Program M

$\{\ x = A \wedge y = B\}$
begin
z := 0;
$\{\ z + xy = AB\}$
while $x \neq 0$ do
  if $even?(x)$
  then
    begin x := $\frac{1}{2}x$; y := $2y$ end
  else
    begin x := $x - 1$; z := $z + y$ end
end
$\{\ z = AB\}$

Program M – Abbreviations

$\{\ x = A \wedge y = B\}$
begin
z := 0;
$\{\ z + xy = AB\}$
while $x \neq 0$ do
  if $even?(x)$
  then
    begin x := $\frac{1}{2}x$; y := $2y$ end
  else
    begin x := $x - 1$; z := $z + y$ end
end
$\{\ z = AB\}$

$$\begin{aligned} \textsc{Pre} &\equiv x = A \wedge y = B \\ \textsc{Inv} &\equiv z + xy = AB \\ \textsc{Post} &\equiv z = AB \end{aligned}$$

Program M – Abbreviations

```
{ Pre}
begin
z := 0;
{ Inv}
while x ≠ 0 do
   if even?(x)
   then
      begin x := ½x ; y := 2y end
   else
      begin x := x − 1 ; z := z + y end
end
{ Post}
```

$$\begin{aligned} \textsc{Pre} &\equiv x = A \wedge y = B \\ \textsc{Inv} &\equiv z + xy = AB \\ \textsc{Post} &\equiv z = AB \end{aligned}$$

Program M – More Abbreviations

```
{ Pre}
begin
z := 0;
{ Inv}
while Test do
   if even?(x)
   then
      Then
   else
      Else
end
{ Post}
```

$$
\begin{aligned}
\textsc{Pre} &\equiv x = A \wedge y = B \\
\textsc{Inv} &\equiv z + xy = AB \\
\textsc{Post} &\equiv z = AB \\
\textsc{Test} &\equiv x \neq 0 \\
\textsc{Then} &\equiv \texttt{begin } x := \tfrac{1}{2}x \texttt{ ; } y := 2y \texttt{ end} \\
\textsc{Else} &\equiv \texttt{begin } x := x - 1 \texttt{ ; } z := z + y \texttt{ end}
\end{aligned}
$$

Program M – More Abbreviations

```
{ Pre}
begin
z := 0;
while Test inv { Inv } do Body
end
{ Post}
```

$$
\begin{array}{rcl}
\textsc{Pre} & \equiv & x = A \wedge y = B \\
\textsc{Inv} & \equiv & z + xy = AB \\
\textsc{Post} & \equiv & z = AB \\
\textsc{Test} & \equiv & x \neq 0 \\
\textsc{Then} & \equiv & \texttt{begin } x := \frac{1}{2}x \texttt{ ; } y := 2y \texttt{ end} \\
\textsc{Else} & \equiv & \texttt{begin } x := x - 1 \texttt{ ; } z := z + y \texttt{ end} \\
\textsc{Body} & \equiv & \texttt{if } \mathit{even}?(x) \texttt{ then } \textsc{Then} \texttt{ else } \textsc{Else}
\end{array}
$$

Program M – Synthesis of Verification Conditions

```
{ Pre}
begin
z := 0;
while Test inv { Inv } do Body
end
{ Post}
```

$$
\begin{array}{rcl}
\textsc{Pre} & \equiv & x = A \wedge y = B \\
\textsc{Inv} & \equiv & z + xy = AB \\
\textsc{Post} & \equiv & z = AB \\
\textsc{Test} & \equiv & x \neq 0 \\
\textsc{Then} & \equiv & \texttt{begin } x := \frac{1}{2}x \texttt{ ; } y := 2y \texttt{ end} \\
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\textsc{Body} & \equiv & \texttt{if } even?(x) \texttt{ then } \textsc{Then} \texttt{ else } \textsc{Else} \\
\textsc{Loop} & \equiv & \texttt{while } \textsc{Test} \texttt{ inv } \{ \textsc{Inv} \} \texttt{ do } \textsc{Body}
\end{array}
$$

Goals:

→ (1) $\{\textsc{Pre}\}$ `begin` z `:=` 1 `;` $\textsc{Loop}$ `end` $\{\textsc{Post}\}$

$$\frac{\{P\}\ S_1\ \{I\} \quad \bullet \quad \{I\}\ \texttt{while}\ T\ \texttt{do}\ \{I\}\ S_2\ \{Q\}}{\{P\}\ \texttt{begin}\ S_1\ \texttt{;}\ \texttt{while}\ T\ \texttt{do}\ \{I\}\ S_2\ \texttt{end}\ \{Q\}}$$

