Quantum Computing Education: A Curricular Perspective

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Abstract

In which we discuss the process of creating custom and data-driven curricular maps for the new knowledge unit that was designed for the CS2023 curricular guidelines report.

1 Introduction

Often confused with the true goal of student learning—attainment of the standards—curriculum is not the end in and of itself. Rather, it serves as a means to an end. In our view (e.g., see [2], [13]) a rigorous curriculum is an inclusive set of intentionally aligned components—clear learning outcomes with matching assessments, engaging and relevant learning experiences, and high-effect-size instructional strategies—organized into sequenced units of study that serve as both the detailed road map and the high-quality delivery system for ensuring that students achieve the desired goal: the attainment of the designated grade- or course-specific standards within a particular content area.

Continuing a process that began more than 50 years ago with the publication of *ACM Curriculum 68*, the ACM and partner organizations[1] have sponsored efforts to establish international curricular guidelines for undergraduate programs in computing on a 10-year cycle. Over the last 15 years, significant advances in quantum technologies have led to a new awareness of their impact on computing. There are now more than 60 companies worldwide that build quantum computers and associated software stack and tools. In the US the Quantum Economic Development Consortium (QED-C) was created in 2018 to accelerate the quantum industry by establishing a robust supply chain and infrastructure, including workforce and standards. However, the continued absence of a committed and structured education in quantum mechanics in a large fraction of traditional US undergraduate programs, including computer engineering and computer science, presents many BS degree STEM graduates with the daunting problem of how to get trained quickly and efficiently to pursue the new opportunities in Quantum Information Science and Technology (QIST). To address this issue, we developed the Quantum Architectures (Q-AR) Knowledge Unit (KU) for the upcoming ACM/IEEE-CS/AAAI CS2023 curricular guidelines. In November 2022 we asked the QED-C members, including industry, academia, national labs, and government agencies, to comment on the proposed competency-based curricular plans along with the selected topics and learning outcomes. At the ACM ITiCSE 2023 conference in Finland [23] we presented the analysis of the data collected during that process.

Specifically, we presented then—and we summarize here—three[2] different curricular maps (aimed for distinct, different instructional purposes) representing the core of our SIGCSE 2023 proposal [19] which was itself the result of a task force working closely with a loosely selected (and somewhat fluid) focus group of experts. Each map shows where within the curriculum student learning outcomes[3] are taught and assessed as well as how student competencies build on each other. The goal of our proposal was to provide a sound and operational foundation on which any school or department could establish a mastery-based curriculum satisfying their specific instructional needs. The work started roughly in February 2021 and was completed by the end of July 2022. From September to November of 2022, QED-C members (industry, academia, national labs, and government agencies) were asked to rate and comment

[1] ACM started in 1968, at some point IEEE Computer Society (IEEE-CS) got involved, and CS2023 brought AAAI.

[2] One is entirely without math but leading into math and it lasts eight weeks. The second is a full semester, 14 weeks long, and entirely based on the linear algebra. The last one is two semesters long and includes weekly, messy quantum hardware labs.

[3] The measurable skills, abilities, knowledge or values that students must demonstrate as a result of completing a course.

on the various aspects of our proposal. We used descriptive and multivariate statistics techniques, such as factor analysis and canonical analysis, to explore the data relationships between proposed topics and associated learning outcomes. The results from both statistical analysis and open-ended answers were then graphically summarized.

2 Quantum Information Science and Technology

Quantum theory and Einstein's general theory of relativity are the two fundamental theories of contemporary physics. Between them, they provide the conceptual framework and the mathematical language in which we express other theories in physics, and they provide the basic principles to which all known laws of nature conform [11]. So, as a fundamental theory, Quantum Mechanics (QM) is great but it is the impact of its applications that drive the economic ecosystem and the investments. Three are the main QM application areas: Sensing, Computing and Networking (also known as the new Quantum Internet). Quantum sensing has been with us for over 50 years and has had incredible, truly remarkable success. Meanwhile, quantum computing (as a theoretical construct) has been around for only a little over 30 years but in the last 15 years, significant advances in quantum technologies have brought about a new awareness about quantum computing. Companies rely on vast amounts of data, and sensitive data needs to be protected. The current protection is through encryption. It has been shown that if a quantum computer with a sufficient number of qubits could operate without succumbing to quantum noise and other quantum-decoherence phenomena, then Shor's algorithm[4] could be used to break public-key cryptography schemes, such as RSA, the Finite Field Diffie-Hellman key exchange, and the Elliptic Curve Diffie-Hellman key exchange. Since companies have large volumes of data, they will need a considerable amount of time to convert it to a quantum-safe format. And with the recently stated goals[5] of million-qubit machines by the end of the decade (or even earlier) a new sense of urgency has emerged. It is fair to say then, that at this time and for the foreseeable future quantum computation will be the driving factor[6] (for investments and such) in the quantum technology ecosystem.

The ACM and IEEE–Computer Society publish, every 10 years, a document [35] that contains a comprehensive set of curricular guidelines in computer science. The report links learning outcomes with knowledge areas with a stated goal of empowering educators to educate students to fit the demands and standards of the modern workplace. The most recent such report [1] is dated 2013 and makes no reference to quantum computing or quantum technologies (and perhaps understandably so). The ACM SIGCSE (Special Interest Group in Computer Science Education) also sponsors an annual and international, technical symposium. SIGCSE 2020 first organized a full-day pre-symposium event [18] entitled "Programming Quantum Computers: Tools and Techniques for Computer Science Undergraduate Faculty". The event was in person with more than half of the keynote speakers giving their talks remotely due to

[4] A very recent paper (see [5], [32]) extends Shor's algorithm to multiple dimensions. In the past 30 years, computer scientists have streamlined Shor's algorithm in preparation for the day that quantum technology matures enough to run it. But a new variant, from the New York University computer scientist, Oded Regev, is faster in a fundamentally new sense. It is the first to improve the relationship between the size of the number being factored and the number of quantum operations required to factor it. The number of elementary logical steps in the quantum part of Regev's algorithm is proportional to $n^{1.5}$ when factoring an n-bit number, rather than n^2 as in the original Shor's algorithm. The algorithm as a whole may not run faster, but speeding up the quantum part by reducing the number of required steps could make it easier to put it into practice. Of course, the time it takes to run a quantum algorithm is just one of several considerations. Equally important is the number of qubits required for such a computation. The number of qubits that Shor's algorithm demands to factor an n-bit number is proportional to n, while Regev's algorithm in its original form requires a number of qubits proportional to $n^{1.5}$—a big difference for 2,048-bit numbers. In classical computing, processing speed is usually a more important consideration than memory, because classical bits are extremely robust: You can save a file on your computer and not worry about it randomly changing when you open it again later. Quantum computing researchers are not always so lucky. Their qubits are constantly trying to fall apart, and we are trying to stop them from falling apart. It is like you are trying to write in the sand and the wind is blowing it away. That means the extra qubits required by Regev's algorithm could be a major drawback. But Regev's paper is not the end of the story. Only six weeks after Regev's paper was published, MIT's Vinod Vaikuntanathan and his graduate student Seyoon Ragavan found a way [31] to reduce the algorithm's memory use. Their variant of Regev's algorithm, similar to Shor's original algorithm, requires a number of qubits proportional to n rather than $n^{1.5}$. This shows the work brings us closer to an implementation that would be more efficient in practice. The broader lesson of Regev's new algorithm, beyond the implications for factoring, is that quantum computing researchers should always be open to surprises, in this very dynamic field, even in problems that have been studied for decades.

[5] https://www.hpcwire.com/2022/04/21/psiquantums-path-to-1-million-qubits-by-the-middle-of-the-decade/

[6] https://www.sciencenews.org/article/quantum-computers-break-internet-save

the apparent onset of the pandemic. Among the speakers were Will Oliver (MIT), James Weaver (IBM), Dan Koch (AFRL), Eric Johnston (PsiQuantum), and Mariia Mykhailova (Microsoft Quantum). At the end of that day the (rest of the) conference was canceled[7] along with several other remaining quantum-related presentations. In 2021 SIGCSE was entirely online; next year (2022) the conference was in person, in Rhode Island. A BOF (Birds-of-a-Feather) session [30] was organized by the Architecture and Organization (AR) sub-committee of the CS2023 Curriculum Task Force with the following topic/title: "Should Quantum Processor Design be Considered a Topic in Computer Architecture Education?" Why should Quantum Computing (QC) be considered to be an Architecture and Organization Topic? It was argued that QC is set to exploit the computational aspects of an entirely new hardware platform (qubits) whose associated (classical) computer architecture and organizational aspects are exceptionally non-trivial.

