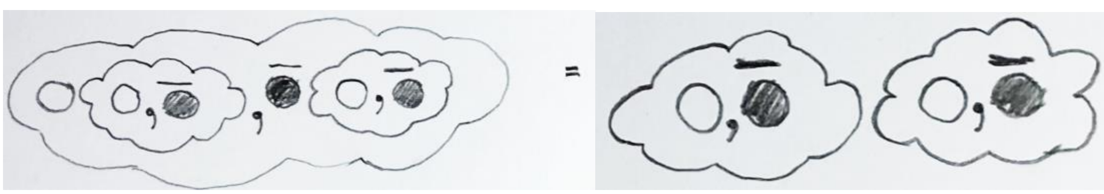
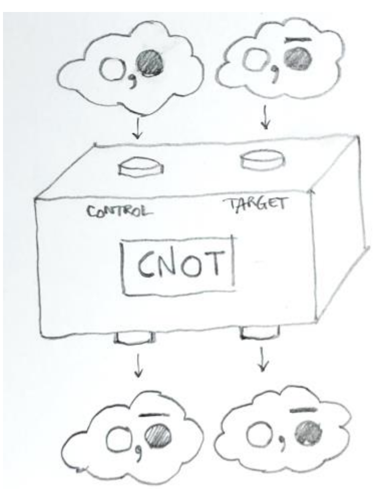
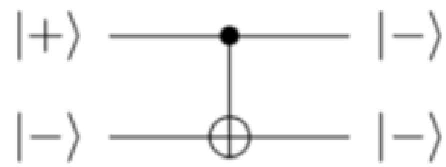


Figure 8. And now we can FOIL back by using the second mist as a common factor.



On the left, the phase kickback with misty states.



Above, the same thing, but in conventional notation [16].

Figure 9. We're done. The summary then of what we did here can be drawn as shown in this picture.

Bernstein-Vazirani

Here's the statement of the problem and its solution:

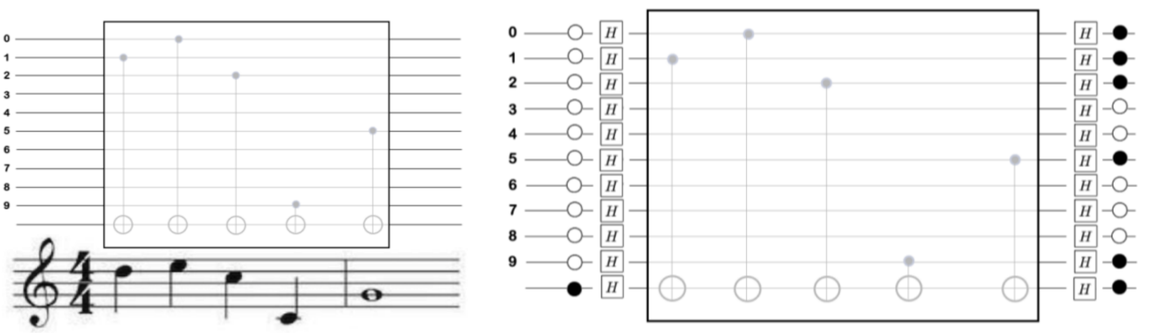
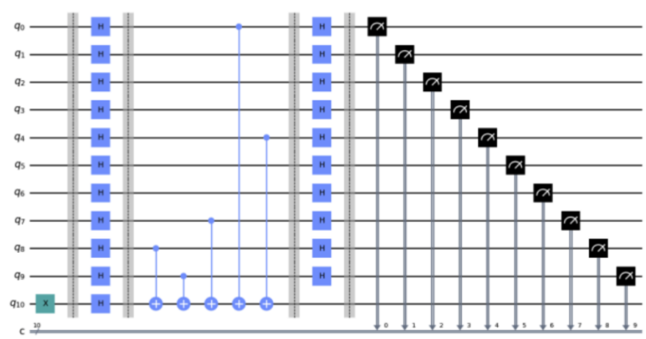


Figure 10. You will recognize that if the gates in the black box were upside down there would be no problem in determining their pattern. As such, though, a classical approach will need to check every single wire one by one (in conjunction with the wire at the bottom) to determine the pattern. Due to our recent discovery (the quantum effect known as phase kickback) we now have a quantum solution that determines the pattern in just one step.