①

Program M – Synthesis of Verification Conditions

```
{ Pre}
begin
z := 0;
while Test inv { Inv } do Body
end
{ Post}
```

$$
\begin{array}{rcl}
\textsc{Pre} & \equiv & x = A \wedge y = B \\
\textsc{Inv} & \equiv & z + xy = AB \\
\textsc{Post} & \equiv & z = AB \\
\textsc{Test} & \equiv & x \neq 0 \\
\textsc{Then} & \equiv & \texttt{begin } x := \frac{1}{2}x \texttt{ ; } y := 2y \texttt{ end} \\
\textsc{Else} & \equiv & \texttt{begin } x := x - 1 \texttt{ ; } z := z + y \texttt{ end} \\
\textsc{Body} & \equiv & \texttt{if } even?(x) \texttt{ then } \textsc{Then} \texttt{ else } \textsc{Else} \\
\textsc{Loop} & \equiv & \texttt{while } \textsc{Test} \texttt{ inv } \{ \textsc{Inv} \} \texttt{ do } \textsc{Body}
\end{array}
$$

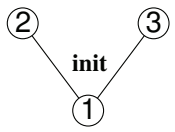
Goals:

(1) $\{\textsc{Pre}\}$ `begin` z `:=` 1 `;` Loop `end` $\{\textsc{Post}\}$

→ (2) $\{\textsc{Pre}\}$ z `:=` 1 $\{\textsc{Inv}\}$

(3) $\{\textsc{Inv}\}$ Loop $\{\textsc{Post}\}$

$$\frac{P \Rightarrow Q\left[\begin{smallmatrix} T \\ v \end{smallmatrix}\right]}{\{P\}\ v := T\ \{Q\}}$$



Program M – Synthesis of Verification Conditions

```
{ Pre}
begin
z := 0;
while Test inv { Inv } do Body
end
{ Post}
```

$$
\begin{array}{rcl}
\textsc{Pre} & \equiv & x = A \wedge y = B \\
\textsc{Inv} & \equiv & z + xy = AB \\
\textsc{Post} & \equiv & z = AB \\
\textsc{Test} & \equiv & x \neq 0 \\
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\textsc{Else} & \equiv & \texttt{begin } x := x - 1 \texttt{ ; } z := z + y \texttt{ end} \\
\textsc{Body} & \equiv & \texttt{if } even?(x) \texttt{ then } \textsc{Then} \texttt{ else } \textsc{Else} \\
\textsc{Loop} & \equiv & \texttt{while } \textsc{Test} \texttt{ inv } \{ \textsc{Inv} \} \texttt{ do } \textsc{Body}
\end{array}
$$

Goals:

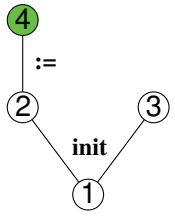
(1) $\{\textsc{Pre}\}$ `begin` z `:=` 1 `;` $\textsc{Loop}$ `end` $\{\textsc{Post}\}$

(2) $\{\textsc{Pre}\}$ z `:=` 1 $\{\textsc{Inv}\}$

→ (3) $\{\textsc{Inv}\}$ $\textsc{Loop}$ $\{\textsc{Post}\}$

(4) $\{\textsc{Pre}\} \Rightarrow \textsc{Inv}\begin{bmatrix}1\\z\end{bmatrix}$

$$\frac{\{I \wedge T\}\ S\ \{I\} \quad \bullet \quad I \wedge \neg T \Rightarrow Q}{\{I\}\ \texttt{while}\ T\ \texttt{do}\ \{I\}\ S\ \{Q\}}$$



Program M – Synthesis of Verification Conditions

```
{ Pre}
begin
z := 0;
while Test inv { Inv } do Body
end
{ Post}
```

$$
\begin{array}{rcl}
\textsc{Pre} & \equiv & x = A \wedge y = B \\
\textsc{Inv} & \equiv & z + xy = AB \\
\textsc{Post} & \equiv & z = AB \\
\textsc{Test} & \equiv & x \neq 0 \\
\textsc{Then} & \equiv & \texttt{begin } x := \frac{1}{2}x \texttt{ ; } y := 2y \texttt{ end} \\
\textsc{Else} & \equiv & \texttt{begin } x := x - 1 \texttt{ ; } z := z + y \texttt{ end} \\
\textsc{Body} & \equiv & \texttt{if } even?(x) \texttt{ then } \textsc{Then} \texttt{ else } \textsc{Else} \\
\textsc{Loop} & \equiv & \texttt{while } \textsc{Test} \texttt{ inv } \{ \textsc{Inv} \} \texttt{ do } \textsc{Body}
\end{array}
$$

Goals:

(1) $\{\textsc{Pre}\}$ `begin` z `:=` 1 `;` $\textsc{Loop}$ `end` $\{\textsc{Post}\}$