We now describe the process in detail.

3 Our Proposal

In 2006 Scott Aaronson remarked (in the presence of Ray Laflamme, at IQC in Waterloo) that quantum mechanics (QM) resembles[8] an operating system on which the rest of Physics is running its application software (except for general relativity "which has not yet been successfully ported to this particular OS").

Prior to that, it took the insight of an educator and eminent computer scientist (Umesh Vazirani) to realize that a complete and consistent introduction to QM can be given via the language of qubits and quantum gates. Closer to the present, it took the profound intuition of another polymath (Terry Rudolph) to realize that the linear algebra normally at the foundation of such an approach can be replaced with a simple rewriting system accessible to middle school students.

Rewriting systems are at the foundation of Computer Science, they are, in fact, the very fabric of it (e.g., Turing machines and lambda calculus), so these are very fortunate developments[9].

Quantum Information Science and Technology (QIST) is inherently interdisciplinary and spans physics, computer science, mathematics, engineering, chemistry, and materials science. We presented three curricular plans for incorporating QIST topics (via Quantum Computing) into the CS undergraduate curriculum. Such plans had been constructed with a preliminary consultation with QED-C members (industry, academia, national labs, and government agencies) asking for comments, suggestions, and general input on these three curricular plans.

We will start with the list of proposed topics, and then move to the proposed set of learning outcomes.

4 List of Proposed Topics

This is the list of topics we proposed for a one-semester class that could be extended to a two-semester sequence if supported by an adequate number of lab sessions:

1. The Wave-Particle Duality Principle.
2. The Uncertainty Principle in the Double-Slit Experiment.
3. Qubits. Superposition. Measurement. Photons as qubits.
4. Basic probability, trigonometry, simple vector spaces.
5. Supporting formalisms: complex numbers, Euler's formula.
6. Systems of two qubits. Entanglement. Bell states. The No-Signaling theorem.
7. Axioms of QM (superposition principle, measurement axiom, unitary evolution of quantum states).
8. Single qubit gates for the circuit model of quantum computation (e.g., X, Z, H, etc.).
9. Two qubit gates and tensor products. Working with matrices.
10. The No-Cloning Theorem. The Quantum Teleportation protocol.
11. Early quantum algorithms: Deutsch-Josza, Bernstein-Vazirani.

[7] See https://legacy.cs.indiana.edu/~dgerman/quantum-computing-for-undergrads/tcNickolas.html
[8] https://www.scottaaronson.com/democritus/lec9.html
[9] Furthermore, a linear algebra prerequisite is now shared firmly in the CS undergraduate curriculum with Machine Learning, a topic that has known a very deep and sudden revival.

12. Simon's algorithm (as a precursor to Shor's algorithm).
13. Implementing Deutsch-Josza with Mach-Zehnder Interferometers.
14. Quantum Factoring (Shor's Algorithm).
15. Quantum Search (Grover's Algorithm).
16. Physical implementation of qubits (there are currently nine qubit modalities).
17. Classical control of a Quantum Processing Unit (QPU).
18. Error mitigation and control. NISQ and beyond.
19. Post-quantum encryption.
20. Quantum Key Distribution (QKD). The Quantum Internet.
21. Adiabatic Quantum Computation (AQC) and Quantum Annealing (QA).

Useful references here are [27, 24, 37, 7]. Each one of these references will support the list of topics above while pleasantly surprising the reader with additional material. For a hardware lab [36] in conjunction with Qiskit Metal would make a great combination.

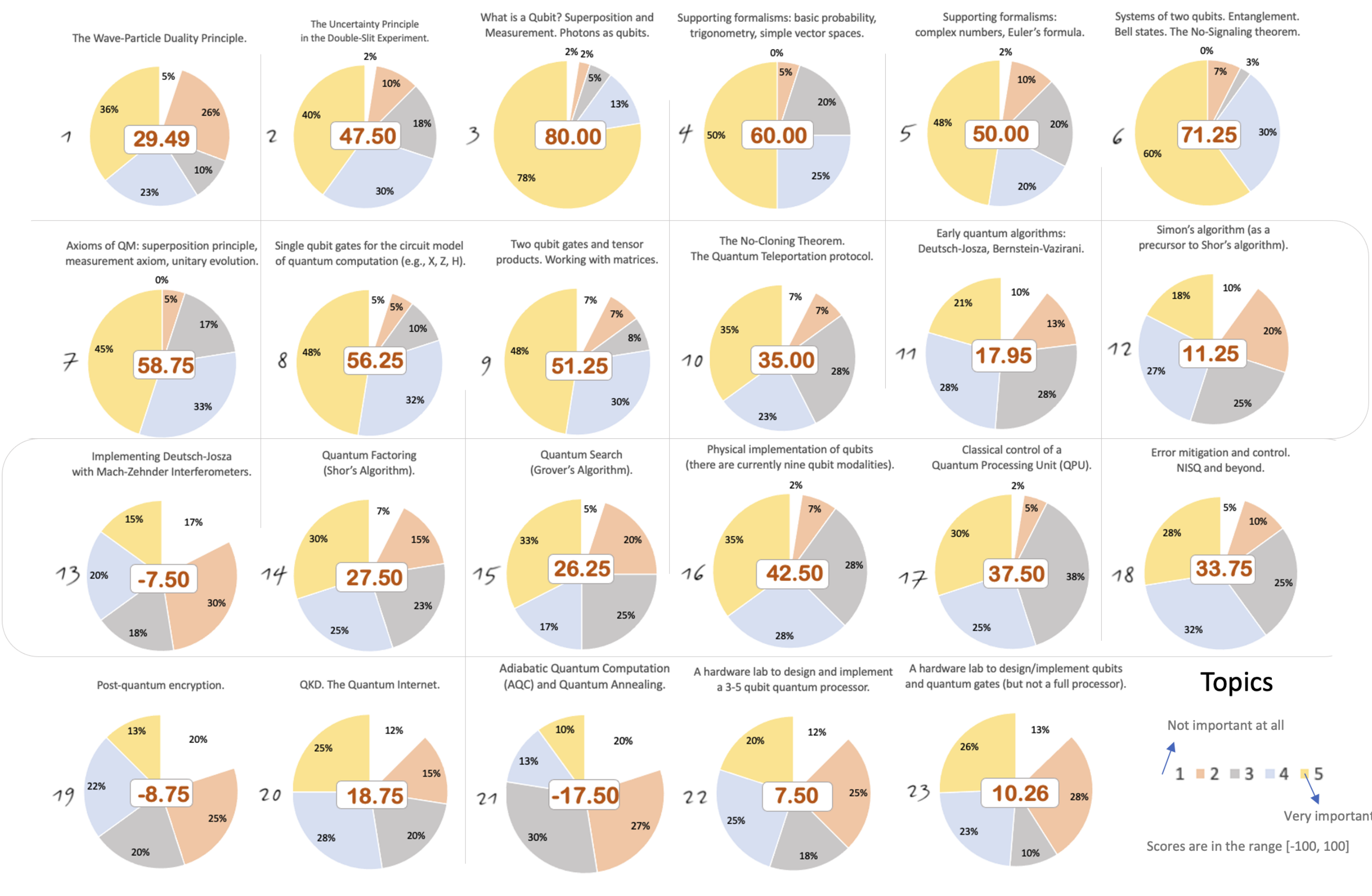


Figure 1: Proposed Topics: Descriptive Statistics

It is probably time to add some essential information about the data and methods used in this study. With respect to the data collected by the QED-C Workforce Development TAC:

- $N = 41$ QED-C member responses were received, with $N = 39$ included in the analytic sample

- 56.4% ($N = 22$) of industry members responding represented hardware companies
- missing data were interpolated with the k-nearest neighbor (KNN) method; after that input, $N = 2$ cases were removed due to having too large of a percent of missing responses (i.e., $> 40\%$)

With respect to the analytical methods used, they were:

- Exploratory Factor Analysis (EFA) using principal factor extraction with promax rotation on 23 topics and 16 desired learning outcomes respectively.
- Canonical Correlation Analysis (CCA) between 5 extracted topic factors and 4 learning outcome factors, based on the factor scores.