Qiskit and Google Colab

The entire effectiveness of this pedagogical approach relies on the assumption that students (and teachers) are already using in their CSCI HS (or middle school) classes Python in Google Colab notebooks. This assumption is more than satisfied in the case of CSCI sophomores who should have no difficulty writing code to solve probability problems of the following kind: throw three dice. What is the likelihood that their sum is not a prime number?



Note that Qiskit uses a reversed convention for the order of the qubits in the circuit so this circuit is a reflection of the one we drew earlier.

```
[18] simulator = Aer.get_backend('qasm_simulator')
     result = execute(circuit, backend=simulator, shots=1).result()
     print(result.get_counts(circuit))

     {'1110010001': 1}
```

The code on the right produces both the circuit above, its diagram and the result of the measurement (which, in one try, is our pattern).

Code used to produce the circuit on the left.

```
[13] !pip install qiskit qiskit-aer
```

```
[14] import qiskit
```

```
[15] from qiskit import *
```

```
[16] secretnumber = '1110010001'
     indices = [1, 0, 2, 9, 5]
     n = len(secretnumber)
     circuit = QuantumCircuit(n+1,n)
     circuit.x(n)
     circuit.barrier()
     circuit.h(range(n))
     circuit.h(n)
     circuit.barrier()
     for index in indices:
       circuit.cx(n - index - 1, n)
     circuit.barrier()
     circuit.h(range(n))
     circuit.barrier()
     circuit.measure(range(n),range(n))
```

Figure11. From Google Colab access to Qiskit is immediate.

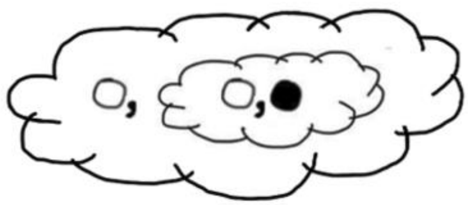
Entanglement

There's no entanglement in the Bernstein-Vazirani algorithm; many entanglement examples are presented in [19, 7].

Eigenvectors of Hadamard Gate

In the original (pure) misty state formalism two or more separate mists can be combined when (and only when) they (a) are at the same level and (b) have the same cardinality. This leads us to states that are irreducible. For example:

$$\{\bigcirc, \{\bigcirc, \bullet\}\} = \cos\frac{\pi}{8}|0\rangle + \sin\frac{\pi}{8}|1\rangle = 0.923879532511...|0\rangle + 0.382683432365...|1\rangle$$



$$\{\bigcirc, \{\bigcirc, \bullet\}\} \approx \{\bigcirc, \bigcirc, \bullet\} = \frac{2}{\sqrt{5}}|0\rangle + \frac{1}{\sqrt{5}}|1\rangle = 0.89|0\rangle + 0.45|1\rangle$$

$$\{\bigcirc, \{\bigcirc, \bullet\}\} \approx \{\{\bigcirc, \bigcirc\}, \{\bigcirc, \bullet\}\} = \{\bigcirc, \bigcirc, \bigcirc, \bullet\} = \frac{3}{\sqrt{10}}|0\rangle + \frac{1}{\sqrt{10}}|1\rangle = 0.95|0\rangle + 0.32|1\rangle$$

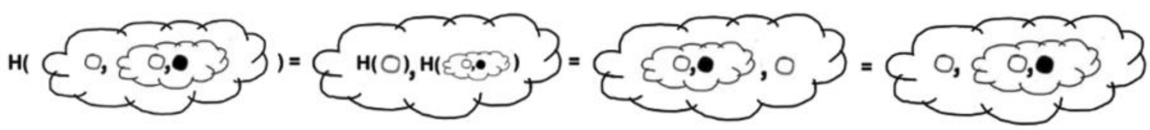


Figure 12. Irreducible misty states; the one presented here is a Hadamard eigenvector.

Figure 12 shows three things: (a) there is a straightforward conversion from a misty state to an algebraic expression; (b) that the original, irreducible state can be approximated reasonably well within the original (pure) misty state formalism and (c) that the state we have chosen (smallest irreducible state with mist inside mist) is in fact one of the eigenvectors of the Hadamard gate. As an aside, but an important one, this eigenvector of the Hadamard gate gives us an alternative path to extending the original (pure) misty state formalism. We just ask the question: “Is there any mist which passes through the Hadamard gate such that the probabilities of observing a white ball or a black ball are unchanged?”. And if we write the equations and solve them we find that the ratio of white to black balls in that state has to be an irrational number. This is an opportunity to tell the story of poor Hippasus as well (math education).

