The Powerset Operator as an Algebraic Tool for Understanding Least Fixpoint Semantics in the Context of Nested Relations

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In this paper, we deal with algebras for nested relations. First, we consider the nested relational algebra augmented with the powerset operator. We show that this augmentation increases the expressiveness of the nested algebra substantially. In particular, we show that in the powerset algebra thus obtained, either the nest or the difference can be removed as a primitive operator. As a consequence, the classical nested relational algebra without the difference is already complete in the sense of Bancilhon and Paredaens. Next, we consider the least fixpoint closure of the nested relational. We prove that this algebra is equivalent to the powerset algebra, thus providing a way to investigate properties of query languages related to the least fixpoint operator by using an algebraic approach. Finally, we show that there exists a fundamental difference between least fixpoint semantics in the context of flat and nested relations respectively, by expressing the parity of a relation in the lfp closure of the nested relational algebra.

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1 Introduction

In the last years, much attention has been paid to structured relations. In order to model some database applications more naturally, Makinouchi proposed to generalize the relational model by removing Codd's first normal form assumption [10], thus allowing relations with set-valued attributes [18]. Subsequently, a generalization of the relational algebra to relations with set-valued attributes was introduced by Jaeschke and Schek [16]. More specifically, they presented the nest and the unnest operator as tools to restructure such relations. Finally, Thomas and Fisher generalized this model by allowing nested relations of arbitrary (but fixed) depth [23]. In [21] a calculus like query language for this model was defined.

Independently, Jacobs, Kuper and Vardi proposed database models, based on database logic[1], which generalize the relational as well as the hierarchical and network models [15,17]. In particular, Kuper and Vardi proposed two query languages, a calculus and an algebra, and established an equivalence result analogous to that established by Codd [11]. In addition, it can easily be seen that database logic incorporates the nested relational model. Recently, it has been observed by Abiteboul, Beeri and Hull that it is possible to express the transitive closure of a binary relation in database logic [1,5,14]. This is neither possible in the flat nor the nested relational model [3,20]. We may thus conclude that database logic is a strict generalization of the nested relational model. More recently, Abiteboul and Beeri addressed other problems concerning the expressiveness of query languages for models such as database logic and the nested relational model [2]. In this paper we investigate related issues.

We begin by defining a nested algebra similar to the one introduced in [23] (Sections 2 and 3). We then consider the powerset operator, which was used in the algebraic language in [17], and add it to the nested algebra (Section 4). We show, by using a combinatorial argument, that this powerset algebra is substantially more expressive than the nested algebra[2], thus giving insight in earlier results. We then proceed by showing that in the powerset algebra either the nest operator or the difference can be considered as redundant. As a surprising consequence of this result, the nested algebra without the difference is complete in the sense of Bancilhon and Paredaens [4,12,19,24]. Furthermore, it follows from our constructions that there exists a close relationship between the nest operator and the difference, which corresponds to an observation made in [6].

When Aho and Ullman [3] realized that the transitive closure was not expressible in Codd's relational algebra and calculus, they proposed to extend these query languages with constructs such as the least fixpoint operator. Chandra and Harel [7,8,9] studied even more powerful generalizations. We consider a similar extension to the nested algebra, by augmenting it with the least fixpoint operator (Section 5). We then derive our main result which is the equivalence between the nested algebra with the least fixpoint operator and the powerset algebra. We feel that this result and the way it is obtained suggests an algebraic approach towards the investigation of semantical properties of query languages supported by the least fixpoint operator.

[1] not to be confused with Datalog

[2] A similar result was obtained independently by Houben, Paredaens and Tahon [13].

We conclude this paper by showing (Section 6) that it is possible to determine the parity of a relation in the least fixpoint closure of the nested algebra, in sharp contrast with a result of Chandra and Harel, establishing the impossibility of doing so in the least fixpoint closure of the flat relational algebra [8,9]. This indicates that the least fixpoint semantics in the context of nested relations is significantly richer than that in the context of flat relations.

2 A Model for Nested Relations

Basically we assume that we have an infinitely enumerable set U of *elementary attributes* and an infinitely enumerable set V of *elementary values*. In this section, we explain how arbitrary attributes and values, relation schemes, relation instances and relations are constructed from these.

First, we define an attribute. Attributes can either be elementary or composed. The latter ones are sets of attributes (which can be composed in turn); the values associated to them are relation instances over that set of attributes, interpreted as a scheme.

Definition 2.1: *The set of all attributes $\mathcal{U}$ is the smallest set containing U such that for each finite subset X of $\mathcal{U}$ in which no elementary attribute appears more than once, $X \in \mathcal{U}$.* ∎

An attribute of U is called an *elementary attribute*; an attribute of $\mathcal{U} \setminus U$ is called a *composed attribute*. The values associated to composed attributes will be called quite fittingly composed values. The definition of a *relation scheme* is now very straightforward:

Definition 2.2: *A relation scheme Ω is a composed attribute, i.e. an element of $\mathcal{U} \setminus U$.* ∎

Before proceeding, we give an example of a nested relation:

Example 2.1: Consider the following relation representing persons, their jobs and their addresses:

PERSON	*JOB*	*STREET*	*NR*	*CITY*
Jeff Willows	professor manager	Broadway	12135	New York
Mary Higgins		Wilshire Blvd.	8125	Los Angeles

Formally, the scheme of this relation is a set of three attributes, the first of which is elementary whereas the others are composed and is represented as $\{JOB\}$ and $\{\{STREET, NR\}, CITY\}$ respectively. The composed values corresponding to them are relation instances over these attributes, considered as schemes. ∎

It should be clear by now that the notions of value, tuple and instance are so closely intertwined that it is easier to define them jointly:

Definition 2.3: *The set $\mathcal{V}$ of all values, the set $\mathcal{I}_X$ of all instances over $X \in \mathcal{U} \setminus U$, the set $\mathcal{T}_X$ of all tuples over $X \in \mathcal{U} \setminus U$ and the set $\mathcal{I}$ of all instances are the smallest sets satisfying:*

1. $\mathcal{V} = V \cup \mathcal{I}$;
2. $\mathcal{I} = \bigcup_{X \in \mathcal{U} \setminus U} \mathcal{I}_X$;
3. $\mathcal{I}_X$ *consists of all finite subsets of* $\mathcal{T}_X$;
4. $\mathcal{T}_X$ *consists of mappings t from X into $\mathcal{V}$, called tuples, satisfying $t(A) \in V$ for all elementary attributes $A \in X \cap U$ and $t(Y) \in \mathcal{I}_Y$ for all composed attributes $Y \in X \setminus U$.* ∎

We now have all the necessary ingredients to formally define a relation:

Definition 2.4: *A relation is a pair (Ω,ω) where $\Omega \in \mathcal{U}\setminus U$ and $\omega \in \mathcal{I}_\Omega$. Ω is called the scheme of the relation and ω is called the instance of the relation. If $\Omega \subseteq U$, then (Ω,ω) is called a flat relation.* ∎

3 The Nested Relational Algebra

In this section, we define a nested algebra based on the model for relations described in the previous section. It is generated by eight operators, defined below.

