THE DEVELOPMENT OF LANGUAGE AND REASONING IN THE CHILD AS CONNECTED WITH MATHEMATICAL LINGUISTICS ANDLLOGIC

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Mathematical Linguistics, Logic and The Development of Language and Reasoning in the Child

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Introduction - In recent years there has been increasing interest in the development of language and reasoning in the child and its relation to mathematical linguistics and logic. Major sources for this two-fold interest are work by Chomsky [5] and Inhelder and Piaget [18].

There are great difficulties in connecting a child's reasoning as described verbally by the child, with some particular formal logic. These difficulties are compounded if it is assumed *a priori* that that logic must be the classical 2-valued logic with which we are most familiar. This is quite evident when one considers the early years of the child's development, or even the preoperational stage of the child before binary logical operations can be performed [32,33].

In recent personal conversation Wescott has suggested that the same considerations which focus interest on protolanguages, as in [35], might also focus interest on "protologics" -- that is, on source constructs from which certain classes of other logics might stem. Such a protologic would find immediate application in research in artificial intelligence where it would suggest a basic natural logic on which to develop a model of adult human reasoning and language understanding. While the search for such protologics must be conducted elsewhere (see, e.g., [21]), an obvious preliminary is to look at the case for contemporary children.

This paper discusses some examples of nonclassical logics which might be incorporated into a mathematical linguistic system related to the development of language and reasoning in the child. Arguments for such strong bonds between the linguistic and logic systems may be found elsewhere (e.g., [14,20]).

<u>Post Language Production Rules for Multiple-valued Logic</u> - Consider those language production rules which involve logical connectives of propositional calculus [19,34]. In what follows, read $\alpha\supset\beta$, $\alpha\&\beta$, $\alpha\vee\beta$, $\sim\alpha$, $\Box\alpha$, $\vdash\alpha$, $W[\alpha]$, and $L\alpha$, as α leads to β, α and β, α or β, it is not the case that α, it is the case that α, α is a theorem, α is a well formed formula and α is a propositional variable, respectively. Where clarity allows, we write $W\alpha$ instead of $W[\alpha]$. The remaining two symbols p, l are used to generate the propositional variables p, pl, pll, plll, pllll, ···, which we denote briefly by p_0, p_1, p_2, p_3, p_4, ···, whenever convenient.

The single axiom Lp and the following production rules yield a logic which cannot have more than two values:

1. $L\alpha\to L\alpha l$
2. $L\alpha\to W\alpha$
3. $W\alpha\to W\sim\alpha$
4. $W\alpha\to W\Box\alpha$
5. $W\alpha,W\beta\to W[\alpha\supset\beta]$
6. $W\alpha,W\beta\to W[\alpha\&\beta]$
7. $W\alpha,W\beta\to W[\alpha\vee\beta]$
8. $W\alpha\to\vdash\alpha\supset\alpha$
9. $W\alpha,W\beta,W\gamma\to\vdash(\alpha\supset\beta)\supset(\gamma\supset(\alpha\supset\beta))$
10. $W\alpha,W\beta,W\gamma\to\vdash(\alpha\supset(\beta\supset\gamma))\supset((\alpha\supset\beta)\supset(\alpha\supset\gamma))$
11. $W\alpha,W\beta\to\vdash(\alpha\&\beta)\supset\alpha$
12. $W\alpha,W\beta\to\vdash(\alpha\&\beta)\supset\beta$
13. $W\alpha,W\beta,W\gamma\to\vdash(\alpha\supset\beta)\supset((\alpha\supset\gamma)\supset(\alpha\supset\beta\&\gamma))$
14. $W\alpha,W\beta\to\vdash\alpha\supset(\alpha\vee\beta)$
15. $W\alpha,W\beta\to\vdash\beta\supset(\alpha\vee\beta)$
16. $W\alpha,W\beta,W\gamma\to\vdash(\alpha\supset\gamma)\supset((\beta\supset\gamma)\supset((\alpha\vee\beta)\supset\gamma)$
17. $W\alpha,W\beta,W\gamma\to\vdash(\alpha\&(\beta\vee\gamma))\supset((\alpha\&\beta)\vee(\alpha\&\gamma)$
18. $W\alpha\to\vdash\alpha\supset\sim\sim\alpha$
19. $W\alpha,W\beta\to\sim\sim(\alpha\supset\beta)\supset(\alpha\supset\beta)$
20. $W\alpha,W\beta\to\vdash(\alpha\supset\sim\beta)\supset(\beta\supset\sim\alpha)$
21. $W\alpha,W\beta\to\vdash\sim\alpha\supset(\alpha\supset\beta)$
22. $W\alpha\to\vdash\Box\alpha\supset((\alpha\supset\alpha)\supset\alpha)$
23. $W\alpha\to\vdash((\alpha\supset\alpha)\supset\alpha)\supset\Box\alpha$
24. $W\alpha\to\vdash\Box\alpha\vee\sim\alpha$
25. $\vdash\alpha,\vdash\alpha\supset\beta\to\vdash\beta$

It will be seen below that production rules for non-2-valued logics are obtained easily from the above by simply altering production rule 24 (without alterations elsewhere), which here is equivalent to the rule corresponding to the classical law of the excluded middle $W\alpha\to\vdash\alpha\vee\sim\alpha$. Note in this case the unary operator $\Box$ simply reduces to the identity operator. In a full system, of course, there would be other linguistic production rules (see [19,34]).