In addition[10] to multiple-choice answers collected we also collected open-ended answers. They are presented below. To emphasize the unique perspective answers collected in this modality are providing we want to point out that topic 21 received an overall low (-17.50) rating which would make us think it is not important. It is incredibly informative (and satisfying) to read that one of the open-ended answers did, in fact, completely anticipate this unfortunate outcome. That comment is marked below and reads:

> "AQC and Quantum Annealing are extremely under-represented in academia, compared to the volume of research publications using this alternative approach to quantum computation. I believe the problem is not enough university and graduate-level introductory materials. Faculty are unfamiliar with the subject and do not have anything to teach from."

We will see this phenomenon manifest itself again, in the section on learning outcomes.

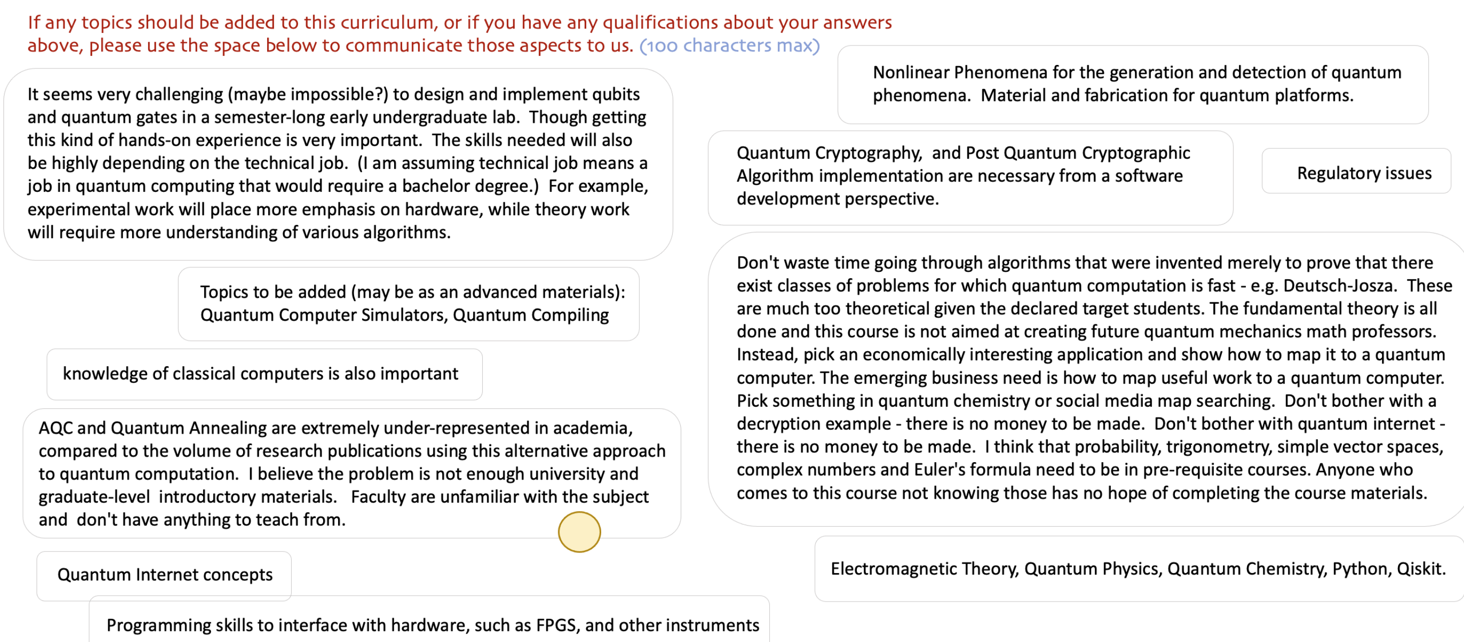


Figure 2: Proposed Topics: Open-Ended Answers Collected

[10] Also added in the multiple-choice section were two questions: (a) should CS undergrads be able to build a 3-5 qubit quantum processor (QPU) in their hardware lab the way they have been building for decades classical devices (counterparts) in the lab? As an example, at IU in the early 1980s we had a hardware sequence (two semesters) in which students would build a PDP-8 from the ground up. Would a 3-5 qubit QPU be too little or too much over the next decade or so? Would the answer depend on the chosen qubit modality? Would it matter that such a small QPU can be effectively simulated in software? and (b) the same question with a less ambitious goal of not implementing a QPU just qubits and quantum gates, maybe even without any kind of error mitigation and/or advanced control.

5 QED-C Survey Responses by Company Nature

In general, it appears that the topics we proposed are closer in spirit to the more academic, theoretical, and enduring aspects of the field. In general the hardware companies downplayed[11] (somewhat) the importance of these topics (though only a little bit, but overall with some consistency) compared with the rest of the QED-C members. Most of these differences were statistically not significant but some of them were and we present those below:

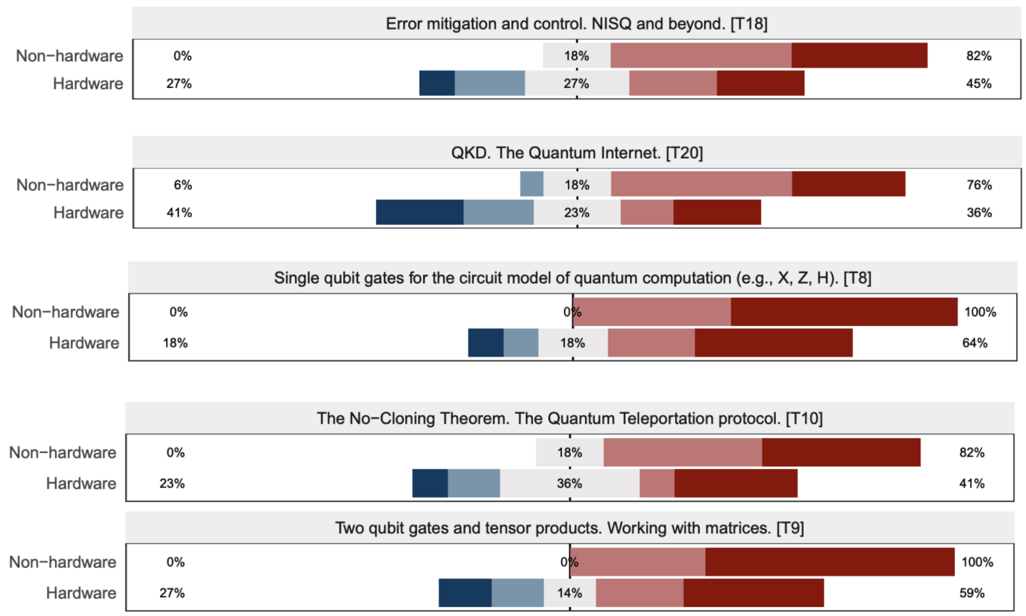


Figure 3: Hardware vs. non-hardware companies position on some topics.

There was only one statistically significant group difference in the learning outcomes and we show it here, in advance:

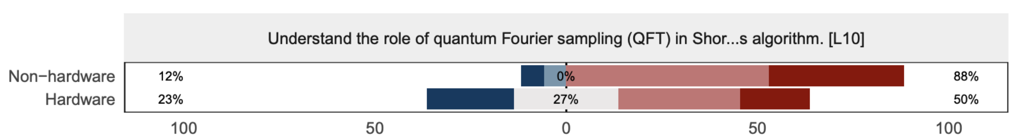


Figure 4: Hardware vs. non-hardware companies position on a proposed learning outcome.