Definition 3.1: *Let (Ω,ω), (Ω,ω_1), (Ω_1,ω_1) and (Ω_2,ω_2) be relations. Suppose that Ω_1 and Ω_2 have no elementary attributes in common.*

- *The union $(\Omega,\omega_1)\cup(\Omega,\omega_2)$ equals $(\Omega,\omega_1\cup\omega_2)$;*
- *The difference $(\Omega,\omega_1)\setminus(\Omega,\omega_2)$ equals $(\Omega,\omega_1\setminus\omega_2)$;*
- *The cartesian product $(\Omega_1,\omega_1)\times(\Omega_2,\omega_2)$ equals (Ω',ω') where $\Omega'=\Omega_1\cup\Omega_2$ and*
$$\omega' = \{t \in \mathcal{T}_{\Omega'} \mid t|_{\Omega_1} \in \omega_1 \ \&\ t|_{\Omega_2} \in \omega_2\}$$
- *Let $\Omega' \subseteq \Omega$. The projection $\pi_{\Omega'}(\Omega,\omega)$ equals (Ω',ω') where $\omega' = \{t|_{\Omega'} \mid t \in \omega\}$;*
- *Let $X \subseteq \Omega$. The nesting $\nu_X(\Omega,\omega)$ equals (Ω',ω') where $\Omega' = (\Omega\setminus X)\cup\{X\}$ and*
$$\omega' = \Big\{t \in \mathcal{T}_{\Omega'} \,\Big|\, \exists t' \in \omega\colon t|_{\Omega\setminus X} = t'|_{\Omega\setminus X} \ \& \\ t(X) = \{t''|_X \mid t'' \in \omega \ \&\ t'|_{\Omega\setminus X} = t''|_{\Omega\setminus X}\}\Big\}$$
- *Let $X \in \Omega\setminus U$. The unnesting $\mu_X(\Omega,\omega)$ equals (Ω',ω') where $\Omega' = (\Omega\setminus\{X\})\cup X$ and*
$$\omega = \{t \in \mathcal{T}_{\Omega'} \mid \exists t' \in \omega\colon t|_{\Omega\setminus\{X\}} = t'|_{\Omega\setminus\{X\}} \ \&\ t|_X \in t'(X)\}$$

Let (Ω,ω) be a relation scheme. Let φ be a permutation on U. φ is extended in the natural way to $\mathcal{U}$, to $\mathcal{I}$ and to $\mathcal{V}$:

- *The renaming $\rho^\varphi(\Omega,\omega)$ equals $(\varphi(\Omega),\varphi(\omega))$;*
- *Assume furthermore that $\varphi(\Omega)=\Omega$. The selection $\sigma^\varphi(\Omega,\omega)$ equals (Ω,ω') where*
$$\omega' = \{t \in \omega \mid \forall X \in \Omega\colon \varphi(t(X)) = t(\varphi(X))\}$$ ∎

Note that the cartesian product is only defined for relations with completely "independent" schemes. This is actually not a heavy restriction: it is indeed always possible to arrange that the schemes of two relations have no elementary attributes in common by performing an appropriate renaming.

We end this discussion about the basic nested algebra operators with a notational issue. In most practical cases, renaming involves only one attribute X at the time. If X is renamed to X', and if, in case X and X' are composed attributes, no ambiguity is possible as to how the renaming is done, we shall denote this operation as $\rho_{X'\leftarrow X}$. We shall use the same notation if X is a set of attributes of the scheme under consideration, and each attribute of X is renamed in a well known way to an attribute of X'. Similarly, if selection comes down to only checking whether the values for composed attributes X and X' are equal upon renaming and if no ambiguity is possible as to how the elementary attributes in X and X' are to be matched, we shall denote this selection by $\sigma_{X=X'}$. Again, we shall use the same notation if X and X' are sets of attributes of the scheme under consideration.

We can now formally define a *nested algebra expression (nae)*:

Definition 3.2:

1. *x, y, z, ... are nae's;*
2. *For all $\Omega \in \mathcal{U} \setminus U$, $(\Omega, \emptyset)$ is an nae;*
3. *For all $\Omega \in \mathcal{U} \setminus U$, $(\{\Omega\}, \{\emptyset\})$ is an nae;*
4. *For all nae's, the basic operators of Definition 3.1 applied to these expressions, are also nae's.* ∎

To avoid extensive use of brackets, we assume the following precedence on nested algebra operators:

1. unary operators;
2. cartesian product;
3. set operators.

The set of all nae's will be denoted by $\mathcal{N}$. If $r, r', \ldots$ is a finite sequence of relations and $E(x, y, \ldots) \in \mathcal{N}$ is an nae with as many variables as there are relations, then $E(r, r', \ldots)$ is interpreted as the relation obtained by substituting every occurrence of a variable in $E(x, y, \ldots)$ by the corresponding relation, whenever this substitution makes sense[3], and undefined otherwise.

Now how powerful is the nested relational algebra? To answer this question, it suffices to consider this question for single relations only, since a database can always be represented as the cartesian product of its non-empty members. A first way to look at this problem is to consider for each relation r the set of relations $\{E(r) \mid E(x) \in \mathcal{N}\}$ that can be derived from r. Van Gucht and Gyssens [12,24] have shown, generalizing results of Bancilhon and Paredaens [4,19], that these sets can be characterized as follows:

Theorem 3.1: *Let r and s be relations. There exists $E(x) \in \mathcal{N}$ such that $s = E(r)$ if and only if s remains invariant for (the natural extension to $\mathcal{I}$) of all permutations on V that leave r invariant.* ∎

Theorem 3.1 is usually summarized by saying that $\mathcal{N}$ is *BP-complete* [4,7,12,19,24]. Intuitively, the philosophy behind BP-completeness is that a query language should manipulate all values as uninterpreted; only their equality or inequality is relevant.