Abstracting and Representing Non-Classicality

Now consider the following well-formed (but irreducible) misty state: {🍎, {🍉, 🍒}}. It says that three types of fruit are possible for a snack: apple, watermelon and cherries. The specific item that we end up with is determined probabilistically (via measurement). We do know it will be one of the three items listed. In this misty state the watermelon and cherries are in equal superposition with each other (let’s call that state s_1) and the apple is in equal superposition with the state s_1. When we estimate the probability of each outcome we find that the probability of receiving an apple is $p(🍎) = \frac{1}{2}$ while for the other two items $p(🍉) = p(🍒) = \frac{1}{4}$ (so each has a probability of 0.25).

Now if we modify the misty state to {🍎, {🍎, 🍒}} we have a minimal non-classical situation. In this new state one would expect the probability of getting an apple to still be 0.75, when in effect it becomes 0.853 (thus lowering the chance of receiving cherries to 0.147). The reason, of course, is that the probability amplitudes add up, but the probabilities do not. The resulting state is (in standard mathematical notation):

$$|\Psi\rangle = \frac{\sqrt{2+\sqrt{2}}}{2}|\text{🍎}\rangle + \frac{\sqrt{2-\sqrt{2}}}{2}|\text{🍒}\rangle = \cos\frac{\pi}{8}|\text{🍎}\rangle + \sin\frac{\pi}{8}|\text{🍒}\rangle$$

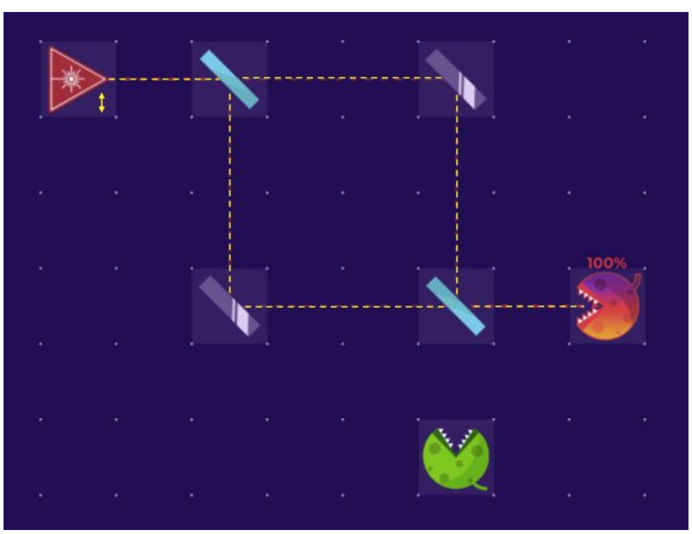
Of course the misty state representation is simpler (though it properly belongs to the extended misty state formalism).

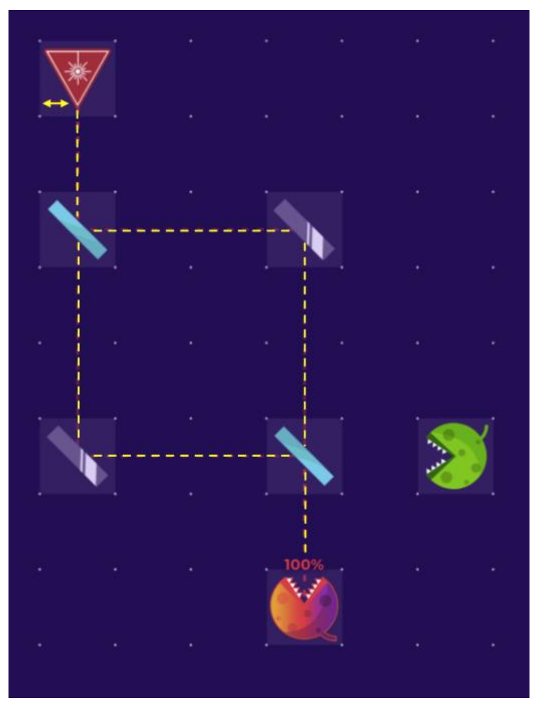
Quantum Flytrap

We have mentioned several times that our brains need to interact with something in order to create a model of it. As Papert puts it: “You can’t think about thinking without thinking about thinking about something.” Lab equipment is expensive, especially for HS and middle schools, especially the kind that would allow students to conceptualize the way quantum objects behave. It’s not enough to tell them that quantum objects are produced as particles, propagate as waves and are detected as particles with the probability distribution of a wave. The Quantum Flytrap [4] is an online system that can facilitate free access to extremely meaningful experiments of the kind we have argued here are needed so the students be able to transition to the kind of thinking required by quantum gates and circuits, QC and QIS.