Rule 24 indicates a bipartite expression with terms $\Box p$ and $\sim p$, and suggests for consideration $2^{2^1}=4$ possible disjunctions of these terms, viz., $\sim p\&\Box p, \sim p, \Box p, \sim p\vee\Box p$. Consideration of two variables, p_1 and p_2, yields 4 terms which through rule 17 are seen to be $\Box p_1\&\Box p_2$, $\sim p_1\&\Box p_2$, $\Box p_1\&\sim p_2$, $\sim p_1\&\sim p_2$, and suggest for consideration $2^{2^2}=16$ possible disjunctions of these terms [24]. Consideration of these full 16 possibilities by Post and Wittgenstein in this century was preceded by a long history of work in which fewer than 16 possibilities were considered. The work of Inhelder and Piaget [18] considers these full 16 possibilities.

Inhelder and Piaget [18] perform experiments in which certain factors $F_1, F_2, \ldots, F_T$ are assumed to play a part in effecting experimental outcomes. A simplifying assumption is that only one factor F_S plays a part, for some S satisfying $1 \leq S \leq T$, and that factors are investigated in pairs (F_i, F_j), for $i=1,2,\ldots,T; j=1,2,\ldots,T$. F_i is then associated with p_1 and F_j with p_2, to obtain each of the 16 possibilities mentioned above.

Inhelder and Piaget establish connection to the full 16 possible disjunctions and classical two-valued logic through these experiments and their protocols [18, pp. 102-104]. For example, in one protocol the experiment concerns apparatus in which there are several boxes and a hidden magnet with other factors including weight, color, distance and content. The disjunction:

$$(\Box p_1 \& \sim p_2) \vee (\sim p_1 \& \Box p_2) \vee (\Box p_1 \& \Box p_2) = \Box p_1 \vee \Box p_2$$

occurs in this protocol when p_1 corresponds to the factor F_i of distance and p_2 corresponds to the factor F_j of content, the subject stating, "It's either the distance or the content." (possibly both) [18, p. 102]. Connections with each of the other 15 disjunctions (using appropriate values of i, j for the factors F_i, F_j) are obtained in similar manner. See also [24].

It is clear that this approach assumes <u>a priori</u> that connections are to be made with 2-valued logic, and that this assumption may tend to bias the protocols toward the establishment of such connections. Some evidence for this is given by Lovell in [22]. In what follows we present Post language production rules for non-2-valued logics, and show their use for such experiments and protocols.

Replacement of rule 24 with rule 24_t given by:

$$24_t \quad W\alpha, W\beta \rightarrow \vdash \sim(\sim\Box\alpha \& \beta) \vee (\alpha \supset \beta)$$

yields a logic with at most three values (see condition (7) of [11]). Replacement of rule 24 with rule 24_m given by:

$$24_m \quad W\alpha, W\beta, W\gamma \rightarrow \vdash (\alpha \supset (\beta \cup \gamma)) \supset ((\alpha \supset \beta) \vee (\alpha \supset \gamma))$$

yields a multiple-valued logic whose number of values need not be bounded. Axioms corresponding to this last system may be found in [12,13].

The immediate interest here is with a law of the included middle which is a consequence of <u>either</u> of these replacements. For the single variable p, this law includes a middle term between the terms $\sim p$, $\Box p$ of the law of excluded middle, and is given by:

$$\sim p \vee \sim(\sim p \vee \Box p) \vee \Box p.$$

Here the included middle term is $\sim(\sim p \vee \Box p)$. This term must be taken into account if either rule 24_t or rule 24_m is used instead of rule 24. Rule 24_t yields a simplifying case in which there is at most one extra value, e, for which $\sim(\sim p \vee \Box p)$ holds , as there is one value t (truth) for which p holds, and one value f (falsity) for which $\sim p$ holds. The presence of additional extra values $e_1, e_2, \ldots$ when rule 24_m is used, does not alter these ideas as the above law of included middle still holds (further details for such systems may be found in [7,10,12,27]).