So, as far as relations are concerned, the nested relational algebra seems sufficiently expressive. We can however ask a much stronger question, which is situated at the level of queries, rather than relations [7,8]: *Let Q be a computable query, i.e. a partial recursive mapping from relations to relations such that whenever $Q(r)$ is defined, $Q(r)$ remains invariant for all permutations on V that leave r invariant. Does there exist $E(x) \in \mathcal{N}$ such that $\forall r: E(r) = Q(r)$?* The answer to this question is no. Although by Theorem 3.1 it is always possible to find an expression that satisfies this equality for a particular relation, there is in general no expression that will do for all relations. Indeed, it was already shown in 1979 that the classical relational algebra is not complete in the sense mentioned above with respect to flat relations. In particular, Aho and Ullman [3] showed that the transitive closure of a binary flat relation is not expressible in the classical relational algebra. Recently it was shown that this query is not even expressible in the nested relational algebra [20].

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[3] where $(\{\Omega\}, \{\emptyset\})$ is short for $(\{\Omega\}, \{t \in \mathcal{T}_{\{\Omega\}} \mid t(\Omega) = \emptyset\})$.

Several attempts have been made to enrich the nested relational algebra. In the following sections, we present some remarkable results in connection with these attempts.

4 Adding the Powerset Operator to the Nested Relational Algebra

Kuper en Vardi introduced the powerset in [17] as one of the primitive operators in their algebraic query language for database logic. Basically the powerset operator generates all subsets of a given relation:

Definition 4.1: *Let* (Ω, ω) *be a relation. The powerset* $\Pi(\Omega, \omega)$ *equals* $(\{\Omega\}, \omega')$ *where* $\omega' = \{t \in \mathcal{T}_{\{\Omega\}} \mid t(\Omega) \subseteq \omega\}$. ∎

We shall first establish that the powerset operator cannot be expressed as a query in the nested relational algebra. Therefore we need a technical lemma of a combinatorial nature:

Lemma 4.1: *Let* $E(x) \in \mathcal{N}$ *be an arbitrary nae. Let* $u(E)$ *be the number of unions and* $p(E)$ *the number of cartesian products in* $E(x)$. *Let for any arbitrary relation* s, $\mathcal{M}(s)$ *be the set of all relations that can be obtained from* s *using nae's in which only the unnest operator appears. Then for each relation* r *we have that for each relation* $r'' \in \mathcal{M}(E(r))$[4]*:*

$$|r''| \leq 2^{u(E)} \max_{r' \in \mathcal{M}(r)} \left(|r'|, 1\right)^{2^{p(E)}}$$

Proof: By straightforward induction on the size of E. ∎

Corollary 4.1: *Let* $E(x) \in \mathcal{N}$ *be an arbitrary nae. Then there exist nonnegative integers* a, b *and* c *such that for all flat relations* r, $|E(r)| \leq a|r|^b + c$. ∎

We can now establish:

Theorem 4.1: *There is no nae that expresses the powerset operator.*

Proof: Suppose at the contrary that there exists $E(x) \in \mathcal{N}$ such that for all relations r, $E(r) = \Pi(r)$. In particular, this equality holds for flat relations. Since $|\Pi(r)| = 2^{|r|}$, Corollary 4.1 cannot possibly hold for this nae, a contradiction. ∎

We now consider the *powerset algebra* whose set of expressions $\mathcal{P}$ is generated by the basic operators of the nested relational algebra defined in Definition 3.1, augmented with the powerset operator. An expression of the powerset algebra is called a *powerset algebra expression (pae)*. Although only one operator is added, $\mathcal{P}$ turns out to be remarkably more expressive than $\mathcal{N}$. We demonstrate this by showing that either the nest operator or the difference can be omitted as a basic operator of $\mathcal{P}$.

Theorem 4.2: *There is a pae in which the nest operator does not occur that expresses the nest operator.*

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[4] For each relation s, $|s|$ denotes the number of tuples in s.

Proof: Let r be the relation (Ω, ω) and let $X \subseteq \Omega$. Let φ_1, φ_2 and φ_3 be permutations on U such that Ω, $\Omega_1 = \varphi_1(\Omega)$, $\Omega_2 = \varphi_2(\Omega)$ and $\Omega_3 = \varphi_3(\Omega)$ have no elementary attributes in common. Let $X_1 = \varphi_1(X)$, $X_2 = \varphi_2(X)$ and $X_3 = \varphi_3(X)$. We now define some pae's the last of which will express $\nu_X(r)$, independent of ω:

$$E_1(x) = \pi_{\Omega\cup\{X_1\}}\sigma_{X=X_2}\mu_{X_2}\sigma_{X_1=X_2}\big(r \times \Pi\pi_{X_1}\rho_{X_1\leftarrow X}(x) \times \Pi\pi_{X_2}\rho_{X_2\leftarrow X}(x)\big)$$

It is readily verified that $s = E_1(r)$ is the relation over the scheme $\Omega \cup \{X\}$ which is empty whenever r is empty, and otherwise satisfies:

1. $\pi_\Omega(s) = r$;
2. $\pi_{X_1}(s) = \Pi\pi_{X_1}\rho_{X_1\leftarrow X}(r)$;
3. For each tuple t in s, $\varphi_1(t|_X) \in t(X_1)$ (and hence $t(X_1) \neq \emptyset$).

$$E_2(x) = \pi_{\Omega\cup\{X_1\}\cup X_2}\sigma_{\Omega\setminus X=\Omega_2\setminus X_2}\sigma_{X_1=X_3}\big(E_1(x) \times \rho_{\Omega_2\leftarrow\Omega}\rho_{X_3\leftarrow X_1}E_1(x)\big)$$

$s = E_2(r)$ is a relation over the scheme $\Omega \cup \{X_1\} \cup X_2$ which is empty whenever r is empty, and otherwise satisfies the conditions 1, 2, 3, and:

