```
\documentstyle[12pt,basics,cla]{report}
\begin{document}
\newcommand{\Bk}{\it presumptions}
\newcommand{\vp}{\vec{r}}
  \newcommand{\Claopssp}[1]{\Claop{\ssp{#1}}}
\newtheorem{defnn}{Definition}
\newtheorem{propp}{Proposition}
\newtheorem{corr}[propp]{Corollary}
$\;$
\vspace{1.5in}
\setcounter{page}{0}
\begin{center}
\Large
State spaces and non-monotonicity\\
$\;$ \\
\large
Jon Barwise\\
$\;$ \\
\normalsize
Departments of Computer Science,\\ Mathematics, and Philosophy\\
\medskip
 Indiana University\\


\medskip

barwise@indiana.edu
\end{center}

\newpage
Over the past decade and more, I have found myself forced to think about
non-monotonicity on several occassions.  In 1986 I wrote a paper on natural language
conditionals where I found myself arguing  that various kinds of irregularities that go
along with reasoning with conditionals, including non-monotonicity, should be
handled by a pragmatic theory of shifting background conditions.  Over the past three
years, Jerry Seligman and I have written {\it Information Flow in Distributed Systems}
[\ref{bar-sel}].  Part of the problem tackled here is to account for the sorts of exceptions
to law-like regularities associated with information flow but also associated with
non-monotonic reasoning.  Most recently, I have become  interested in the
general problem of reconciling everyday common sense theories with scientific theories.
Again, non-monotonicity becomes an issue one has to face: if human reasoning is basically
non-monotonic and scientific reasoning is classical, and hence monotonic, there seems to
be a serious obstacle to reconciling the two.

This paper weaves
together these three stands of work.  In the first section I introduce the
basic issues and review some machinery from Barwise-Seligman~[\ref{bar-sel}].  In the
second section, I use this to model the logical reasoning implicit in the use of
state spaces in science and applied mathematics, including the use of
background conditions.  Finally, in the third section, I return to compare the model of
background conditions in state space logics with the informal theory of such
conditions I put forward ten years ago, comparing their similarities and differences.


\subsection*{Introduction}

A bumper sticker recently in vogue advocated that one should ``Think globally, act
locally.''   While the sentiment behind this slogan is a worthy one, what it advocates
is, strictly speaking, impossible.  Thinking and reasoning is inherently local, located
in space and time, taking place against certain prevailing background conditions. We are
all reliant on the environment in which we live, and on local background
conditions being in certain ways that are built into our everyday thinking and
reasoning.
```

Sometimes we have to revise our reasoning by recognizing that the background
conditions we have been relying on, often unknowingly, have changed.
I have a certain common sense
understanding of our home's heating system, the furnace, thermostat, vents, and the way
they function to keep the house warm. My understanding gives rise to inferences like the
following.
\begin{quote} The thermostat is set between 65 and 70. (p_1) \\ The room temperature is
58. (p_2)\\ $\entcla{}{}{}$ Hot air is coming out of the vents. (q)
\end{quote}
This reasoning takes place against a set of background conditions and, for most
of the winter does just fine.   But sometimes the reasoning
breaks down. If during a blizzard I add to the
above inference the premise
\begin{quote}
The power is off (p_3)
\end{quote}
then the inference fails: $\nent{p_1, p_2, p_3}{}{q}$.

In this paper I explore a formal, pragmatic account of non-monotonicity. The
basic idea is that reasoning is basically classical but ``local,'' based on certain
background conditions that hold locally, and that failures
cause us to not change the classical forms of reasoning, but rather to change logics by
adapting background conditions in the light of new information. I present this theory
within the context of the theory of local logics developed in
Barwise-Seligman~[\ref{bar-sel}].

\subsubsection*{A review of local logics}

In this section we review   some material from~[\ref{bar-sel}]. (This
material will be spelled out more leisurely in the full version of the paper.)