6 Focus Group Quotes

We asked the following (composite) question of any expert we ran into during the initial phase:

- Do you think that CS undergrads should be able to build (now, in five years, or in ten years) a 3-5 qubit quantum computer in their hardware lab the way they have been building decades of classical devices (counterparts) in their (CS undergraduate) labs?

[11] This may indicate that the hardware aspect of this technology is not yet streamlined, on the whole, something we will in fact address in the next section, or that the specific area in which a hardware company operates (lasers, enabling technologies, cryogenics, plus the differences in chosen qubit modalities) is more correlated with the contents of another survey that QED-C ran at the same time, for the National Quantum Technician Consortium (a bit more vocational).

- Would implementing a 3-5 qubit quantum computer in the hardware lab be too little or too much (as a recommendation and curricular goal) over the next decade or so?
- Would the answer depend on the chosen qubit modality? Would it matter that a small quantum computer (3-5 qubits) can be effectively simulated in software?

We started with Michael J. Biercuk, CEO and Founder of Q-CTRL. His company abstracts over all qubit modalities; also because in his TEDx Sydney talk [4] he pointed out that within the new set of emerging quantum technologies, humanity will for the first time have to rely on a fundamentally different kind of approach, that is, fabrication from the bottom up, atom by atom. His answer was straightforward:

> "To answer your query, I think the task you've set out is quite challenging for senior academics, and regrettably borderline impossible for an undergrad lab. The underlying technology is simply not there yet, and most of the supporting infrastructure must be customer-built/assembled. For reference, most senior academic teams work in the 1-5 qubit device regime. I think ambitions should be scaled back to something more realistic, like trapping an ion or measuring the properties of a superconducting device derived from a foundry using COTS[12] tools. Good luck!" (July 28, 2022, via e-mail.)

Prof. Anthony Laing from the Physics Department of the University of Bristol, Co-Director of The Quantum Engineering and Technology (QET) Laboratories at the Centre for NanoScience and Quantum Information was also extremely kind to provide the following rather extensive answer:

> "Assembling a quantum processor, even a small one, is a significant undertaking, normally involving a research group. I could envisage undergraduate students calibrating a quantum device, or programming it. This is certainly already possible for photonics; AQT, an Austrian company, is building a[nd] delivering rack-mounted ion trap QCs, and this could also work in an undergrad lab—they (AQT) have likely developed these ideas. [Implementing a 3-5 qubit quantum computer in the hardware lab] would allow students to perform some basic tomography tests and run a very simple quantum algorithm, like factoring 15. So this size fits the scope of an undergraduate lab, though I expect the size could be increased as the technology develops. [... A]s I said, photonics is the most suitable for this. Rack-mounted ion traps are possible. Dilution fridge superconducting qubits may be more of a challenge. It's not a problem that the circuit could be simulated with a classical computer—for an undergraduate experiment, this verification might be desirable. But I would not advocate running the whole project with a classical emulator; I think there is value and inspiration in engaging with real quantum hardware. [New question: Would it be worthwhile to maybe recommend instead[13] implementing quantum gates with Mach-Zehnder interferometers [12] in the lab (e.g., using bulk optics)? Or focus on classical control? Answer:] On-chip photonics would be a better solution, focusing on calibration, quantum interference experiments, basic gates, and a small algorithm. The main cost currently for photonics is the cryogenic detection and laser pumping, possibly approximately $100K. This could come down significantly in the next 5 years as many detectors on-chip becomes a possibility. [New question: Trapping an ion or measuring properties of a superconducting device from a foundry with COTS tools have been suggested. Would photonics [6] provide a more accessible path? Answer:] Yes it would. But SC and ions are important and should be part of the undergrad experience if possible." (August 7, 2022, via e-mail.)

The last expert quote we include here is a composite answer from IBM's Zlatko Minev[14] and Thomas George McConkey[15]. IBM, of course, is a leading member of the superconducting qubit community. The answer emphasizes and supports some of the opinions expressed earlier and proposes a very concrete alternative based on the new Qiskit Metal:

> "[Question: Do you think that within 5-10 years CSCI undergrads should be able to build a 3-5 qubit quantum computer in their hardware lab the way they have been building classical

[12] Commercial, off-the-shelf tools.

[13] Our main concern was the ratio feasibility/cost so if it's prohibitive now but likely accessible in 10 years we'd still recommend it. And we indicated that we were at the time reaching out to experts in various qubit modalities.

[14] `https://www.zlatko-minev.com/`

[15] Senior Microwave/Quantum Design Engineer at IBMQuantum

devices for decades in their hardware labs? Answer:] Yes if there was a lot of plug and play. [My colleague,] Thomas George McConkey with the team did something like this with grad students and postdocs last summer with Qiskit Metal and CMCL[16]. The fab alone is time-consuming and tricky, so they would have to have access to a fab or be able to send it out for fab. The measurements could be set up for an OK cost in 10 year[s], the hardest part is the dil fridge—if they had a shared one, but this is still very expensive. [...] If they had a lot of plug and play, support, I could see it work. [...] On the other hand, it might be easier for most institutions to focus on skip the fab and dil fridge/hands-on measurements and use the type of Qiskit Metal design analysis and virtual emulation of the QPU, such as with the dynamics of what you programmed—basically, you can do the pulses measurements almost all virtually in software. [New question: Do you think Qiskit Metal would provide a complete quantum hardware experience for a CS undergraduate in the lab of a Quantum Architectures knowledge unit? Answer:] Yes, Qiskit Metal aims to complete the full stack from the quantum hardware layout to the pulses and time dynamics experiments. Hence, it emulates and predicts with high precision what you would actually find in an experiment. At the level of a few qubits, this is very numerically tractable. [...] You could design a three-qubit chip and ask them to design for instance a microwave Mach Zender gate, etc. [...] Indeed, it will be great to give a broad foundation. [When the old Hill and Peterson book that introduced AHPL was brought up along with the new book on the Principles of Superconducting Quantum Computers [36] by Stancil and Byrd:] On the textbook subject, I can share that we are underway writing a textbook with Michel Devoret and Jens Koch—credited with inventing the field [of] superconducting circuits and the transmon qubit, respectively—on macroscopic atoms and microwave photons. We hope to make the textbook an easy pathway into this field, and one that can stand for a long time, while on top of that, we can pair up with some more hands-on current material via Qiskit Metal." (August 17-18, 2022, via e-mail.)

Part of Thomas George McConkey's answers (below) made us realize (or, rather, reminded us of) the following truth: very little actual physics is necessary to teach quantum computation topics, as had already been pointed out by Scott Aaronson almost two decades ago. An appreciation of quantum mechanics is necessary, as a fundamental level of very desirable literacy. However, detailed knowledge of quantum mechanics is not needed for teaching quantum computation topics, much like we never think of any kind of semiconductor physics if we have to write down some code in Java, Python, or C. However, for those students who have to build something (i.e., the quantum engineers) the knowledge of physics becomes a *sine qua non*. Thus, Thomas eventually asks:

"Is the design the primary learning outcome, or the implementation?"