4. $\rho_{X\leftarrow X_2}\pi_{X_2}(s) = \pi_X(r)$;
5. For each tuple t in s, $\varphi_1\varphi_2^{-1}(t|_{X_2}) \in t(X_1)$.
6. For each tuple t in s, the tuple t' over Ω defined by $t'|_{\Omega\setminus X} = t|_{\Omega\setminus X}$ and $\varphi_2(t'|_X) = t|_{X_2}$ is in r.

$$E_3(x) = \pi_{\Omega\cup\{X_1\}\cup X_2}\sigma_{X_1=X_3}\big(E_1(x) \times \rho_{\Omega_2\leftarrow\Omega}\rho_{X_3\leftarrow X_1}E_1(x)\big) \setminus E_2(x)$$

$s = E_3(r)$ is a relation over the scheme $\Omega \cup \{X_1\} \cup X_2$ which is empty whenever r is empty, and otherwise satisfies the conditions 1, 2, 3, 4, 5 and:

6′. For each tuple t in s, the tuple t' over Ω defined by $t'|_{\Omega\setminus X} = t|_{\Omega\setminus X}$ and $\varphi_2(t'|_X) = t|_{X_2}$ is not in r.

$$E_4(x) = E_1(x) \setminus \pi_{\Omega\cup\{X_1\}}E_3(x)$$

$s = E_4(r)$ is a relation over the scheme $\Omega \cup \{X_1\}$ which is empty whenever r is empty, and otherwise satisfies the conditions 1, 2, 3 and:

7. For each tuple t in s, each tuple t' over Ω satisfying $t'|_{\Omega\setminus X} = t|_{\Omega\setminus X}$ and $\varphi_1(t'|_X) \in t(X_1)$ is in r.

The projection of $E_4(r)$ onto $(\Omega \setminus X) \cup \{X_1\}$ can alternatively be described as the relation defined by the property that for each tuple t it contains, there exists a tuple t' in $\nu_X(r)$ such that $t|_{\Omega\setminus X} = t'|_{\Omega\setminus X}$ and $t(X_1) \subseteq \varphi_1(t'(X))$. So, what we still need to do is select those tuples t from $E_4(r)$ for which $t(X_1)$ is maximal with respect to inclusion. The remaining constructions in this proof have as a purpose to perform this selection.

$$\begin{aligned}E_5(x) = {} & \pi_{(\Omega\setminus X)\cup\{X_1,X_3\}}\sigma_{\Omega\setminus X=\Omega_2\setminus X_2}\big(E_4(x) \times \rho_{X_2\leftarrow X}\rho_{X_3\leftarrow X_1}E_4(x)\big) \\ & \setminus \pi_{(\Omega\setminus X)\cup\{X_1,X_3\}}\sigma_{\Omega=\Omega_2}\big(E_4(x) \times \rho_{X_2\leftarrow X}\rho_{X_3\leftarrow X_1}E_4(x)\big)\end{aligned}$$

$E_5(r)$ is a relation over $(\Omega \setminus X) \cup \{X_1, X_3\}$ which can be defined by the property that for each tuple t it contains there exists a tuple t' in $\nu_X(r)$ such that $t|_{\Omega\setminus X} = t'|_{\Omega\setminus X}$, $\emptyset \neq t(X_1) \subseteq \varphi_1(t'(X))$, $\emptyset \neq t(X_3) \subseteq \varphi_3(t'(X))$ and $\varphi_1^{-1}(t(X_1)) \cap \varphi_3^{-1}(t(X_3)) = \emptyset$.

Obviously a tuple t in the projection of $E_4(r)$ onto $(\Omega \setminus X) \cup \{X_1\}$ is not in $\rho_{X_1 \leftarrow X}\, \nu_X(r)$ if $t(X)$ is not maximal, i.e. if t is the projection of $E_5(r)$ onto $(\Omega \setminus X) \cup \{X_1\}$. We may conclude:

$$\nu_X(x) = \rho_{X \leftarrow X_1}\big(\pi_{(\Omega \setminus X) \cup \{X_1\}} E_4(x) \setminus \pi_{(\Omega \setminus X) \cup \{X_1\}} E_5(x)\big) \qquad \blacksquare$$

A careful examination of the proof of Theorem 4.2 reveals that it centered around an extensive and subtile use of the difference. Therefore it is a logical question to ask whether the nest operator and the difference might be interchangeable in the powerset algebra. Surprisingly enough, the answer is yes! In order to make the proof more legible, we first establish the following technical lemmas:

Lemma 4.2: *Let $r = (\Omega, \omega)$ be a relation. Let $N(r)$ denote the one tuple relation*

$$\big(\{\Omega\}, \{t \in \mathcal{T}_{\{\Omega\}} \mid t(\Omega) = \omega\}\big)$$

There exists a pae in which the difference does not occur that expresses $N(r)$, independent of ω.

Proof: First note that nesting over the entire scheme yields the desired answer if $\omega \neq \emptyset$ and that the powerset operator gives the right answer if $\omega = \emptyset$. An expression that always returns the correct result is:

$$N(x) = \pi_{\{\Omega\}}\, \mu_{\{\Omega\}}\, \sigma_{\{\Omega_1\} = \{\Omega\}}\big(\nu_{\{\Omega\}}\, \Pi(x) \times \nu_{\{\Omega_1\}}(\{\Omega_1\}, \{\emptyset\})\big) \;\cup\; \nu_\Omega(x) \qquad \blacksquare$$

Lemma 4.3: *Let $r = (\Omega, \omega)$ be a relation and let $X, X' \in \Omega \setminus U$ be composed attributes. Suppose there exists a permutation φ on U such that $X' = \varphi(X)$. φ is extended to $\mathcal{I}$ in the usual way. Denote by $\sigma^{\varphi}_{X \cap X' = \emptyset}(r)$ (or $\sigma_{X \cap X' = \emptyset}(r)$ if φ is understood) the relation*

$$\big(\Omega, \{t \in \omega \mid \varphi(t(X)) \cap t(X') = \emptyset\}\big)$$

There exists an nae (and hence a pae) in which the difference does not occur that expresses $\sigma^{\varphi}_{X \cap X' = \emptyset}(r)$, independent of ω.