Results for two variables p_1, p_2 are now obtained as they were in the previous section. Since there are 3 terms in the above law of included middle, these now generate $3^2=9$ conjunctions -- namely, $\sim p_1 \& \Box p_2$, $\sim p_1 \& [\sim(\sim p_2 \vee \Box p_2)]$, $\sim p_1 \& \Box p_2$, $[\sim(\sim p_1 \vee \Box p_1)] \& \sim p_2$, $[\sim(\sim p_1 \vee \Box p_1)] \& [\sim(\sim p_2 \vee \Box p_2)]$, $[\sim(\sim p_1 \vee \Box p_1)] \& \Box p_2$, $\Box p_1 \& \sim p_2$, $\Box p_1 \& [\sim(\sim p_2 \vee \Box p_2)]$, $\Box p_1 \& \Box p_2$. It is easily seen that the number of possible disjunctions of these 9 conjunctions is $2^{3^2}=512$. Further details may be found in [8].

There is a likewise easy extension for the work of Inhelder and Piaget. It is evident that these 512 disjunctions contain the 16 disjunctions of the previous section. While it is not clear that the simple experiments of Inhelder and Piaget are appropriate for such extension (for one thing, the conditions of each experiment were time invariant) it is easy to observe that no distinction was made between factors F_j which were untested as opposed to factors F_k which were tested and subsequently excluded from the experiment at hand. In other words $\Box p_1$, could just as easily be the result of an exclusion of a tested factor, obtained through $[\Box p_1 \& \sim p_2] \vee (\Box p_1 \& \Box p_2) = \Box p_1$, as the result of failing to take p_2 into account. In the system of this section, $[\Box p_1 \& \sim p_2] \vee [\Box p_1 \& \Box p_2]$ does not so reduce. There is provision for expressing the extraneity of an untested factor through

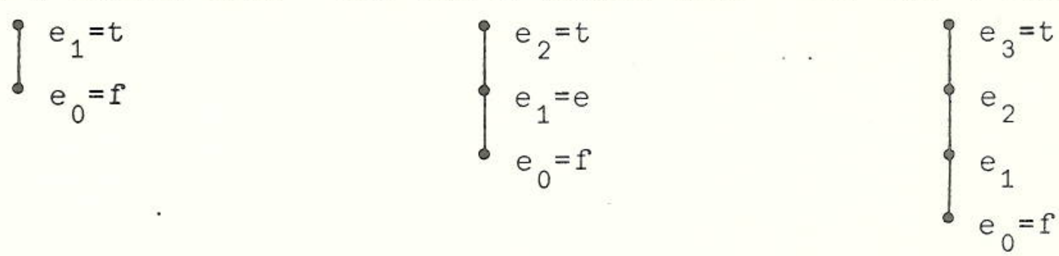
$\sim(\sim p_2 \vee \Box p_2)$, or through a third value e for which $\sim(\sim e \vee \Box e)$ evaluates to the value t.

Regardless of whether this extension is appropriate for these simple experiments, some such extension is required to account for the fact that children face experiments and decision-making situations which daily become more and more complex in nature. The fact that there may be as many as 512 possible disjunctions to consider for two propositional variables in the 3-valued case, as opposed to 16 possible disjunctions for two propositional variables in the 2-valued case, may be discouraging for those who incline toward the fewer and easier number, but should be more than sobering for those who reject the balance of these 496 possibilities in experiments or decision-making situations as a matter of simple convenience, or on the assumption that these possibilities are irrelevant or without ramifications (see [8]).

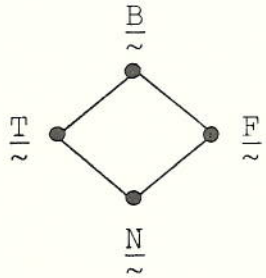
Yet a different alteration in the above production rules yields a system of rules for a relevance logic which is related to computerized question-answering (QA) systems [3,31]. This is presented in the next section.

Post Language Production Rules for a Relevance Logic - Let us make some observations about the systems given above. First, observe that each production rule contains exactly one occurrence of the production symbol $\to$, whereas there is no such limitation on the number of occurrences of the symbol $\supset$ on the right-hand side of these rules. Second, contradictions lead to any proposition variable whatever, the corresponding rule being $W\alpha, W\beta \to \vdash (\alpha \& \sim\alpha) \supset \beta$. Third, the Hasse lattice diagrams for the logical multiple values in these systems are as follows:

for the 2-valued case for the 3-valued case for the 4-valued case



The relevance logic about to be given offers a counterpoint to each of the above observations. This is the so-called relevance logic of tautological entailments [1, Ch. III] with corresponding 4-valued Hasse diagram:



This uses denotations for the values as suggested in [2].