(2) $\{\textsc{Pre}\}$ z `:=` 1 $\{\textsc{Inv}\}$

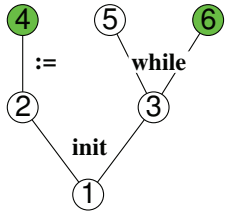
(3) $\{\textsc{Inv}\}$ $\textsc{Loop}$ $\{\textsc{Post}\}$

(4) $\{\textsc{Pre}\} \Rightarrow \textsc{Inv}\begin{bmatrix}1\\z\end{bmatrix}$

→ (5) $\{\textsc{Inv} \wedge x \neq 0\}$ $\textsc{Body}$ $\{\textsc{Inv}\}$

(6) $\textsc{Inv} \wedge (x = 0) \Rightarrow \textsc{Post}$

$$\frac{\{P \wedge T\}\ S_1\ \{Q\} \quad \bullet \quad \{P \wedge \neg T\}\ S_2\ \{Q\}}{\{P\}\ \texttt{if } T \texttt{ then } S_1 \texttt{ else } S_2\ \{Q\}}$$



Program M – Synthesis of Verification Conditions

```
{ Pre}
begin
z := 0;
while Test inv { Inv } do Body
end
{ Post}
```

$$
\begin{array}{rcl}
\textsc{Pre} & \equiv & x = A \wedge y = B \\
\textsc{Inv} & \equiv & z + xy = AB \\
\textsc{Post} & \equiv & z = AB \\
\textsc{Test} & \equiv & x \neq 0 \\
\textsc{Then} & \equiv & \texttt{begin } x := \tfrac{1}{2}x \texttt{ ; } y := 2y \texttt{ end} \\
\textsc{Else} & \equiv & \texttt{begin } x := x - 1 \texttt{ ; } z := z + y \texttt{ end} \\
\textsc{Body} & \equiv & \texttt{if } even?(x) \texttt{ then } \textsc{Then} \texttt{ else } \textsc{Else} \\
\textsc{Loop} & \equiv & \texttt{while } \textsc{Test} \texttt{ inv } \{ \textsc{Inv} \} \texttt{ do } \textsc{Body}
\end{array}
$$

Goals:

(1) $\{\textsc{Pre}\}$ `begin` z `:=` 1 `;` $\textsc{Loop}$ `end` $\{\textsc{Post}\}$

(2) $\{\textsc{Pre}\}$ z `:=` 1 $\{\textsc{Inv}\}$

(3) $\{\textsc{Inv}\}$ $\textsc{Loop}$ $\{\textsc{Post}\}$

(4) $\{\textsc{Pre}\} \Rightarrow \textsc{Inv}\begin{bmatrix}1\\ z\end{bmatrix}$

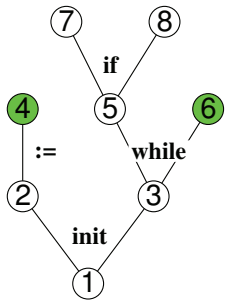
(5) $\{\textsc{Inv} \wedge x \neq 0\}$ $\textsc{Body}$ $\{\textsc{Inv}\}$

(6) $\textsc{Inv} \wedge x = 0 \Rightarrow \textsc{Post}$

→ (7) $\{\textsc{Inv} \wedge x \neq 0 \wedge even?(x)\}$ $\textsc{Then}$ $\{\textsc{Post}\}$

(8) $\{\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x)\}$ $\textsc{Else}$ $\{\textsc{Post}\}$

$$\frac{\{P\}\ S\ \{Q\begin{bmatrix}T\\ v\end{bmatrix}\}}{\{P\}\ \texttt{begin}\ S\ \texttt{;}\ v\ \texttt{:=}\ T\ \texttt{end}\ \{Q\}}$$



Program M – Synthesis of Verification Conditions

```
{ Pre}
begin
z := 0;
while Test inv { Inv } do Body
end
{ Post}
```

$$
\begin{array}{rcl}
\textsc{Pre} & \equiv & x = A \wedge y = B \\
\textsc{Inv} & \equiv & z + xy = AB \\
\textsc{Post} & \equiv & z = AB \\
\textsc{Test} & \equiv & x \neq 0 \\
\textsc{Then} & \equiv & \texttt{begin } x := \tfrac{1}{2}x \texttt{ ; } y := 2y \texttt{ end} \\
\textsc{Else} & \equiv & \texttt{begin } x := x - 1 \texttt{ ; } z := z + y \texttt{ end} \\
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\textsc{Loop} & \equiv & \texttt{while } \textsc{Test} \texttt{ inv } \{ \textsc{Inv} \} \texttt{ do } \textsc{Body}
\end{array}
$$

Goals:

(1) $\{\textsc{Pre}\}$ `begin` z `:=` 1 `;` $\textsc{Loop}$ `end` $\{\textsc{Post}\}$