Proof: Let φ_1 and φ_2 be permutations on U such that Ω, $\Omega_1 = \varphi_1(\Omega)$ and $\Omega_2 = \varphi_2(\Omega)$ have no elementary attributes in common. Let $X_1 = \varphi_1(X)$, $X_1' = \varphi_1(X')$ and $X_2' = \varphi_2(X')$. First consider the expression:

$$E_1(x) = \nu_{X_1'}\, \pi_{\Omega \cup X_1'}\, \sigma_{X_1 = X_1'}\, \mu_{X_1}\, \mu_{X_1'}\, \sigma_{\Omega = \Omega_1}\big(x \times \rho_{\Omega_1 \leftarrow \Omega}(x)\big)$$

$E_1(r)$ is a relation over $\Omega \cup \{X_1\}$ containing those tuples t that satisfy simultaneously:

1. $t|_\Omega$ is a tuple of r;
2. $\varphi_1\varphi\big(t(X)\big) \cap \varphi_1\big(t(X')\big) = t(X_1')$;
3. $t(X_1') \neq \emptyset$.

What we need however are those relations for which $\varphi\big(t(X)\big) \cap t(X') = \emptyset$. We can obtain this set fairly easy by the following "marking" procedure:

$$E_2(x) = \nu_{\{X_1'\}}\, \mu_{\{X_1'\}}\big(\nu_{\{X_1'\}}\, E_1(x) \;\cup\; x \times \nu_{\{X_1'\}}(\{X_1'\}, \{\emptyset\})\big)$$

Let t be a tuple of this relation. Then $t\big(\{X_1'\}\big)$ consists of two relations if $\varphi\big(t(X)\big) \cap t(X') \neq \emptyset$ and of one relation, namely $\big(\{X_1'\}, \{\emptyset\}\big)$, in the other case. Hence we have:

$$\sigma^{\varphi}_{X \cap X' = \emptyset}(x) = \pi_\Omega\, \sigma_{\{X_1'\} = \{X_2'\}}\big(E_2(x) \times \nu_{\{X_2'\}}(\{X_2'\}, \{\emptyset\})\big) \qquad \blacksquare$$

Theorem 4.3: *There is a pae in which the difference does not occur that expresses the difference.*
Proof: Let $r = (\Omega, \omega)$ and $r' = (\Omega, \omega')$ be relations over the same scheme. Since $r \setminus r' = (r \cup r') \setminus r'$, we may assume without loss of generality that $\omega' \subseteq \omega$. Let φ_1 and φ_2 be permutations on U such that Ω, $\Omega_1 = \varphi_1(\Omega)$ and $\Omega_2 = \varphi_2(\Omega)$ have no elementary attributes in common. We now define some pae's the last of which will express the nest operator. Throughout our comments, we assume that $r \neq r'$, i.e. $r \setminus r' \neq \emptyset$; we invite the reader to check that the expressions below yield the correct result in the other case too.

$$E_1(x,y) = \nu_{\Omega_1}\, \mu_{\Omega_1}\big(\sigma_{\Omega=\Omega_1}(\Pi(x) \times \rho_{\Omega_1 \leftarrow \Omega}\Pi(x)) \ \cup\ \Pi(x) \times \rho_{\Omega_1 \leftarrow \Omega}\, \nu_\Omega(y)\big)$$

$E_1(r, r')$ is the relation over the scheme $\{\Omega, \Omega_1\}$ consisting of those tuples t for which $t(\Omega)$ is a subset of ω, $t(\Omega_1) = \varphi_1\big(t(\Omega) \cup \omega'\big)$ and $t(\Omega_1) \neq \emptyset$.

$$E_2(x,y) = \pi_{\{\Omega\}}\sigma_{X_1 = X_2}\left(E_1(x,y) \times \rho_{\Omega_2 \leftarrow \Omega}\, \nu_\Omega(x)\right)$$

It is readily verified that $E_2(r,r') = \big(\{\Omega\}, \{t \in \Pi(r) \mid t(\Omega) \cup \omega' = \omega\}\big)$. Hence $E_1(r, r')$ basically consists of the non-empty subrelations of r that contain $r \setminus r'$. Hence the desired result is the smallest among these relations, i.e. the one that has no tuples in common with r':

$$x \setminus y = \mu_\Omega\, \pi_{\{\Omega\}}\sigma_{\Omega \cap \Omega_1 = \emptyset}\big(E_2(x,y) \times \rho_{\Omega_1 \leftarrow \Omega} N(y)\big) \qquad ∎$$

As an equally remarkable corollary to this surprising result, we have:

Corollary 4.2: *The set of nae's in which difference does not occur is BP-complete.*
Proof: Let $r = (\Omega, \omega)$. We first show that for all nonnegative integers i, there exists an nae $E_i(x)$ in which difference does not occur such that

$$E_i(r) = \big(\{\Omega\}, \{t \in \mathcal{T}_{\{\Omega\}} \mid t(\Omega) \subseteq \omega \ \&\ |t(X)| \leq i\}\big)$$

For notational convenience, we only write down $E_2(x)$ explicitly; the generalization is obvious. We assume that φ_1 and φ_2 are permutations on U such that Ω, $\Omega_1 = \varphi_1(\Omega)$ and $\Omega_2 = \varphi_2(\Omega)$ have no elementary attributes in common.

$$E_2(x) = \pi_{\{\Omega\}}\, \nu_\Omega\big(\sigma_{\Omega=\Omega_1}(x \times \rho_{\Omega_1 \leftarrow \Omega}(x) \times \rho_{\Omega_2 \leftarrow \Omega}(x)) \\ \cup\ \sigma_{\Omega = \Omega_2}(x \times \rho_{\Omega_1 \leftarrow \Omega}(x) \times \rho_{\Omega_2 \leftarrow \Omega}(x))\big) \ \cup\ \big(\{\Omega\}, \{\emptyset\}\big)$$

Clearly, $\Pi(r) = \bigcup_{i=0}^{|r|} E_i(r)$. Note that this expression depends on the size of r, which should be the case, by Theorem 4.1. Corollary 4.2 now immediately follows from Theorem 4.3 and Theorem 3.1. ∎

5 The Least Fixpoint Closure of the Nested Relational Algebra

Another powerful tool to model relational queries is the *least fixpoint (lfp) operator* [3,8,9]. Actually this operator does not work on relations but on queries; it transforms them into other ones. A classical example of a query that can be constructed from a flat relational algebra query using the lfp operator, is the *transitive closure* of a flat binary relation. As this query cannot even be expressed in the nested relational algebra [3,20], it follows that there are nae's for which the lfp operator applied to it cannot be expressed as an nae. Therefore we shall study the *lfp closure of the nested algebra*. We show that it is equivalent to the powerset algebra, thus proving once again the expressiveness of the latter one.