He then elaborates:

"[Question: What is the minimum set of topics (and skills/competencies) that a CS undergraduate should be taught in a Quantum Architectures course? Answer:] Rather dependent on the intended outcome. One doesn't **need** to understand Hamiltonians to design the microwave circuitry for a qubit chip. Just knowing that one needs to hit certain parameters, and what physical stuff changes those parameters. In that scenario, just basic circuits would be useful (R, L, C, etc.). Would say the same minimum is needed to take a microwave engineering course (or maybe control systems). This is thinking from a superconducting/trap/dot implementation. [Question: What is the interface for this knowledge unit (where input refers to the required pre-requisite skills, and output to the desired learning outcomes)? Answer:] Ideal outcome would be having the capability to design a qubit chip to spec, using the relevant tools (be they Metal or other) to do so. Further, having some basic understanding of the earlier (e.g. Hamiltonian/circuit design) and later (e.g. clean room fabrication) so as to understand their requirements/restrictions and how those needs impact the design. [Question: What is the theoretical minimum that captures the essence without overwhelming with details diving into physics and maths? Answer:] Approaching it primarily from an electrical engineering perspective, such as the microwave engineering components. Quantum physics (and related math) can be for the most part enabled via software tools, with only really needing to understand the quantum parameters and what classical electrical circuitry they relate to (e.g. from

[16] https://www.cmc.ca/qscitech-quantumbc-workshop-jul-2021

the dispersive readout χ to the capacitive coupling between the qubit and a resonator). [New question: How can we ensure a 10-15 years horizon shelf life for this knowledge unit (including recommendations for a CS undergraduate quantum hardware lab)? Answer:] Probably would want to cover a few different hardware implementations. It is difficult to clearly state a 15-year window given how rapidly the technology is developing. [Question: Do you think that within 5-10 years CS undergrads should be able to build a 3-5 qubit quantum computer in their hardware lab the way they have been building classical devices for decades in their hardware labs? Answer:] Perhaps with optics? Anything requiring cryogenics would likely be impractical and very costly. The fabrication steps and operation are still quite advanced for most implementations. They could certainly design such a chip, but fabricating and operating the chip in a dilution fridge with all the relevant electronics would be another matter. Perhaps for a very small class in their last year and [if] there is an active research lab with the necessary equipment. [New question: Would implementing a 3-5 qubit quantum computer in the hardware lab be too little or too much (as a recommendation and curricular goal) over the next decade or so? Perhaps the technology is not there yet? Answer:] Would not be too little. For basic functionality, even 2 qubits could be good (so it allows single and two-qubit gates). The feasibility of this is another matter. [Question: Would the answer depend on the chosen qubit modality? Would it matter that a small quantum computer (3-5 qubits) can be effectively simulated in software? Would a chip make a difference? Answer:] Simulation might be the more viable route for the undergrad level, though it depends on the desired experience. **Is the design the primary learning outcome, or the implementation?**" (August 18-19, 2022 also via e-mail.)

Answers like the ones we listed here were exceptionally informative and greatly helped provide an additional degree of focus for our group. We hope the manner of quoting these experts does not in any way introduce any degree of misrepresentation[17] and/or ambiguity in the very valuable information they were kind enough to share with us.

7 The End Game

Interest in incorporating Q-AR topics in the traditional CS curriculum remains high.

In our SIGCSE 2023 paper[19], we aimed to:

(a) determine the minimum set of topics for an effective knowledge unit

(b) determine its interface (set of prerequisite skills vs. learning outcomes)

(c) express/quantify any tension between the various disciplines involved,

(d) identify the theoretical minimum, to avoid overwhelming the students and

(e) ensuring a 10-15 years horizon shelf life for the proposed knowledge unit.

In that paper we have argued that there is more than one way to achieve that goal, and we reviewed many available resources. And while none of the questions we ask[ed] at the beginning of that paper admits a clear-cut answer, we did try to summarize there our position as follows:

1. the CS undergraduate should have a proper appreciation for the quantum mechanical nature of our world. They should know that there is more than one way to implement a qubit, that we are currently in the NISQ era, and there is a gate model as well as an alternative, adiabatic model of quantum computation.
2. there are many entry points in such a program and consequently an equal number of associated prerequisites. The main prerequisite should be a certain intellectual versatility, manifested in a willingness to be exposed to information from more than one domain/discipline.
3. in quantum computation labs will be quintessential, and they will rely on computer-assisted mathematics (e.g., Wolfram Alpha, `numpy`, Qiskit, `matplotlib`, etc.) software emulation (Qiskit Metal) traditional maths (Google Colab and LaTeX) access to actual quantum computers via various cloud

[17] Where there is any doubt we assume responsibility entirely for any potential ambiguity and we remain very grateful to those experts that found time to help us with their kind and thoughtful comments; transdisciplinary individuals are intellectual risk-takers and institutional transgressors (i.e., genuine, legitimate trail-blazers).

platforms (Amazon Braket, IBM Q, Xanadu Borealis, etc.) and occasionally access to a physics lab, fab, or foundry.

4. a genuine interdisciplinary program can only be built if the faculty has wide general support towards such a goal. Cross-campus, inter-departmental communication and cooperation may not be trivial, and faculty need to know that they might have to make a concerted effort to achieve such a beneficial desiderate.
5. incorporating material about all qubit modalities in the curriculum will ensure the material will remain relevant over a reasonably long period. Some qubit modalities (e.g., photons via bulk optics) might allow more accessible experimental setups than others (trapped ions or superconducting qubits). However, the widespread opinion is that students should be exposed to more than one qubit modality, including the design and implementation of qubits (e.g., via Qiskit Metal) and error mitigation and (classical) control.

We need to keep an open mind and prepare our students for their possible futures. This naturally also leads us to the section about the proposed, associated learning outcomes.

8 Illustrative Learning Outcomes

With this list of topics in mind, we said that at the end of the course, we would want students to

Understand that:

- a quantum object (a) is produced as a particle, (b) propagates like a wave, and (c) is detected as a particle with a probability distribution that corresponds to a wave
- at the quantum level nature is inherently probabilistic.
- entanglement can be used to create non-classical correlations, but there is no way to use quantum entanglement to send messages faster than the speed of light.
- nature is inconsistent with any local hidden variable theory.
- quantum gates implement time evolution of a quantum state.

Become aware of the following:

- the power and idiosyncrasies of quantum communication
- the power of quantum parallelism and the role of constructive vs destructive interference in quantum algorithms given the probabilistic nature of measurement(s).

Understand that:

- quantum computation breaks the extended Church-Turing thesis but does not violate the original Church-Turing thesis and what the difference is
- quantum computation already occurs in nature. We are just trying to get better at harnessing it.

Understand:

- the role of quantum Fourier sampling and quantum Fourier transform (QFT) in Shor's algorithm
- the classical components/aspects in Shor's algorithm
- the mechanisms of phase inversion and inversion around the mean in Grover's algorithm.

Be able to:

- enumerate, compare, and contrast the implementation-level specifics of each qubit modality (e.g., trapped ion, superconducting, silicon spin, photonic, quantum dot, neutral atom, topological, color center, electron-on-helium, etc.)
- pinpoint differences between adiabatic quantum computing (AQC, QA) and the gate model of quantum computation and which kind of problems each is better suited to solve.

Understand that:

- a QPU is a heterogeneous multicore architecture, similar to a FPGA or a GPU
- the building blocks of a quantum computer are: a quantum algorithm, a quantum language, a compiler, arithmetic, instruction set, micro-architecture, a quantum to classical conversion and a quantum chip (but see open-ended comments about this learning outcome in the next section).

We can now share what our industrial members thought about these proposed learning outcomes.