Figure 13. How we demonstrate in Quantum Flytrap that two Hadamard gates stacked leave the input unchanged.

By convention the horizontal direction is equivalent with a white ball in the misty state formalism. Beam splitters act as Hadamard gates. The vertical direction stands for a black ball.





Quantum entanglement produces instantaneous (superluminal) correlations. While in itself that's not enough to be able to send a message faster than the speed of light (students need to be aware of the no-signaling principle, along with a few other very important learning outcomes CSC2023 puts forth [11]) the phenomenon needs to be observed and interacted with to create a minimal understanding, or intuition. John Preskill [18]: "Perhaps kids who grow up playing quantum games will acquire a visceral understanding of quantum phenomena that our generation lacks." Thus:

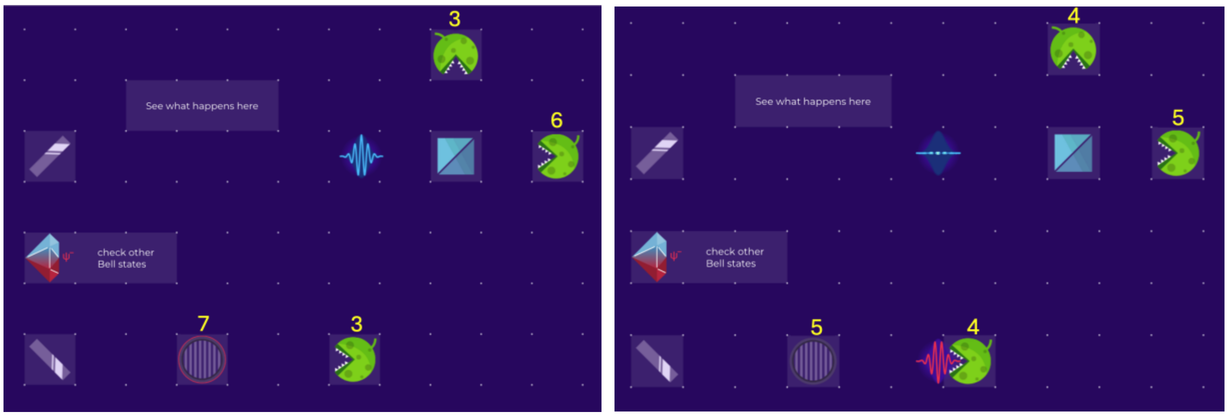
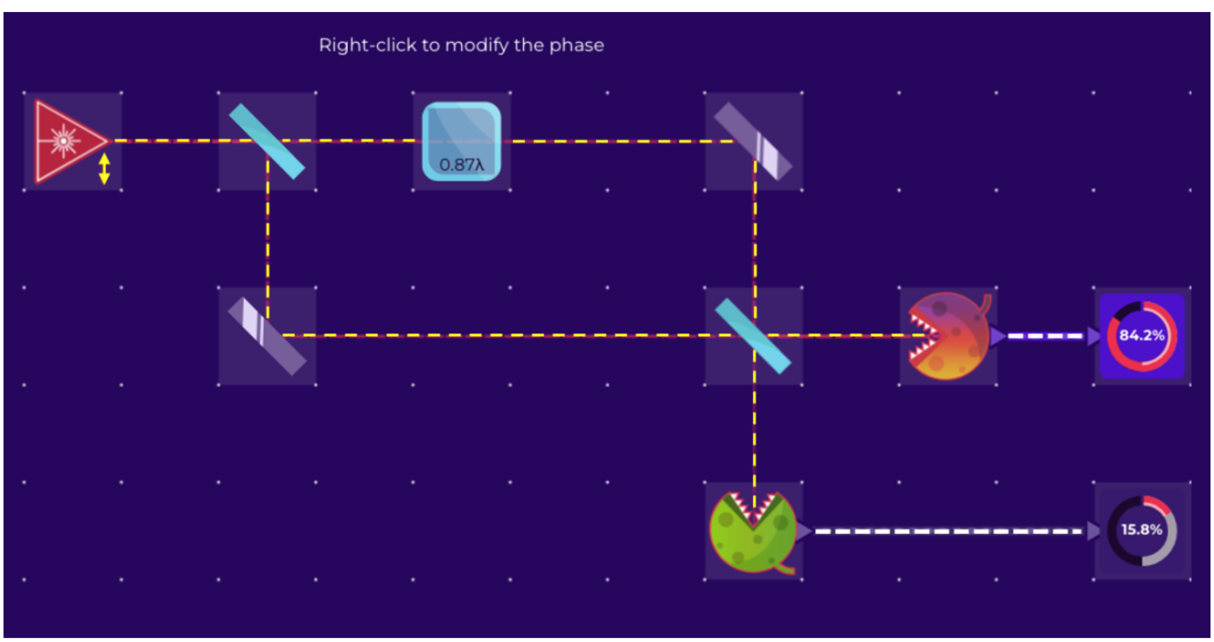