First, we formally define the lfp closure of the nested relational algebra. Therefore, we need to observe that the lfp operator makes only sense for *unary monotone expressions*. We say that an nae or pae $E(x)$ is monotone if, for all relations r and s for which $E(r)$ and $E(s)$ are defined, $r \subseteq s$ implies $E(r) \subseteq E(s)$.

Definition 5.1: *The lfp closure $\mathcal{N}^*$ (respectively $\mathcal{P}^*$) of the nested algebra $\mathcal{N}$ (respectively the powerset algebra $\mathcal{P}$) is the smallest set of expressions satisfying:*

1. $\mathcal{N} \subseteq \mathcal{N}^*$ *(respectively $\mathcal{P} \subseteq \mathcal{P}^*$);*
2. f*or each scheme preserving expression $E(x)$, $E^*(x)$ is also an expression.* ∎

If $E(x)$ is a monotone expression and r is a relation, then $E^*(r)$ is defined if and only if $E(r)$ is defined and must in that case be interpreted [22] as the smallest relation s containing r for which $E(s) = s$. Now let $\widehat{E}(x) = x \cup E(x)$. A straightforward argument then shows that $E^*(r) = \bigcup_{i=1}^{\infty} \widehat{E}^i(r)$, where, for all positive i, $\widehat{E}^i(x)$ stands for

$$\underbrace{\widehat{E} \ldots \widehat{E}}_{i \text{ times}}(x)$$

Note that for each relation r for which $E(r)$ is defined, $E^*(r)$ can always be computed, since there must exist some positive integer k for which $\widehat{E}^k(r) = \widehat{E}^{k+1}(r) = \widehat{E}^{k+2}(r) = \cdots$. $\mathcal{N}^*$ will be called the *lfp nested algebra for short*; an expression of $\mathcal{N}^*$ will be called an *lfp nested algebra expression (lnae)*. A similar terminology will be used for $\mathcal{P}^*$.

We first show that the powerset operator can be expressed in the lfp nested algebra:

Theorem 5.1: *There exists an lnae that expresses the powerset operator.*

Proof: Let r be the relation (Ω, ω). Let φ_1 and φ_2 be permutations on U such that Ω, $\Omega_1 = \varphi_1(\Omega)$ and $\Omega_2 = \varphi_2(\Omega)$ have no elementary attributes in common. Since $\Pi(r)$ is a relation over $\{\Omega\}$, we first need an nae $E_1(x)$ such that $E_1(r)$ is a relation with scheme $\{\Omega\}$:

$$E_1(x) = \pi_{\{\Omega\}}\, \nu_\Omega\, \sigma_{\Omega=\Omega_1}\big(x \times \rho_{\Omega_1 \leftarrow \Omega}(x)\big) \;\cup\; \big(\{\Omega\}, \{\emptyset\}\big)$$

Clearly, $E_1(r)$ consists of all singletons of r and the empty set, i.e. of all subsets of r of size at most 1. We now write down an expression $E_2(x)$, defined on relations with scheme $\{\Omega\}$:

$$E_2(x) = \pi_{\{\Omega\}}\, \nu_\Omega\, \mu_\Omega\Big(\sigma_{\Omega=\Omega_1}\big(x \times \rho_{\Omega_1 \leftarrow \Omega}(x) \times \rho_{\Omega_2 \leftarrow \Omega}(x)\big) \cup \sigma_{\Omega=\Omega_2}\big(x \times \rho_{\Omega_1 \leftarrow \Omega}(x) \times \rho_{\Omega_2 \leftarrow \Omega}(x)\big)\Big) \;\cup\; \big(\{\Omega\}, \{\emptyset\}\big)$$

If s consists off all subsets of r up to size i, then $E_2(s)$ consists of all subsets of r up to size $2i$. Since $E_2(x)$ is monotone, we may conclude:

$$\Pi(x) = E_2^* E_1(x)$$ ∎

As an immediate corollary to this result, we have:

Corollary 5.1: *$\mathcal{N}^*$ and $\mathcal{P}^*$ are equivalent, i.e. they express the same class of queries.*

We are now going to show that for each lpae there exists an equivalent pae, thus proving that the lfp nested algebra and the powerset algebra are equivalent. Therefore we need three technical lemmas. The first one is of a notational nature.

Lemma 5.1: *Let $r = (\Omega, \omega)$ be a relation and let $X, X' \in \Omega \setminus U$ be composed attributes. Suppose there exists a permutation φ on U such that $X' = \varphi(X)$. φ is extended to $\mathcal{I}$ in the usual way. Denote by $\sigma^{\varphi}_{X \subseteq X'}(r)$ (or $\sigma_{X \subseteq X'}(r)$ if φ is understood) the relation*

$$\big(\Omega, \{t \in \omega \mid \varphi(t(X)) \subseteq t(X')\}\big)$$

There exists an nae (and hence a pae) that expresses $\sigma^{\varphi}_{X \subseteq X'}(r)$, independent of ω.

Proof: Let φ_1 be a permutation on U such that Ω and $\Omega_1 = \varphi_1(\Omega)$ have no elementary attributes in common. Let $X_1 = \varphi_1(X)$ and let $X_1' = \varphi_1(X')$. We invite the reader to check that the following nae satisfies the requirements of Lemma 5.1:

$$\sigma^{\varphi}_{X \subseteq X'}(r) = \pi_{\Omega}\sigma_{X=X_1}\,\nu_{X_1}\,\pi_{\Omega \cup X_1}\sigma_{X_1 = X_1'}\,\mu_{X_1'}\,\mu_{X_1}\,\sigma_{\Omega=\Omega_1}(x \times \rho_{\Omega_1 \leftarrow \Omega}(x))$$
$$\cup\ \pi_{\Omega}\sigma_{\Omega=\Omega_1}\big(x \times (\{X_1\}, \{\emptyset\})\big) \quad \blacksquare$$

in Lemmas 5.2 and 5.3 we introduce some constructs necessary to prove that the lfp operator can be expressed in the powerset algebra.