A {\it (Boolean) classification}
$\cla{A} =
\tuple{S,
\Si,
\models,
\wedge, \neg, }$ consists of a non-empty set S of objects called {\it situations}, a set
$\Si$ of objects called {\it propositions}, a binary relation $\models \subseteq S \times
\Si$, a binary operation $\wedge: \Si \times \Si \rightarrow \Si$, and a
unary operation $\neg: \Si \rightarrow \Si$ satisfying the
following conditions:
\begin{enumerate}
\item $s \models p_1 \wedge p_2$ iff $s \models p_1$ and $s
\models p_2$.
\item $s \models \neg p$ iff $s \not \models p$.
\end{enumerate}
\medskip

In [\ref{bar-sel}], propositions and
situations are called ``types'' and ``tokens,'' respetively.\footnote{The reader who is
sceptical about situations is welcome to think of these situations as possible
worlds.}   [\ref{bar-sel}] adopts the basic Gentzen sequent calculus approach to logic,
so that
$\entcla{\Ga}{}{\De}$ means roughly that the conjunction of the propositions in $\Ga$
entails the disjunction of the propositions in $\De$. This is not crucial to the
account, but I will follow it here. By a {\it sequent}, I mean simply a pair
$\tuple{\Ga, \De}$ where $\Ga$ and $\De$ are sets of propositions.
\medskip