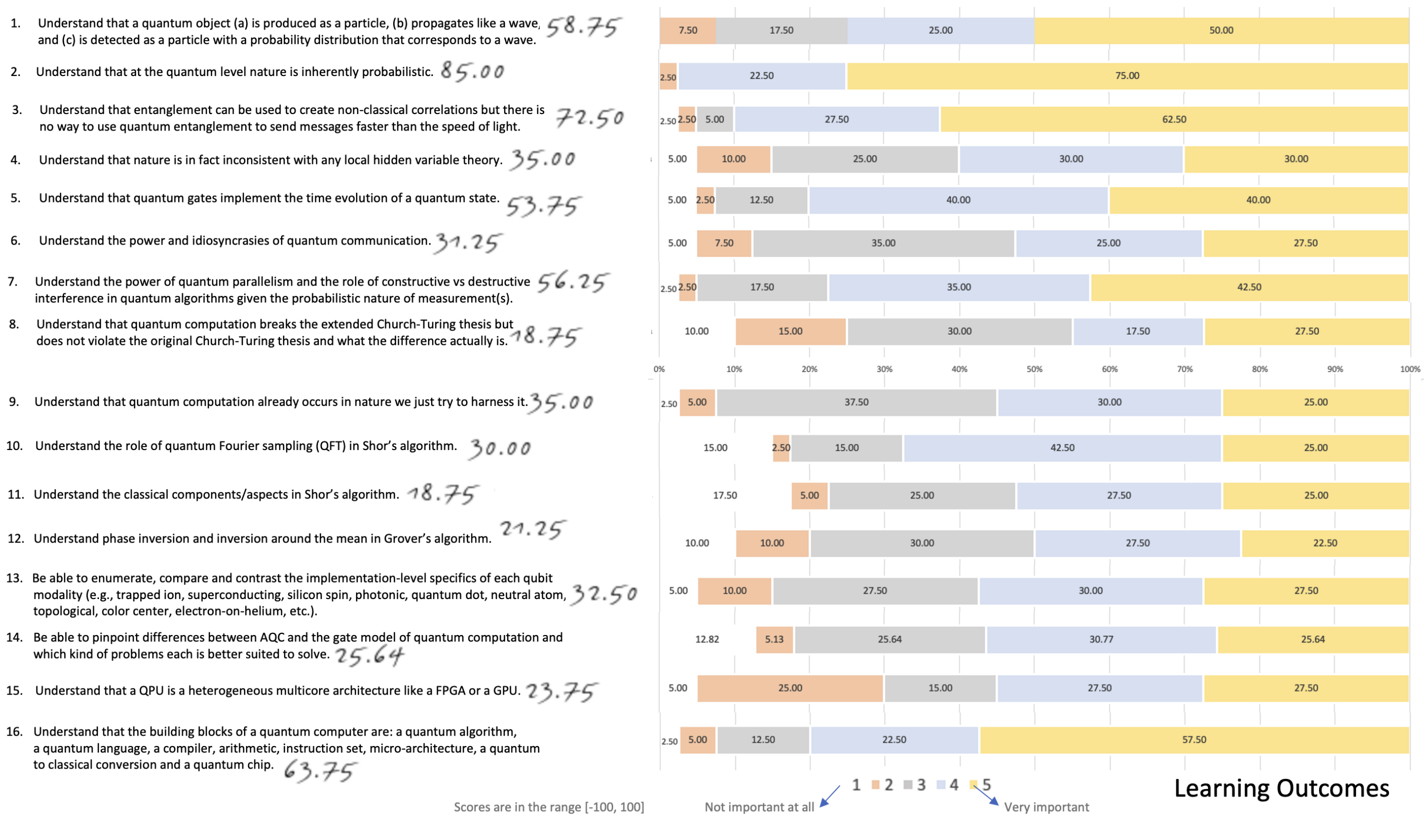


Figure 5: Learning Outcomes: Descriptive Statistics

In one of the previous sections, we emphasized a rather paradoxical situation: one of the proposed topics did not receive a high-enough (cumulative, aggregate) score through the multiple-choice votes collected. Instead, the vote of the community was rather modest on the given scale ($-17.50 \in [-100, 100]$).

What was incredibly satisfying was that one of the open-ended answers collected entirely anticipated and independently explained that effect. We will see something similar here: one of the highest-ranked learning outcomes[18] appears to be a little flawed and/or biased (i.e., it emphasizes too much a certain approach at the expense of other, promising, emerging approaches). Fortunately, three of the open-ended answers collected in this section immediately seem to "catch on" to that aspect (or, rather, omission).

You can see a summary of the open-ended answers collected as listed on the next page in the top half of Figure 6. Answers marked with a blue[19] circle refer to learning outcome 16 and point out the implicit bias:

- Why are you assuming there is a "quantum chip" vs a trap or atom array?
- The last learning outcome is not quite accurate. In Measurement-Based Quantum Computing, there is no single "quantum chip" that contains qubits. Also, it is arguable whether ion traps, cold atoms, diamond centres, etc, are in chips, or just housed in some other kind of packaging.
- That last statement about components of a quantum computer does not fit AQC/Quantum Annealing. [Yet a]n[other] example of how this topic is under-represented

The very last comment, refreshingly, also reminds us of the point that was made in an earlier section.

[18] Item 16 with an aggregate ranking of $63.75 \in [-100, 100]$

[19] Some answers are also color-coded (a little) based on the aspect they seem to address.

If any learning outcomes should be added, or if you would like to qualify any of your answers above, please use the space below to communicate that to us.. (100 characters max)

Platforms and costs for diverse materials. Waste management. Philosophical questions about the meaning of entanglement. Ethical consequences and implications of quantum generation of devices and networks.

Why are you assuming there is a "quantum chip" vs a trap or atom array?

Quantum Internet (Nitrogen Vacancy Centers or other transportation techniques)

An important concept is that qubits are described by amplitudes, and that we interfere amplitudes that are indistinguishable. If they are distinguishable, nothing interesting normally an be done, you simply have classical probability. the fact the amplitudes are complex gives rise to the interesting quantum effects

The last learning outcome is not quite accurate. In Measurement Based Quantum Computing, there is no single "quantum chip" that contains qubits. Also it is arguable whether ion traps, cold atoms, diamond centres, etc, are in chips, or just housed in some other kind of packaging.

Native quantum language fluency.

That last statement about components of a quantum computer does not fit AQC/Quantum Annealing. [Yet a]n[other] example of how this topic is under-represented.

Helping students develop an intuitive feel for Quantum Mechanics' non-intuitive idiosyncrasies is very important. Only then will students understand how to fully leverage Quantum Mechanical principles to solve problems not solvable by classical computers today. Some level of complexity theory is necessary as well. To fully grasp this concept.

