Postscript and Further Reading
(for Pollack chapter)

Since this paper was published in 1991, there has been some further work in the area. Several researchers have focussed simply on the task of inducing finite state machines from data, often using the Tomita datasets or languages as a benchmark (Giles et al 1992; Watrous & Kuhn 1992; Angeline, Saunders & Pollack 1994; Zeng, Goodman & Smyth 1993). In some of this work, the issue of how to prevent the kind of state blurring we observed in this paper has become a central issue, and techniques ranging from symbolic analysis to clustering states, to using discrete state thresholding have been developed.

We've studied the issue of where fractal limit sets, such as the "magic mushroom" of Figure 7g, comes from and have found deeper connections between the dynamics of recurrent networks and iterated function systems, which makes dubious the idea of getting a finite information state idea from nonlinear recurrent networks at all (Kolen 1994a). The fact that there is a deep connection between the dynamics of recurrent networks and fractals, led to work using recurrent networks as the basis for mental imagery (Stucki and Pollack, 1992).

Finally, the hypothesis in the conclusion of the paper, which related the question of where a decision boundary was located in an attractor to what kind of generative complexity such a dynamical recognizer might obtain, has been further developed into question of whether a dynamical system can even have generative capacity on its own. A simple demonstration suffices to show that in the case of physical systems, complexity implicates the observer who establishes the measurement framework for the sequence of symbols and the boundaries between sentences (Kolen & Pollack, 1994). These questions, which go to the heart of the modern synthesis of cognitive science, are fully explored in John Kolen's Ph.D. dissertation (Kolen, 1994b).

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