A {\it (Boolean) local logic} $\log{L} = \tuple{\cla{A},
\vdash, N}$ consists of a Boolean classification $\cla{A} = \tuple{S, \Si, \models, \wedge,

```
}$ together with a binary relation
$\vdash$ relating sets of propositions, and a set
$N \subseteq S$ of situations called the {\it normal situations} satisfying the
following additional conditions:
\begin{description}
\item[Entailment] The relation $\vdash$ satisfies all the usual structural and
propositional rules associated with classical propositional logic, including the
rules of Identity, Cut, and {\bf Monotonicity:}\begin{quote} If
$\entcla{\Ga}{}{\De}$,
$\Ga
\subseteq \Ga'$,  and   $\De \subseteq \De'$, then  $\entcla{\Ga'}{}{\De'}$.\footnote{The ot
out in detail in the full paper.} \end{quote}
\item[Normal situations] If $s$ is a normal situation, $\Ga \vdash \De$, and $s \models
p$ for all $p \in \Ga$ then $s \models q$ for some $q \in \De$.
\end{description}
\medskip
The local logic $\log{L}$ is {\it sound} if every $s \in S$ is normal.
$\log{L}$ is {\it complete} if for all sets $\Ga, \De$ of propositions, if $\Ga \not
\vdash
\De$ then there is a normal situation $s$ such that $s \models p$ for every $p \in \Ga$
and $s \models \neg q$ for every $q \in \De$.
\medskip

The partial ordering $\logleq{}{}$ on local logics $\log{L_1}$ and
$\log{L_2}$  on a fixed classification $\cla{A}$
is defined by:
$\log{L_1} \logleq{}{} \log{L_2}$ iff
\begin{enumerate}
\item For all sets $\Ga, \De$ of propositions, $\entlog{\Ga}{L_1}{\De}$ entails
$\entlog{\Ga}{L_2}{\De}$ and
\item every situation of $\cla{A}$ that is normal  in $\log{L_2}$ is  normal
in $\log{L_1}$.
\end{enumerate}
It is important to notice that these inclusions go the opposite directions.  The
more entailments one has the fewer normal situations.  This
reversal is intimately connected with the issue of non-monotonicity. It is not difficult to
local logics on a classification form a complete lattice under this partial
ordering.

In the theory developed in~[\ref{bar-sel}], the central notion is that of an
``infomorphism'' between classifications or between local logics. Space here does not
permit us to go into this notion and its uses here, though they are quite relevant for
the topic under discussion. Rather, in the next section we examine one mechanism that
gives rise to shifting between local logics on a given classification.

\subsection*{State spaces and their logics}

The prevailing methodology in the sciences is that of state spaces. We refer the reader
to Casti~(\ref{casti}) for an excellent two-volume introduction to state spaces and their
use in the physical sciences, life sciences, and social sciences. The overwhelming
success of this methodology in these sciences makes it surprising how little is known
about the relationship between logic and state spaces. It seems worth
investigating this relationship for at least three reasons. First, the proven utility of
the state space approach makes understanding the logical aspects of state space models
interesting in its own right.  Second, it could be that such an approach will allow one,
at least in some instances, to automate inference by means of libraries of numerical
routines.  Finally, and most relevantly for this conference, it could be that we can
learn something about some aspects of non-monotonicity by seeing how the same kinds of
problems are handled in other parts of science.


\begin{defnn} \rm \label{defn.sta}
A {\em state space system} $\cla{S} = \tuple{\Omega, S, state, f_i}_{i
```

```
\in I}$ consists of a set
$\Omega$ of {\it states}, a set $S$ of {\it situations}, a function
$\fun{state}{S}{\Omega}$, and an indexed family of functions
$\fun{f_i}{\Omega}{\cla{R}}$.\footnote{We use
$\cla{R}$ for the set of real numbers. The fact
that observables take real numbers as values is not particularly relevant
for the theory developed here, but it is relevant for its applications. There is
an enormous literature full of applications of state spaces to various problems in
physics, biology, and the social  sciences, all of which use observables taking real
values.  Secondly, the theory developed here suggests a way to reduce common sense
inference problems to calculations that could be implemented using Tarski's decision
procedure for $\cla{R}$.  For these reasons we stick to having real-valued observables.} The
the set of {\it observables} and the function
$f_i$ is called the $i$-th {\it observation function} of the system. If
$\si = state(s)$ then $\si$ is called the {\it state} of $s$ and $f_i(\si)$ is
called the value of the $i$-th observable at $s$.
\end{defnn}

 The intuitive idea is that the state $\si$ of a situation $s$ gives one total
information about the situation, relevant to the issues under investigation, and
that the various observables give one a measure of some aspect of $s$.


\medskip \noindent
{\bf Example:} We use my heating system as a running example and so start by
describing a state space $\ssp{S_{hs}}$ for this heating system. For situations,  take
some set of objects without additional mathematical structure.  Intuitively, these are
to be instances of my complete heating system at various times, including the vents, the
thermostat, the furnace, and the ambient air in the room where the thermostat is
located, and so forth.  For observables, we are going to have the following:
\begin{description}
\item[Thermostat setting:] some real number $r_1$ between 55 and 80.
\item[Temperature] in Fahrenheit of the ambient air at the thermostat: a real number
$r_2$ between 20 and 110.
\item [Power:] $r_3 = $ 1 (on) or 0 (off).
\item [Outside vents:] $r_4 =$ 1 (plugged) or 0 (unplugged).
\item [Operating condition:] $r_5 = $ -1 (cooling), 0 (off), or 1 (heating).
\item [Running:] $r_6 =$ 1 (on) or 0 (not on).
\item [Output air temperature:] a real number $r_7 = $  between 20 and 110.
\end{description}  For states we let $\Omega$ be the set of vectors $\vp =
\tuple{r_1,\ldots,r_7}
\in
\cla{R}^7$ satisfying certain equations and inequalities. (These will be made explicit
in the full version of the paper.) We have seven observation functions, $f_1, \ldots,
f_7$, with
$f_i(\vp) = r_i$.
 We let
$\ssp{S}_{hs}$ consist of these situations and states, with some total
function state mapping situations into states.

\begin{defnn} \rm For any state space $\ssp{S}$, the
Boolean classification $\Claopssp{S}$ is defined by:
\begin{itemize}
\item The situations are those of the state space system $\ssp{S}$.
\item The propositions are sets $p \subseteq \Omega$ of states.
\item A proposition
$p$ is true of a situation $s$, $\oftcla{s}{}{p}$, if $state(s) \in
p$.
\item
$p \wedge q = p \cap q$ and $\neg p = \Omega - p$.
\end{itemize}
\end{defnn}
```

```
\noindent
The propositions used in our example are
represented in $\Claopssp{S_{hs}}$ by:
 \[ \begin{array}{lcl}
p_1 & = & \set{\si \in \Omega_{hs}}{65 \leq f_1(\si)
\leq 70}\\
 p_2 & = & \set{\si \in \Omega_{hs}}{ f_2(\si)
= 58}\\
p_3 & = & \set{\si \in \Omega_{hs}}{ f_3(\si) = 0} \\
q & = & \set{\si \in \Omega_{hs}}{ f_6(\si) = 1 \mbox{ and } f_7(\si) > f_2(\si)}
\end{array}\]

In what follows we assume we are working relative to a fixed state space system
$\ssp{S}$. We want to define a family of local logics on $\Claop{S}$ to model the
way we reason using the state space. Given a set $J$ of observables,
we let
$f_J(\si) =
\tuple{f_i(\si)}_{i \in J}$ be the sequence of values of the observables on the
observables in $J$.  We say that states
$\si_1$ and
$\si_2$ are {\it identical on
$J$}, written
$\si_1 \equiv_J
\si_2$, if
$f_J(\si_1) = f_J(\si_2)$. Notice that $\equiv_J$ is an equivalence relation on states,
and that the larger the set $J$, the finer grained the equivalence relation
$\equiv_J$.

\subsubsection*{Background conditions}



Casti~[\ref{casti}] assumes the set $I$ of observables is partitioned into three
parts,
$$I = Parameters
\cup Input \cup Output.$$  The idea is that the output observables are each
determined by some function of the remaining observables, and the parameters
are held fixed. In our example it would be natural
to take the observables thermostat setting and room temperature as inputs, those of
running and output air temperature as output, and the remaining three (power, outside
vents, and operating condition) as background parameters.  In this paper the distinction
between output and other variables will not play an important role, so we consider
partitions $I = P
\cup J$ where
$P$ is a set of observables thought of as ``parameters'' and
$J$ is a set of ``variable'' observables.  A {\it
parameter setting} on
$P$ is a sequence
$\vp =
\tuple{r_i}_{i
\in P}$ of real numbers, indexed by $P$.
We use $\epsilon$ to denote the unique parameter setting on the empty set of
observables.

\begin{defnn} \rm A {\it background condition} $B = \tuple{P,
V}$ consists of a set $P$ of observables, called the parameters of $B$, and a set $V$ of
parameter settings on
$P$.  Given background conditions $B_0 =
\tuple{P_0, V_0}$ and $ B_1 = \tuple{P_1, V_1}$, we say that $B_1$ {\it strengthens}
$B_0$, written $B_0 \leq B_1$, if $P_0 \subseteq P_1$ and for every parameter setting
$\vp \in V_1$, its restriction  $\res{\vp}{P_0}$ to $P_0$ is in $B_0$.
\end{defnn}
```