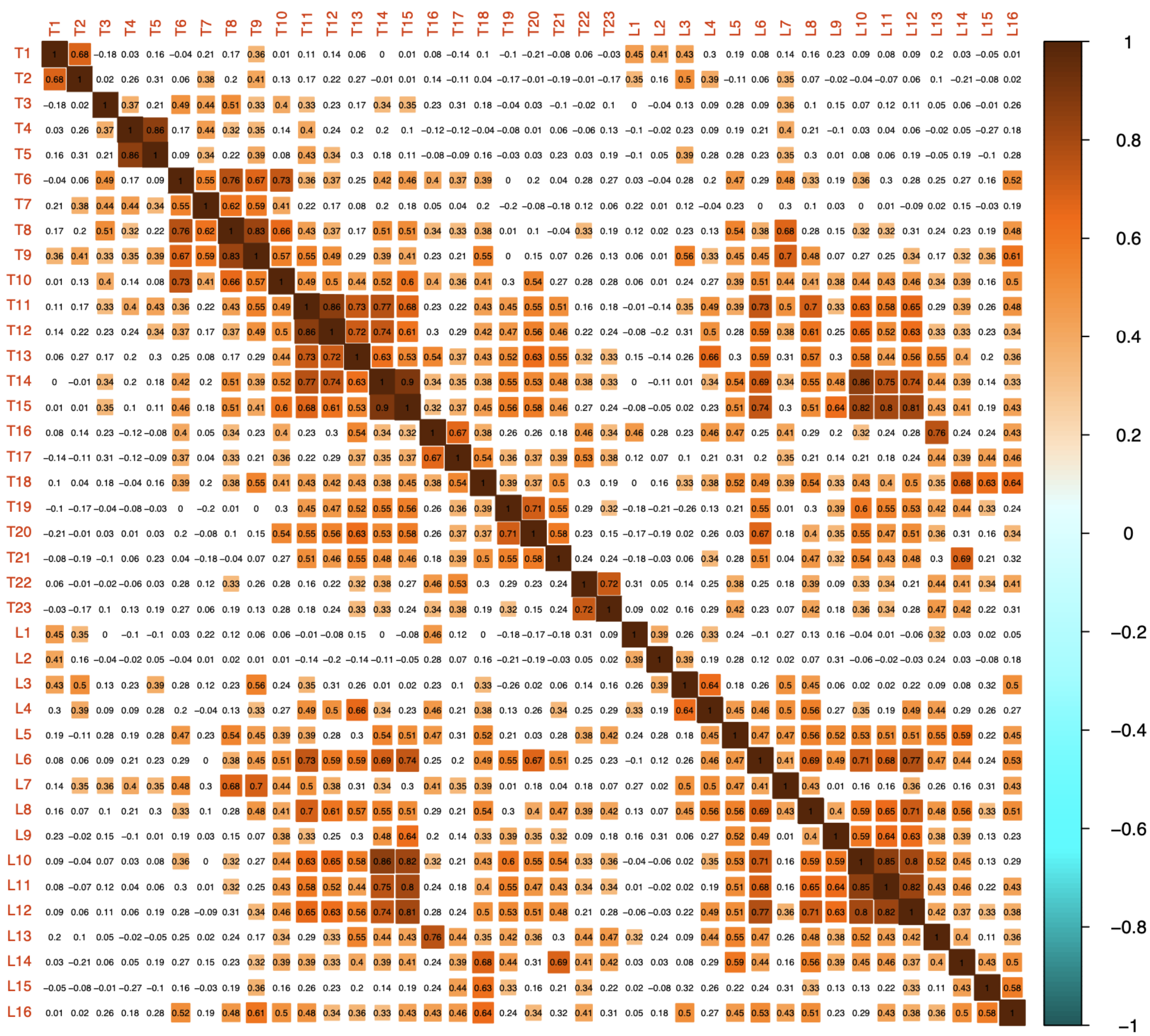
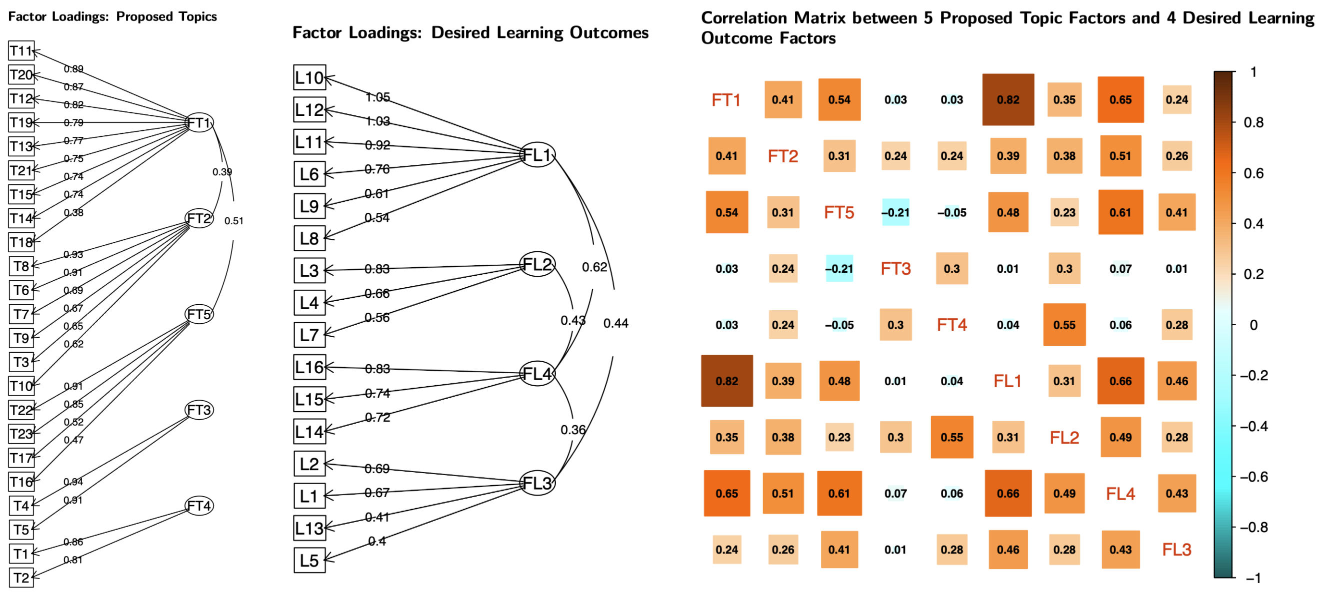


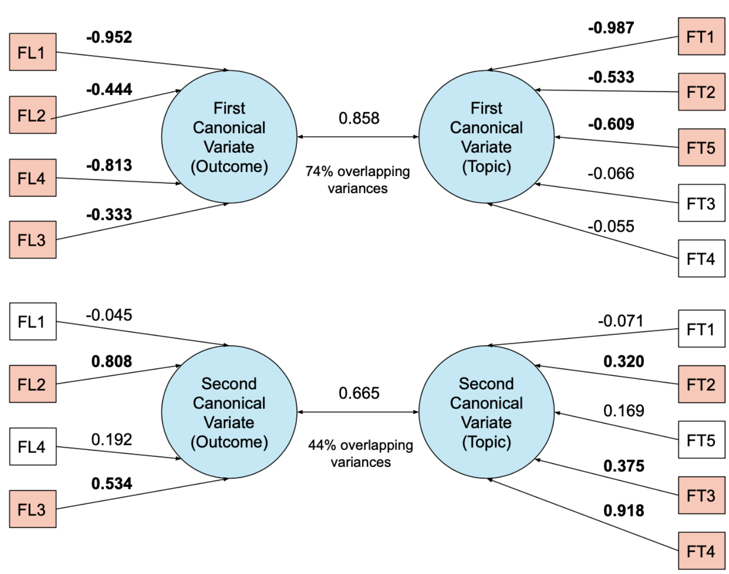
Figure 6: Top: Open-Ended Answers (Learning Outcomes). Bottom: Correlation Matrix.

9 Factor Analysis and Factors Correlation Matrix



Factor analysis is a statistical method used to describe variability among observed, correlated variables in terms of a potentially lower number of unobserved variables called factors. Factor analysis searches for such joint variations in response to unobserved latent variables. Simply put, the factor loading of a variable quantifies the extent to which the variable is related to a given factor.

10 Canonical Analysis



Canonical analysis is a multivariate technique that is concerned with determining the relationships between groups of variables in a data set. It focuses on finding linear combinations that account for the most correlation in two datasets.

11 Custom Curricular Maps for the Knowledge Unit

Consider the following three syllabi, represented as chronological lists of topics to be addressed:

Eight-week class	Semester-long class	Two semesters (sequence)
Qubits	Overview	Superposition
Phase, Interference	Mathematical preliminaries	Quantization
One-qubit gates	Quantum interference	Spin
Entanglement	Qubits	Qubits
Two-qubit gates	Quantum gates	Entanglement
Quantum circuits	Measurements	Quantum Key Distribution
Phase kickback	Quantum entanglement	Quantum gates
Quantum Teleportation	Density matrices	Teleportation
Superdense coding	Quantum channels	Superdense coding
Deutsch-Josza	Stabilisers	Tensor products
Bernstein-Vazirani	Quantum cryptography	Quantum parallelism
Grover's	Bell's theorem	Computational complexity
The GHZ game	Quantum algorithms	Deutsch algorithm
W-entangled states	Approximation	Grover algorithm
Quantum abacus break-even point	Decoherence, basic error correction	Shor algorithm
Intro to traditional QIS maths	Quantum fault tolerance	Precession
	Further topics, selected reading	Electron spin resonance
		Two state dynamics
		Implementing two-qubit gates
		DiVicenzo criteria
		Nuclear magnetic resonance
		Solid state spin qubits
		Quantum dots
		Trapped ions
		Superconducting qubits
		Neutral atoms
		Color (NV) centers
		Adiabatic Quantum Computing
		Optical QC (Photonics)
		Quantum error correction
		Topological quantum computing
		Classical control

11.1 The Quantum Abacus

The first syllabus is based on a method developed by Terry Rudolph [34]. In 2017 he proposed a method of teaching quantum mechanics and quantum computing using only the simple rules of arithmetic to students as early as sixth grade. The method is incredibly effective and in a series of papers ([20], [22], [16], [15], [17], [14], [21]), we showed how we use it to introduce superposition, phase, interference, and entanglement with virtually no mathematical overhead. Furthermore, we showed that a complete eight-week introductory course (for computer science sophomores) has been built around this approach with the following milestones: quantum gates and circuits, phase kickback, the Deutsch-Josza algorithm, Bernstein-Vazirani and the extended Church-Turing thesis, the GHZ game, and quantum teleportation. There is general consensus that the actual mathematics behind quantum computation is an inevitable and desirable destination for our students. But for those students that lack an adequate mathematical background (HS and younger students), one can reliably use Terry's method (i.e., computing with misty states, also referred to as The Quantum Abacus) to communicate a visual and entirely operational understanding of key quantum computing concepts without resorting to complex numbers or matrix multiplication. In [14] a submission to the APS March Meeting 2024, we present concrete evidence that the approach can create a genuine bridge to the actual mathematics behind quantum computation. We start[ed] with superdense coding and Grover's algorithm (to illustrate how effective the system is) then we identify an elementary break-even point when creating a W–entangled state. Terry's Abacus is based on a paper by Shih that Toffoli plus Hadamard gates are universal. When trying to create the W–entangled state we need to accommodate rotations and we must use controlled Hadamard gates. And this is what allows for a break-even point: a Hadamard gate controlled by the output of another Hadamard gate breaks the ubiquitous symmetry in Terry's system, and from then on one has to carry around (i.e., specify) the actual probability amplitudes in misty states. This means that students can proceed to develop, in parallel, with (extended) misty states and Dirac notation. And after crossing that bridge we have an entirely conventional Quantum Computation course, but the intuition we acquired while computing with misty states remains with us.