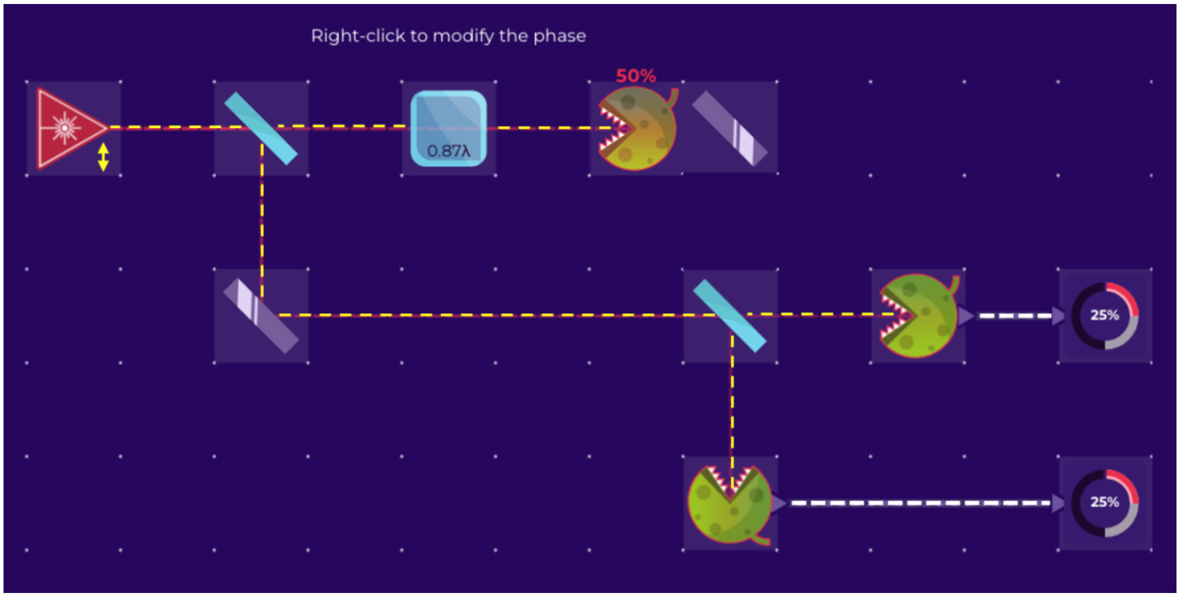
Figure 14. Snapshots of entanglement in action in the Quantum Flytrap. Must see this as an animation to appreciate. In the experiment above first a measurement on the lower branch resulted in the photon being absorbed (as the 7-th such photon on that branch). That instantaneously collapsed the state of the other photon which will now enter the detector on the right (and change its counter from 6 to 7—in this environment no photon is ever lost). The alternative situation is when the photon on the bottom branch does not get absorbed, instead passes through: The state of the other photon immediately collapses to its other state. The photon on that branch will be collected in the detector at the top, whose counter will change from 4 to 5—something that will happen with the detector at the very bottom, as well.

Figure 15. Here now is how we can demonstrate experimentally the quantum snack discussed earlier:



The vertical beam represents cherries, while the horizontal beam stands for apple.

Figure 16. Here is the same setup without interference:



These experiments figure prominently in both Scarani books and some of his papers on one particle interferometry. As an exercise for the reader please prove that the following quantum circuit (this time represented in Wolfram Quantum Framework notation) generates both eigenvectors of the Hadamard gate as follows: start by measuring the first qubit. If it's 0 then one of the eigenvectors is on the other line. If it's 1 then the other eigenvector is on line 2.