Lemma 5.2: *Let $E(x)$ be a pae defined on relations r with scheme Ω. Let Ω^E be the scheme of the resulting relations $E(r)$. Let φ_1 be a one to one mapping from U to itself such that no elementary attribute of Ω is contained in the range of φ_1. For all $X \in \mathcal{U}$, let $X_1 = \varphi_1(X)$. Then there exists a pae $\widetilde{E}(x)$ defined on relations with scheme $\{\Omega\}$ such that for each relation $s = (\{\Omega\}, \omega)$:*

$$\widetilde{E}(s) = \Big(\{\Omega, \Omega_1^E\}, \big\{t \in \mathcal{T}_{\{\Omega,\Omega_1^E\}} \,\big|\, t(\Omega) \in \omega \ \&\ t(\Omega_1^E) = \varphi_1\big(E\big(t(\Omega)\big)\big)\big\}\Big)$$

Proof: By induction on the size of $E(x)$. As an example, let $E(x) = E_1(x) \times E_2(x)$. Let φ_2 be a permutation on U such that $\Omega_2 = \varphi_2(\Omega)$, $\Omega_2^{E_1} = \varphi_2(\Omega_1^{E_1})$ and $\Omega_2^{E_2} = \varphi_2(\Omega_1^{E^2})$ have no elementary attributes in common with Ω, $\Omega_1^{E_1}$ or $\Omega_2^{E_2}$. Then:

$$\widetilde{E}(x) = \pi_{\{\Omega, \Omega_1^{E_1} \cup \Omega_1^{E_1}\}}\,\nu_{\Omega_1^{E_1} \cup \Omega_1^{E_2}}\,\mu_{\Omega_1^{E_1}}\,\mu_{\Omega_1^{E_2}}\,\sigma_{\Omega=\Omega_2}(\widetilde{E}_1(x) \times \rho_{\Omega_2 \leftarrow \Omega}\widetilde{E}_2(x))$$
$$\cup\ \pi_{\{\Omega\}}\sigma_{\Omega_1^{E_1} = \Omega_2^{E_1}}\Big(\widetilde{E}_1(x) \times \big(\{\Omega_2^{E_1}\}, \{\emptyset\}\big)\Big) \times \big(\{\Omega_1^{E_1} \cup \Omega_1^{E_2}\}, \{\emptyset\}\big)$$
$$\cup\ \pi_{\{\Omega\}}\sigma_{\Omega_1^{E_2} = \Omega_2^{E_2}}\Big(\widetilde{E}_2(x) \times \big(\{\Omega_2^{E_2}\}, \{\emptyset\}\big)\Big) \times \big(\{\Omega_1^{E_1} \cup \Omega_1^{E_2}\}, \{\emptyset\}\big)$$

We leave it to the reader to write similar expressions for the other nested relational algebra operators as well as for pae's of size 1, which are all fairly straightforward. Finally, let $E(x) = \Pi E_1(x)$ and let φ_2 be a permutation on U satisfying the same conditions for Ω and $\Omega_1^{E_1}$ as above. Then:

$$\widetilde{E}(x) = \nu_{\{\Omega_1^{E_1}\}}\,\pi_{\{\Omega, \Omega_1^{E_1}\}}\sigma_{\Omega_1^{E_1} \subseteq \Omega_2^{E_1}}\,(\Pi\pi_{\Omega_1^{E_1}}\,\mu_{\Omega_1^{E_1}}\,\widetilde{E}_1(x) \times \rho_{\Omega_2^{E_1} \leftarrow \Omega_1^{E_1}}\,\widetilde{E}_1(x)) \quad \blacksquare$$

Lemma 5.3: *Let $r = (\Omega, \omega)$ be a relation. Define by $\Omega(r)$ the relation:*

$$\big(\{\Omega\}, \{t \in \mathcal{T}_{\{\Omega\}} \mid \textit{all elementary values in } t \textit{ also appear somewhere in } \omega\}\big)$$

There exists a pae $E_{\Omega}(x)$ which is independent of ω such that $E_{\Omega}(r) = \Omega(r)$.

Proof: Rather than writing down the expression $E_\Omega(x)$ which is very involved, we explain how it is constructed. First, we construct an expression which yields a one attribute relation in which all the elementary values of r appear. This expression is obtained by a sequence of unnestings, followed by projection, renaming and union. Let $E_1(x)$ be this expression. Now, consider for each elementary attribute A in Ω the expression $E_A(x)$ yielding a relation over $\{A\}$ obtained from $E_1(x)$ by an appropriate renaming. We now construct $E_\Omega(x)$ inductively as follows. Let $X = \{X_1, \dots, X_k\}$ be a set of attributes. Then $E_X(x) = \Pi\big(E_{X_1}(x) \times \cdots \times E_{X_k}\big)$. Obviously, $E_\Omega(x)$ satisfies the requirements of Lemma 5.3. ∎

We are now ready to state our result:

Theorem 5.2: *Each lpae can be expressed by a pae.*

Proof: Let $E(x)$ be a monotone pae. We show that there exists a pae that expresses $E^*(x)$. Therefore, let $r = (\Omega, \omega)$ be a relation on which $E(x)$ is defined. Let φ_1 be as in Lemma 5.3 and let φ_2 and φ_3 be permutations on U such that Ω, $\Omega_2 = \varphi_2(\Omega)$ and $\Omega_3 = \varphi_3(\Omega)$ have no elementary attributes in common. We now introduce some pae's the last of which expresses $E^*(x)$.

$$E_1(x) = \pi_{\{\Omega\}}\sigma_{\Omega=\Omega_1^{E_1}}\widetilde{E}E_\Omega(x)$$

Clearly, $E_1(r)$ consists of the set of all relations s over Ω satisfying:

1. the elementary values of s occur in r;
2. $E(s) = s$.