The background condition
$B =
\tuple{P, V}$ is the formal analogue of taking the observables in P parameters and
assuming that we are dealing with situations
s such that $f_P(state(s)) \in V$.   (In the case of our motivating example,
intuitively it seems that the inference $\ent{p_1, p_2}{}{q}$ takes place against
the condition that the power is on, the outside vents are unplugged, and the
operating condition is heating. We leave it to the reader to find the mathematical
analogue of this condition.)

\begin{propp} The set of background conditions with the $\leq$ ordering is a complete
lattice.
\end{propp}
\begin{proof} It is easy to see that $\leq$ is a partial ordering. If $X = \{
\tuple{P_k,V_k} \mid k \in K \}$ then
\[\begin{array}{lcl} \bigvee X & = & \tuple{ \bigcup_{k \in K} P_k, \{ \vp \mid
\res{\vp}{P_k}
\in V_k
\mbox{ for all } k \in K \} }\\ \bigwedge X & = & \tuple{ \bigcap_{k \in K} P_k, \{
\res{\vp}{\bigcap_{k \in K} P_k}
\mid
\vp \in V_k
\mbox{ for some } k \in K \} }, \end{array} \] as is not difficult to verify.
\end{proof}

The smallest background condition in this ordering, namely $T = \tuple{\emptyset,
\{\epsilon\} }$, puts no restrictions on any observables.
The largest background condition is $F = \tuple{ I, \emptyset}$; this condition is, of
course, impossible to satisfy.