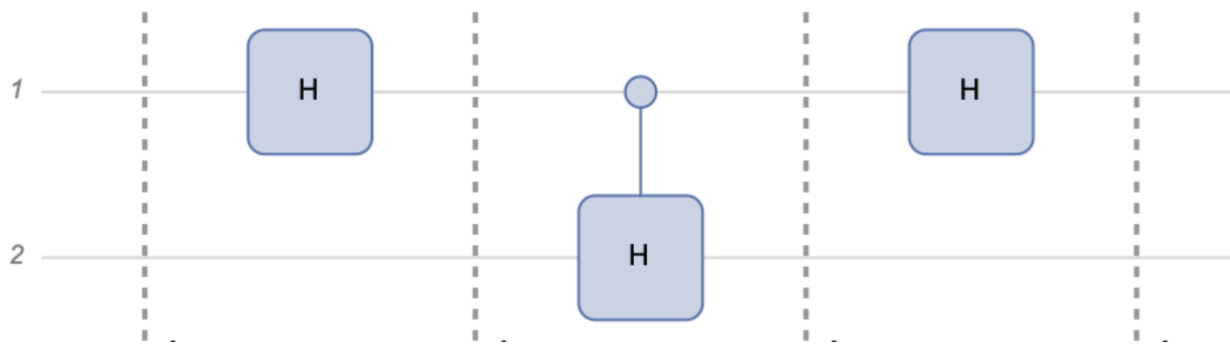


Figure 17. The simplest quantum circuit that produces both eigenvectors of the Hadamard gate.

The proof[1] is immediate using the misty state formalism developed and introduced by Terry Rudolph in [19].

CS2023

Continuing a process that began 57 years ago with the publication of Curriculum 68 (computing in 1968 was perhaps ahead of where quantum computing is today, but not by whole a lot) the three major professional societies in computing (ACM and IEEE Computer Society, now joined by AAAI) have sponsored 6 efforts to establish international curricular

---

[1] https://tinyurl.com/eigenmist

guidelines for undergraduate programs in computing on a roughly 10-year cycle. The last report came out in 2023 and is the first to include recommendations, learning outcomes and topics associated with an optional knowledge unit on quantum computing architectures. The unit was developed as part of the Architecture and Organization (AR) knowledge area because, it was argued, QC is set to exploit the computational aspects of an entirely new hardware platform (qubits) and because the associated (classical) computer architecture and organizational aspects are non-trivial. From 2021-2023 the QED-C Workforce Development Technical Advisory Committee (TAC) worked closely with the ACM and IEEE Boards of Education assisting in the proper, accurate inclusion of QC topics and learning outcomes [11] in the CS2023 Curricular Guidelines. We have referenced some of the materials associated with the CS2023 project above. The Quantum Economic Development Consortium (QED-C) is a broad international group of stakeholders from industry, academia, national labs and professional organizations that aims to enable and grow the quantum industry and its associated supply chain. QED-C was established with support from NIST as part of the federal strategy for advancing QIST as per the National Quantum Initiative Act in 2018.

Conclusion

CS2023 proposes three curricular plans at various levels of detail and accessibiliy: (a) an eight week plan/class that is accessible to middle school, HS and CSCI sophomores (based on Terry Rudolph's misty state formalism as introduced in the book "Q is for Quantum"); (b) a full semester class that's basically promoting the structure and goals of the very well known Berkeley CS191 (also known as the Vazirani) course; and (c) a two semester course that relies on a significant quantum hardware lab component and/or software tools like Qiskit Metal and is meant to remain relevant for the foreseeable future until the next iteration of the curricular guidelines project in ten years. There is extensive bibliography that supports proposals (b) and (c) and rely heavily on the traditional mathematics associated with QC and QIS. In this paper we aimed to introduce the core (and extensions) associated with Terry Rudolph's system as its mathematical requirements are reduced to a minimum (pre-algebra). To support student agency we used the material in [7] which introduces seven cases of quantum advantage in six modules. Each one of these modules can be addressed by a teacher independently and unconditionally as a function of their specific students' interest, in any order, and using concrete quantum circuits modeled in Qiskit and programmed in Python within Google Colab notebooks. It is assumed that the reader of this paper has access to, and is reasonably familiar with Terry's book [19].