Clearly, of all these relations, $E^*(r)$ is the smallest containing r. Therefore, let (cfr. Lemma 4.2):

$$E_2(x) = \pi_{\{\Omega\}}\sigma_{\Omega_2\subseteq\Omega}\big(\rho_{\Omega_2\leftarrow\Omega}N(x) \times E_1(x)\big)$$

$E_2(r)$ consists of all the relations of $E_1(r)$ that contain r. Since the smallest of these is characterized by its containment in all the relations of $E_2(r)$, we finally have:

$$E^*(x) = \mu_\Omega\,\pi_{\{\Omega\}}\sigma_{\Omega_2=\Omega_3}\big(\nu_{\Omega_2}\,\sigma_{\Omega\subseteq\Omega_2}(E_2(x) \times \rho_{\Omega_2\leftarrow\Omega}E_2(x)) \times \nu_{\Omega_3}\,\rho_{\Omega_3\leftarrow\Omega}E_2(x)\big)$$ ∎

As an immediate corollary to Theorems 5.1 and 5.2, we have:

Corollary 5.2: *$\mathcal{P}$, $\mathcal{N}^*$ and $\mathcal{P}^*$ are all equivalent, i.e. they all express the same class of queries.*

6 The Expressiveness of the Lfp Nested Algebra

In this final section, we discuss two issues related to the expressiveness of the lfp closure of the nested relational algebra. First, we would like to make a comment about the transitive closure of a binary relation. Since it can obviously be expressed by an lnae, one might wonder whether the lfp algebra can be generated from the nested relational algebra and the transitive closure. The answer is no, since:

Theorem 6.1: *There is no expression built from the nested relational algebra operators and the transitive closure of a binary relation that expresses the powerset operator.*

Proof: Let $r = (\{A,B\},\omega)$ be a binary relation and let $\tau(r)$ be the transitive closure of r. Then $\tau(r) \subseteq \pi_{\{A\}}(r) \times \pi_{\{B\}}(r)$. The theorem follows from this observational and a combinatorial argument, similar to that of Lemma 4.1, Corollary 4.1 and Theorem 4.1. ∎

So, the lfp operator is significantly more powerful than the transitive closure.

The other issue we want to address in this section concerns a comparison between the expressive power of nested and flat algebras. Indeed, several results point out that there is no fundamental difference between the flat and the nested relational algebra as to this matter [12,20,24]. There is however a major difference between their lfp closures. To see this, consider:

Definition 6.1: *Let $r = (\Omega,\omega)$ be a relation. Then:*

$$\text{even}(r) = \begin{cases} r & \text{if } |r| \text{ is even} \\ (\Omega,\emptyset) & \text{if } |r| \text{ is odd} \end{cases}$$

∎

As pointed out by [8,9], this query applied to flat relations cannot be expressed in the lfp closure of the flat relational algebra. We have though:

Theorem 6.2: *Let $r = (\Omega,\omega)$ be a relation. There is an lpae that expresses* even(r)*, independent of ω.*

Proof: Let φ_1 and φ_2 be permutations on U such that Ω, $\Omega_1 = \varphi_1(\Omega)$ and $\Omega_2 = \varphi_2(\Omega)$ have no elementary attributes in common. First, consider the expression[5]:

$$\begin{aligned} E_1(x) = \pi_{\{\Omega\}}\,\nu_\Omega\big(&\sigma_{\Omega=\Omega_1}\sigma_{\Omega_1\neq\Omega_2}(x \times \rho_{\Omega_1\leftarrow\Omega}(x) \times \rho_{\Omega_2\leftarrow\Omega}(x)) \\ &\cup \sigma_{\Omega=\Omega_2}\sigma_{\Omega_1\neq\Omega_2}(x \times \rho_{\Omega_1\leftarrow\Omega}(x) \times \rho_{\Omega_2\leftarrow\Omega}(x))\big) \;\cup\; (\{\Omega\},\{\emptyset\}) \end{aligned}$$

$E_1(r)$ is the relation over $\{\Omega\}$ that consists of all subrelations of r of even size not greater than 2. Now let $E_2(x)$ be the following expression defined on relations with scheme $\{\Omega\}$ (cfr. Lemma 4.3):

$$\begin{aligned} E_2(x) = \pi_{\{\Omega\}}\,\nu_\Omega\,\mu_\Omega\big(&\sigma_{\Omega=\Omega_1}\sigma_{\Omega_1\cap\Omega_2=\emptyset}(x \times \rho_{\Omega_1\leftarrow\Omega}(x) \times \rho_{\Omega_2\leftarrow\Omega}(x)) \\ &\cup \sigma_{\Omega=\Omega_2}\sigma_{\Omega_1\cap\Omega_2=\emptyset}(x \times \rho_{\Omega_1\leftarrow\Omega}(x) \times \rho_{\Omega_2\leftarrow\Omega}(x))\big) \;\cup\; (\{\Omega\},\{\emptyset\}) \end{aligned}$$

Clearly, if s is the relation over $\{\Omega\}$ consisting of the subrelations of r of even size not greater than $2i$ for some positive integer i, then $E_2(s)$ is the relation over $\{\Omega\}$ consisting of the subrelations of r of even size not greater than $2i+2$. Since $E_2(x)$ is obviously monotone, we can express:

$$E_3(x) = E_2^* E_1(x)$$

$E_3(r)$ is the relation over $\{\Omega\}$ consisting of all subrelations of r of even size. Obviously, we now have (cfr. Lemma 4.2):

$$\text{even}(x) = \pi_\Omega\,\mu_\Omega\,\sigma_{\Omega=\Omega_1}(E_3(x) \times \rho_{\Omega_1\leftarrow\Omega}N(x))$$ ∎

Hence the lfp closure of the nested relational algebra is fundamentally more expressive that the lfp closure of the flat relational algebra.

[5] Note that selection on inequality can be expressed by selection on equality followed by complementation.

7 Conclusion

In this paper, we considered the powerset algebra. Not only does this algebra turn out to be substantially more powerful than the nested relational algebra, it is even equivalent to the lfp closure of the nested relational algebra. Finally, we showed that there is a fundamental difference between the lfp closures of the flat and the nested relational algebra respectively. Now what is causing the lfp nested algebra to be so significantly more powerful than the lfp closure of the flat relational algebra? Basically, we feel that, whereas the flat algebra can express first order logic queries, the powerset algebra can also deal with second order logic queries, since in this algebra, it is possible to manipulate sets. This is we think an issue that certainly needs further investigation.

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