For any background condition $B = \tuple{P,V}$, say that a state $\si$ {\it satisfies}
B if $f_P(\alpha) \in V$, and we say that a situation s satisfies B if $\sta{s}{}$
satisfies B. Thus every situation satisfies T and no situation satisfies
F. Notice that if $B \leq B'$ then every state or situation that satisfies B' also
satisfies B. We say that a background condition B is non-vacuous
if there is some state $\si$ that satisfies it.   We now come to the main definition
of this paper.

\begin{defnn} \rm For each background condition B
define the {\em local logic $\log{L}(B)$ supported by B} to the the logic on
$\Claopssp{S}$ given by:
\begin{itemize}
\item If $\Ga, \De$ are
sets of propositions, $\ent{\Ga}{B}{\De}$ iff for every
state
$\si$ satisfying B, if $\si \in p$ for all $p \in \Ga$ then $\si \in q$ for some
$q
\in
\De$.
\item The normal situations are those situations that satisfy B.
\end{itemize}
We let $\log{L}(\ssp{S})$ be the local logic corresponding to the smallest background
condition T.
\end{defnn}

This definition codifies the intuition
behind taking a background condition for granted.   It is not difficult to see that
$\log{L}(B)$ is a Boolean local logic, in particular, that the consequence relation
$\ent{}{B}{}$ satisfies all the rules of classical (infinitary) propositional
logic, including Monotonicity.

```
\begin{propp}  Let $B$ be a background condition.
\begin{enumerate}
\item The logic $\log{L}(B)$ is sound iff every situation satisfies $B$.  In particular, the
logic $\log{L}(\ssp{S})$ is sound.
\item The logic $\log{L}(B)$ is complete iff every state that satisfies $B$ is the state
of some situation.
\end{enumerate}
\end{propp}
\begin{proof}
The first is just a restatement of the definitions. The second is almost as obvious.
\end{proof}

In general the logic $\log{L}(B)$ is not sound. The most unsound local logic of all
is the strongest,
$\log{L}(F)$ since $\ent{\Ga}{F}{\De}$, for every
$\Ga$ and $\De$, and there are no normal situations.

Being classical, these logics have the
property that an entailment $\ent{\Ga}{B}{\De}$ can hold vacuously, simply because there
are no states satisfying $B$ that are in all the propositions in $\Ga$.  We say that
$\ent{\Ga}{B}{\De}$ {\it non-vacuously}, and write $\Ga
\Rightarrow_B\De$, if $\ent{\Ga}{B}{\De}$ and there are states satisfying $B$ that
are in all the propositions in $\Ga$.  (This does not necessarily entail that
all the propositions in $\Ga$ are true of some situation, let alone of some normal
situation.)

The aim in the remainder of this paper is to examine various mechanisms that allow us to
compare logics based on different background conditions.  The following result
corresponds to the intuition that the more you assume in the way of background
conditions, the more you will be able to infer, but the less applicable will be your
results.

\begin{propp} \label{prop.mono} Let $B_1, B_2$ be background conditions with $B_1 \leq
B_2$. Then
$\logleq{\log{L}(B_1)}{\log{L}(B_2)}$; that is, if
$\ent{\Ga}{B_1}{\De}$ then $\ent{\Ga}{B_2}{\De}$, and every normal situation
of $\log{L}(B_2)$ a normal situation of $\log{L}(B_1)$.
\end{propp}
Note, by contrast, that  $\Ga \Rightarrow_{B_1} \De$ does not in general imply
 $\Ga \Rightarrow_{B_2} \De$. The reason is that the stronger conditions
imposed by
$B_2$ might result in $\ent{\Ga}{B_2}{\De}$ holding vacuously.

Another way to state the first of the two conclusions of this proposition, one
that is perhaps more immediately relevant to the phenomena that inspires non-monotonic
logic, is as follows: if $\ent{\Ga}{B}{\De}$ but $\nent{\Ga'}{B'}{\De'}$, where $\Ga
\subseteq
\Ga'$ and $\De \subseteq \De'$ then $B \not \leq B'$.  In other words, on this way of
looking at things, the reason the logically valid rule of monotonicity {\it seems} to
fail is because the implicit background conditions of the conclusions are not contained
in the background conditions of the premise.


 We would like to know when  we can go from reasoning in a strong logic
to reasoning in a weaker logic, one that has fewer background assumptions.  There are
a couple of conditions under which this is possible.  Any background condition $P$ gives ris
follows:
\[ [B] \quadeq \{ \si \in \Omega
\mid \si \mbox{ satisfies } B  \} \]
It is clear that if $B_1 \leq B_2$ then $[B_2] \subseteq [B_1]$, that $[T] = \Omega
$ and that $[F]= \emptyset$.  Define a sequent
$\tuple{\Ga,
\De}$ to {\it enforce
```