The custom curricular map example presented later in this section refers to this syllabus.

11.2 Standard Approaches

In this section, we include syllabi modeled by classes taught by Umesh Vazirani[20] at Berkeley, John Watrous[21] at IBM Quantum and Artur Ekert[22] at the University of Oxford. Chapter 10 in [10] presents a self-contained, but very short version of the Vazirani Lectures on Quantum Mechanics and Quantum Computing. This[23] website contains the (almost finished) electronic version of Artur Ekert's book. And the lectures by John Watrous in written form are available here[24].

Other useful references (books) with similar approach/contents: [37], [7], [3], [28], [26], [33].

11.3 Two-Semester Class (with Weekly Hardware Lab)

The references here are [27], [24], [29].

We now turn our attention to creating custom curricular maps. We will describe a process that applies equally well to all classes, whether any one of the three we mentioned or any other kind of class that the reader(s) might put together (depending on what is needed).

A curriculum map[25] shows where within a curriculum student learning outcomes are taught and assessed. It can be used to ensure that alignment exists between the expected learning outcomes and what is taught in a curriculum. A curriculum map can be used as a planning tool when a curriculum is initially developed and it can also be developed for an existing curriculum in an accreditation-like process.

11.4 How to Create Custom Curricular Maps

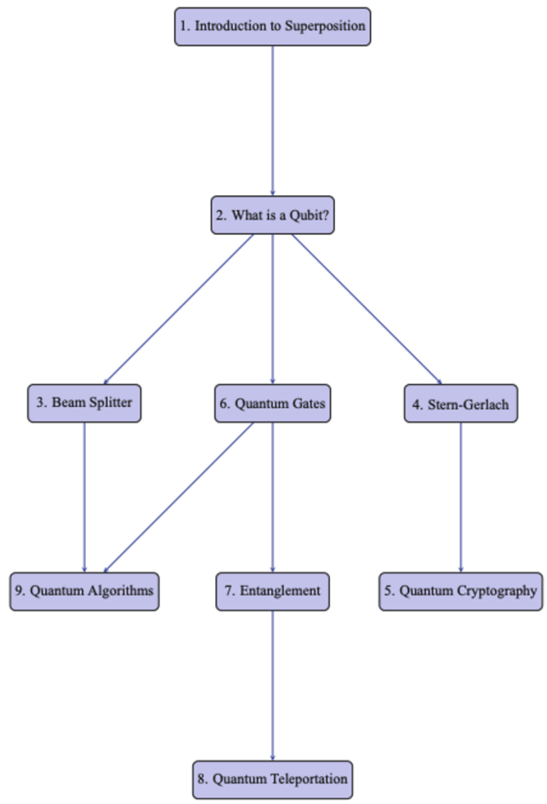
Here's the curricular map for an eight-week introductory class using the quantum abacus[26]:

Topics↓	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	← Outcomes ↓ Related Topics
1	0.45	0.41	0.43														
2	0.35		0.5	0.39			0.35										
3							0.36									0.52	
6					0.47		0.48	0.33		0.36							
7																	2, 3, 4, 5, 6, 8, 9, 10 (see correlation matrix)
8					0.54	0.38	0.68			0.32	0.32					0.48	
9			0.56	0.33	0.45	0.45	0.70	0.48				0.34		0.32	0.36	0.61	
10					0.39	0.51	0.44	0.41	0.38	0.44	0.43	0.46	0.34	0.39		0.50	
11			0.35	0.49	0.39	0.73	0.50	0.70	0.33	0.63	0.58	0.65					

We now describe the process of building (and using) such a map.

[20] https://simons.berkeley.edu/news/online-course-quantum-computing
[21] https://www.youtube.com/watch?v=0Av89fZenSY
[22] https://www.arturekert.org/iqis
[23] https://qubit.guide/using-the-e-book
[24] https://learning.quantum-computing.ibm.com/catalog?content=courses
[25] https://www.unco.edu/center-enhancement-teaching-learning/pdf/assessment/program-curriculum-mapping-quick-guide.pdf
[26] https://legacy.cs.indiana.edu/~dgerman/2023/fie-2023/05-22/the-paper.pdf

A curricular map is essentially a matrix. We start by identifying the topics that we want to teach. As an example we have a flow diagram describing the purpose and contents (with dependencies) in [25]. This little beautiful book has been tested successfully on HS students and has somewhat similar goals with our eight week class based on the quantum abacus. To build the associated curricular map (via the correlation matrix shown earlier) we list as columns all the learning outcomes and topics as rows. Then we write down all the significant correlations between selected topics and corresponding learning outcomes. In the example shown on the previous page we immediately see that all learning outcomes are backed up by strong correlations, except perhaps learning outcome #13 (experimental). We also see that topic #7 does not seem to strongly correlate with any learning outcome, a situation that seems a bit puzzling. Careful consideration further reveals that topic #7 is foundational and strongly correlated with eight other topics (all part of the syllabus, and clearly indicated in the table) so it's worth remembering that sometimes correlations are indirect, in two steps.



Here's the curricular map of the exact same class but using topics (FT1, FT2 and FT4) and learning outcomes (FL1, FL2, FL3, FL4) from the factor analysis. It's a meaningful exercise to name the factors that result from the factor analysis (the relation between a factor and a topic is the same between the title of a chapter in a book and the title of a section in that chapter). So in this table we have topics and outcomes identified by the factor analysis. The table is sound/complete even though we did not print explicitly the correlations for all the repeating lines in the table.

		FL1	FL2	FL3	FL4
1	FT4	0.04	0.55	0.06	0.28
2	FT4				
3	FT2	0.39	0.38	0.51	0.26
6	FT2				
7	FT2				
8	FT2				
9	FT2				
10	FT2				
11	FT1	0.82	0.35	0.65	0.24

12 Quantum in Pictures

In the past 2-3 years a new method of introducing quantum processes and implicit the basics of quantum computation to teenagers via graph rewriting techniques has gained traction[27]. The method has been developed (over a long period of time) at the University of Oxford and has been promoted by Quantinuum and the University of Oxford. It is closely related to the ZX calculus. We would like to capture this exciting development so we describe it here briefly. Note that this is an orthogonal approach to the method developed by Terry Rudolph.

[27]https://qsec.sitehost.iu.edu/news/seminar-information/

Diagrams can be used effectively to represent tensor networks providing beginners with an operational tool that avoids standard mathematics. Using this approach one can just rely on wires, boxes, and spiders to communicate the basic idea of the compositional structure of quantum processes[28]. Starting from naïve teleportation one introduces gate-based QC, and then measurement-based (one-way) QC. Next, we can introduce relativistic causality and the idea that certain quantum processes are fundamentally non-deterministic. We can then show how the diagrams can be used to compute probabilities for the outcomes of quantum computations and protocols. We can then extend our diagrammatic language to handle explicitly both quantum systems and classical systems. We finally talk about measurements, decoherence, the no-broadcasting principle and quantum key distribution, non-locality, QAOA, and the simulation of quantum dynamics.

The references here are the beautiful, slim book [8] by Stefano Gogioso and Bob Coecke and the bigger, encyclopedic book [9] written by Aleks Kissinger and Bob Coecke.

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