References

1. Aaronson, S. et al. Key Concepts for Future Quantum Information Science Learners (NSF Workshop). Workshop output available at: https://qis-learners.research.illinois.edu/ Published May 13, 2020.
2. Bernstein, E. and Vazirani, U. 1993, Quantum Complexity Theory in Proceedings of the Twenty-Fifth Annual ACM Symposium on Theory of Computing (STOC '93), ACM, New York, 11-20.
3. Burd, B. *Quantum Computing Algorithms. Discover how a Little Math Goes a Long Way*. Packt Publishing, September 22, 2023.
4. Ekert, A. (scientific advisor) The Quantum Flytrap: A no-code IDE for Quantum Computing. Freely available online at https://quantumflytrap.com/

5. Ellwanger, D. *A Quantum Computing Pamphlet* Paperback, independently published, Sept 8, 2019.
6. Fayer, M.D. *Absolutely Small: How Quantum Theory Explains Our Everyday World*. Hardcover. Amacom Books. Published on January 1, 2010.
7. German, A., The Little Qubitzer: Introduction to Quantum Advantage https://tinyurl.com/qubitzer
8. German, A. Quantum Computing Education: A Curricular Perspective. Extended Abstract. Available as part of the CS2023 Report (pp. 435-437) online at https://dl.acm.org/doi/book/10.1145/3664191 also available separately at https://legacy.cs.indiana.edu/~dgerman/2024/quantum-computing-education-abstract.pdf
9. German, A. and Pias, M. and Xiang, Q. On the Design and Implementation of a Quantum Architectures Knowledge Unit for a CS Curriculum. In Proceedings of the 54th ACM Technical Symposium of Computer Science Education, V. 1, pp. 1150-1156. Available online at https://doi.org/10.1145/3545945.3569845
10. German, A. The Quantum Abacus: A Break-Even Point. Talk at APS March Meeting 2024, Session K61: Teaching Quantum Information at All Levels I https://meetings.aps.org/Meeting/MAR24/Session/K61.4
11. German, A. Quantum Computing Education: A Curricular Perspective. Available online (under Programmatic Considerations) from https://csed.acm.org/curricular-practices-volume/ also available online from this URL: https://legacy.cs.indiana.edu/~dgerman/2023/curricular-maps/cs2023-quantum.pdf
12. Grumbling, E. and Horowitz, M. (Editors). Quantum Computing. Progress and Prospects. The National Academies of Science, Engineering and Medicine: Quantum Computing, Progress and Prospects available at https://nap.nationalacademies.org/catalog/25196/quantum-computing-progress-and-prospects
13. Hore P. and Mouritsen H. and Buckby C, Quantum Birds project site at https://www.quantumbirds.eu/
14. Kumar A. and The Joint Task Force on Computer Science Curricula, ACM, IEEE-CS, AI. The CS2023 Report, published January 2024, available at https://dl.acm.org/doi/pdf/10.1145/3664191
15. Marinescu D.C. and Marinescu, G.M. *Approaching Quantum Computing*. Pearson, September 2004.
16. Matuschak, A. and Nielsen, M. https://quantum.country/ Quantum Computing for the Very Curious (advertised as a free introduction to Quantum Computing and Quantum Mechanics that is "presented in a new mnemonic medium which makes it almost effortless to remember what you read.")
17. Orzel, C. *How to Teach Quantum Physics to Your Dog.* Scribner, paperback (December 7, 2010)
18. Preskill, J. Online tweet on X. https://x.com/preskill/status/1820926759094931655?lang=en
19. Rudolph, T. *Q is for Quantum*. Paperback. Published by Terence Rudolph, July 19, 2017.
20. Scarani, V. with Yang, S. and Chua, L (HS students). *Six Quantum Pieces. A First Course in Quantum Physics*. Paperback. World Scientific Publishing Company (September 27, 2010).
21. Scarani, V. *Quantum Physics: A First Encounter: Interference, Entanglement, and Reality.* Paperback. Oxford University Press, 1st edition, February 16, 2006.
22. Watrous, J. Understanding Quantum Information and Computation. Playlist of videos by the IBM Technical Director of Education available on the IBM Qiskit YouTube channel via https://tinyurl.com/watrous
23. Wong, T.G. *Introduction to Classical and Quantum Computing*. Rooted Grove, Omaha, Nebraska, 2022. Available at https://www.thomaswong.net/introduction-to-classical-and-quantum-computing-1e4p.pdf