B_2 given B_1} iff
$$\bigcap \Ga \cap [B_1] \subseteq \bigcup \De \cup [B_2].$$ Intuitively, this
means that the sequent makes no claims on the parameters of B_2 that are not
justified by the background condition B_1.

\begin{propp} \label{prop-reduction}
Let $B_1 \leq B_2$ and let $\tuple{\Ga, \De}$ be any
sequent that enforces B_2 given B_1. If
$\ent{\Ga}{B_2}{\De}$ then $\ent{\Ga}{ B_1}{\De}$.
\end{propp}

\begin{proof} We could give a straightforward proof, but it is more instructive to go by way
the following useful and natural observation. First, note that if B is a background
condition, then $[B]$ is a proposition, and that for all sets $\Ga, \De$ of
propositions, $$\ent{\Ga}{ B}{\De} \mbox{ iff }\ent{\Ga, [B]}{\ssp{S}}{\De}.$$
 Now assume
$\ent{\Ga}{ B_2}{\De}.$ Then
$\ent{\Ga , [B_2] }{\ssp{S}}{\De}.$ But since $\tuple{\Ga, \De}$ is a sequent
that enforces B_2 given B_1, we have
$\ent{\Ga, [B_1] }{\ssp{S}}{\De, [B_2]}.$ By Cut (one of the structural rules
that holds in all local logics), this gives
$\ent{\Ga, [B_1] }{\ssp{S}}{\De},$
which is equivalent to our desired conclusion $\ent{\Ga}{ B_1}{\De}.$
\end{proof}

Enforcement relative to a sequent is a rather difficult condition to check. In
practise, the following notion is much more intuitive and useful.

Let J_1 and J_2 be sets of observables. J_2 is {\it independent
of J_1 given} B if the observables in J_1 can vary arbitrarily, maintaining the
background condition B, without effecting the observables in J_2; more
precisely, if for every state
$\si$ satisfying B and every parameter setting $\vp
\in rng(f_{J_1})$ there is a state $\si'$ satisfying B such that that $f_{J_1}(\si') =
\vp$ and
$f_{J_2}(\si') = f_{J_2}(\si)$. In systems where the observables are
naturally divided into inputs and outputs, it is not uncommon to find that the inputs are
independent of one another.   (In our heating system, every subset of the
first five observables is independent of the remaining five.)

A proposition p {\it lives on a set} J of observables if $\si \in p$ and $\si
\equiv_J
\si'$ implies $\si' \in p$. (The propositions p_1, p_2 in our motivating
example live on the set J of the first two observables, but p_3 does not.)