In what follows there is exactly one occurrence of the symbol $\supset$ on the right-hand side of any symbol $\vdash$. This captures the corresponding property of the symbol $\to$ within these same rules, as well as the property of single step-by-step reasoning shown by children within the protocols of Inhelder and Piaget. Also, contradictions do not lead to any proposition variable whatsoever. That is, in what follows, the corresponding rule $W\alpha,W\beta\to\vdash(\alpha\&\sim\alpha)\supset\beta$ does not hold, although the rule $W\alpha\to\vdash(\alpha\&\sim\alpha)\supset\alpha$, for example, does. It is supposed that some such relaxing of the rule $W\alpha,W\beta\to\vdash(\alpha\&\sim\alpha)\supset\beta$ is required to prevent the breakdown of these formal systems in the presence of contradictions which occur during the prelogical stage, or even later. The single axiom is again Lp and production rules are as follows:

1. $L\alpha\to L\alpha 1$
2. $L\alpha\to W\alpha$
3. $W\alpha\to W\sim\alpha$
4. $W\alpha,W\beta\to W[\alpha\&\beta]$
5. $W\alpha,W\beta\to W[\alpha\vee\beta]$
6. $W\alpha,W\beta\to\vdash(\alpha\&\beta)\supset\alpha$
7. $W\alpha,W\beta\to\vdash(\alpha\&\beta)\supset\beta$
8. $W\alpha,W\beta\to\vdash\alpha\supset(\alpha\vee\beta)$
9. $W\alpha,W\beta\to\vdash\beta\supset(\alpha\vee\beta)$
10. $W\alpha,W\beta,W\gamma\to\vdash(\alpha\&(\beta\vee\gamma))\supset((\alpha\&\beta)\vee\gamma)$
11. $W\alpha\to\vdash\alpha\supset\sim\sim\alpha$
12. $W\alpha\to\vdash\sim\sim\alpha\supset\alpha$
13. $\vdash\alpha\supset\beta,\vdash\beta\supset\gamma\to\vdash\alpha\supset\gamma$
14. $\vdash\alpha\supset\beta,\vdash\alpha\supset\gamma\to\vdash\alpha\supset(\beta\&\gamma)$
15. $\vdash\alpha\supset\gamma,\vdash\beta\supset\gamma\to(\alpha\vee\beta)\supset\gamma$
16. $\vdash\alpha\supset\sim\beta\to\vdash\beta\supset\sim\alpha$

The only theorems which involve the logical operation $\supset$ alone are of the form

$$\vdash p_i\supset p_i, \quad i=0,1,2,\cdots$$

As above, there is associativity and commutativity for each of the operations &,$\vee$, with distributivity of each operation across the other, and again both of DeMorgan's laws hold for these two operations. However, $\vdash(p_1\&(\sim p_1\vee p_2))\supset p_2$ does not hold, and $\vdash p_1\supset(p_2\vee\sim p_2)$

also fails to hold. Again, in a full system there would be other linguistic production rules [19,34].

Here we focus attention on the 4-valued Hasse diagram just given. In simplest terms, consider any question-answering or interactive system in which $\underset{\sim}{\underline{T}}$ represents that one component (human, entry of computer memory, etc.) of the system has received information that p is true, $\underset{\sim}{\underline{F}}$ represents that one component of the system has received information that p is false, $\underset{\sim}{\underline{B}}$ represents that one component of the system has received information (from 2 or more sources) that p is both true and false, and $\underset{\sim}{\underline{N}}$ represents that one component of the system hasn't received any information (from any source) whether p is true or false -- that is, none of true or false has been received.

This is of interest not only for complex systems involving one or more computer subsystems [2,31], but also for experimental situations with children in which the testing of irrelevant as well as relevant factors is encouraged, or where the experiment is such that contradictory test results are generated.

It is the actual case that such situations cannot be ignored by real life experimenters in a laboratory. As a simple example, the time-invariance hypothesis for an experiment may fail just because the conditions of the experiment were accidentllly changed by some third party during the temporary absence of the experimenters. Under this circumstance, what was relevant may become irrelevant, what was irrelevant may become relevant, and new experimental outcomes may contradict old experimental outcomes.

The logic just described, however, is only one of various kinds of relevance logics which might have been considered in this section. The system in [4], for example, does not fix any limit on the number of occurrences of the logical operation $\supset$. There are, in fact, numerous variations for thse systems (see [6,9,29] for multiple-valued logics and [1] for relevance logics). There are still other kinds of nonclassical logic which have not been mentioned, for reasons of brevity. There are a number of systems which are obtained through the introduction of unary operations which obey certain production rules because of semantic or hypothesized interpretations of these operations. In particular, the operation □ appears in modal logics [16]. These are discussed by Lakoff [20] and Hintikka [15]. Among other logics, we mention temporal logics [26,30], spatial logics [28, pp. 229-249] and epistemic logic [28, pp. 40-53]. See [25], [17], and [23], respectively, for work by Piaget in these areas.

Summary - We have suggested that it is unnecessary to use the classical 2-valued logic alone, when establishing relations to the development of language and reasoning in the child. We have presented some alternatives.

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