\begin{propp} Let B be a non-vacuous background condition such that
$\ent{\Ga}{B}{\De}$. If the parameters P of B are independent of some set J of
observables, given some background condition B_0, and every proposition in
$\Ga
\cup
\De$ lives on J, then $\ent{\Ga}{B_0}{\De}$.
\end{propp}
\begin{proof} Let $B= \tuple{P, V}$ and assume $\ent{\Ga}{B}{\De}$. Let $\si$ be a state
satisfying
B_0, $\si \in \bigcap \Ga$. We need to show $\si \in \bigcup \De$. Since B
is non-vacuous, there is a parameter setting $\vp \in V \cap rng(f_P)$. Since P is
independent of J given B_0, there is a state $\si'$ such that $f_P(\si') = \vp$
and
$f_{J}(\si') = f_{J}(\si)$. Since every proposition in
$\Ga$ lives on J, $\si' \in \bigcap \Ga$. But since $f_P(\si') = \vp$, $\si'$
satisfies B, so, by the assumption, $\si' \in \bigcup \De$. But then since every proposit
$\De$ lives on J, $\si \in \bigcup \De$, as desired.
\end{proof}

```latex
The next result entails that there is always a weakest background condition $B$ that
makes a given sequent a theorem of the logic $\log{L}(B)$.

\begin{propp}  Let $X$ be a non-empty set of background conditions.  If
$\ent{\Ga}{B}{\De}$ for each $B \in X$ then $\ent{\Ga}{\wedge X}{\De}$.  \end{propp}
\begin{proof} Assume $\ent{\Ga}{ B}{\De}$ for each $B \in X$.  Then
$\ent{\Ga, [B]}{\ssp{S}}{\De}$ for each $B \in X$, by the lemma. Hence
$\ent{\Ga, \wedge X}{\ssp{S}}{\De}$ by the conjunction laws.   Hence
$\ent{\Ga}{\wedge X}{\De}$. \end{proof}

\begin{corr}
For every sequent $\tuple{\Ga, \De}$ there is a least background condition $B$ such that
$\ent{\Ga}{ B}{\De}$.
\end{corr}
\begin{proof} Let $X$ be the set of all background conditions $B'$ such that
$\ent{\Ga}{ B'}{\De}$. $X$ is non-empty since $F \in X$. Let $B =
\wedge X$.  By the proposition, $B \in X$ so $B$ is least.
\end{proof}

Pragmatically, weaking fails when an added premise contradicts the least background
against which an entailment can be established.  In the case of our motivating example,
the least
$B$ such that
$\ent{p_1, p_2}{B}{q}$, is easily seen to be the condition that the power is on, the
outside vents are unplugged, and the operating condition is heating, and $p_3$
contradicts this background condition.


Space limitations prevent us from including any other results along these lines.

\subsection*{Retrospective}

In Barwise~[\ref{bar1}], I argued that the properties of natural
language conditionals that have seemed to call for different rules of logic should be
handled by a pragmatic theory of shifting background conditions. I gave a set of five
informal axioms on such background conditions, and argued that these five axioms were
indicated by semantic phenomena involving conditionals.

More recently, after I developed the model of background conditions and
their logics presented above, I remembered my old paper and so went back to see
how well its theory and the present model fit.   Of the five axioms postulated there,
four hold in the model developed  here.  (The fifth had to do with setting parameters in
propositions and so does not really make sense in the current context.)  Here are the
four axioms that hold, together with a justification of each based on the above results.

\begin{enumerate}
\item {\it If $p \Rightarrow_B q$ and $q \Rightarrow_B r$ then $p \Rightarrow_B r$.}
\item[] This is a special case of Cut in the logic $\log{L}(B)$.
\item {\it If $p \Rightarrow_B q$ and $B \leq B'$ where $B'$ is compatible with $p$
then $p \Rightarrow_{B'} q$.}  \item[]  This is a consequence of
Proposition~\ref{prop.mono}.
\item {\it If $p \Rightarrow_B q$ then $B$ is compatible with $p$.} \item[] This
follows from the definition of $\Rightarrow_B$.
\item {\it If  $p \Rightarrow_B q$ and the situation $s$ satisfies $B$ then $p
\Rightarrow q$ is actual. } \item[]  The term ``actual'' is not defined in this paper,
but we could take it to mean the sequent $\ent{p}{}{q}$ holds of the situation $s$.
Then the result follows from the fact that the hypothesis insures us that the
situation $s$ is a normal situation in the logic $\log{L}(B)$.
\end{enumerate}
```

```
Whether this nice fit between theory and model is evidence for both, or for the
thought that the author was under the same misguided ideas in 1986 and 1996 we leave the
reader to decide.  We close with a quote from the first chapter in Casti~[\ref{casti}]
and the inspiration for the theory presented in this paper:
\begin{quote}
\small
error or surprise always involves
a discrepancy between the objects [situations] {\it open} to interaction and the
abstractions [states] {\it closed} to those same interactions.  In
principle, the remedy for closing this gap is equally clear: augment the description
by including more observables to account for the unmodeled interactions.
(Casti~[\ref{casti}], p. 25)
\end{quote}


\subsection*{References}
```


```
\begin{enumerate}
\item \label{bar1} Jon Barwise, ``Conditionals and conditional information,'' in {\it On
Conditionals}, ed. by Traugott {\it et. al.}, Cambridge University Press, 1986.
Reprinted in {\it The Situation in Logic}, by Jon Barwise, CSLI Lecture Notes No 17,
1987.
\item \label{bar-sel} Jon Barwise and Jerry Seligman, {\it Information Flow in Distributed
Systems}, Cambridge University Press, to appear.
\item \label{casti} John Casti, {\it Reality Rules, vols. I, II}, Wiley Interscience, 1992
\end{enumerate}
```


```
\end{document}
```