(2) $\{\textsc{Pre}\}$ z `:=` 1 $\{\textsc{Inv}\}$

(3) $\{\textsc{Inv}\}$ $\textsc{Loop}$ $\{\textsc{Post}\}$

(4) $\{\textsc{Pre}\} \Rightarrow \textsc{Inv}\begin{bmatrix}1\\ z\end{bmatrix}$

(5) $\{\textsc{Inv} \wedge x \neq 0\}$ $\textsc{Body}$ $\{\textsc{Inv}\}$

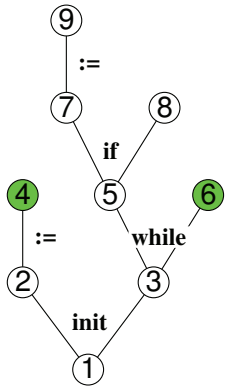
(6) $\textsc{Inv} \wedge x = 0 \Rightarrow \textsc{Post}$

(7) $\{\textsc{Inv} \wedge x \neq 0 \wedge even?(x)\}$ $\textsc{Then}$ $\{\textsc{Post}\}$

→ (8) $\{\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x)\}$ $\textsc{Else}$ $\{\textsc{Post}\}$

(9) $\{\textsc{Inv} \wedge x \neq 0 \wedge even?(x)\}$ x `:=` $\frac{1}{2}x$ $\{\textsc{Post}\begin{bmatrix}2y\\ y\end{bmatrix}\}$

$$\frac{\{P\}\ S\ \{Q\begin{bmatrix}T\\ v\end{bmatrix}\}}{\{P\}\ \texttt{begin}\ S\ \texttt{;}\ v\ \texttt{:=}\ T\ \texttt{end}\ \{Q\}}$$



Program M – Synthesis of Verification Conditions

```
{ Pre}
begin
z := 0;
while Test inv { Inv } do Body
end
{ Post}
```

$$
\begin{array}{rcl}
\textsc{Pre} & \equiv & x = A \wedge y = B \\
\textsc{Inv} & \equiv & z + xy = AB \\
\textsc{Post} & \equiv & z = AB \\
\textsc{Test} & \equiv & x \neq 0 \\
\textsc{Then} & \equiv & \texttt{begin } x := \tfrac{1}{2}x \texttt{ ; } y := 2y \texttt{ end} \\
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\textsc{Loop} & \equiv & \texttt{while } \textsc{Test} \texttt{ inv } \{ \textsc{Inv} \} \texttt{ do } \textsc{Body}
\end{array}
$$

Goals:

(1) $\{\textsc{Pre}\}$ `begin` z `:=` 1 `;` $\textsc{Loop}$ `end` $\{\textsc{Post}\}$

(2) $\{\textsc{Pre}\}$ z `:=` 1 $\{\textsc{Inv}\}$

(3) $\{\textsc{Inv}\}$ $\textsc{Loop}$ $\{\textsc{Post}\}$

(4) $\{\textsc{Pre}\} \Rightarrow \textsc{Inv}\begin{bmatrix}1\\ z\end{bmatrix}$

(5) $\{\textsc{Inv} \wedge x \neq 0\}$ $\textsc{Body}$ $\{\textsc{Inv}\}$

(6) $\textsc{Inv} \wedge x = 0 \Rightarrow \textsc{Post}$

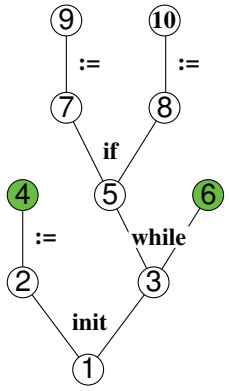
(7) $\{\textsc{Inv} \wedge x \neq 0 \wedge even?(x)\}$ $\textsc{Then}$ $\{\textsc{Post}\}$

(8) $\{\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x)\}$ $\textsc{Else}$ $\{\textsc{Post}\}$

→ (9) $\{\textsc{Inv} \wedge x \neq 0 \wedge even?(x)\}$ x `:=` $\tfrac{1}{2}x$ $\{\textsc{Post}\begin{bmatrix}2y\\ y\end{bmatrix}\}$

(10) $\{\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x)\}$ x `:=` $x - 1$ $\{\textsc{Post}\begin{bmatrix}z+y\\ z\end{bmatrix}\}$

$$\frac{\{P\}\ S\ \{Q\begin{bmatrix}T\\ v\end{bmatrix}\}}{\{P\}\ \texttt{begin}\ S\ \texttt{;}\ v\ \texttt{:=}\ T\ \texttt{end}\ \{Q\}}$$



Program M – Synthesis of Verification Conditions

```
{ Pre}
begin
z := 0;
while Test inv { Inv } do Body
end
{ Post}
```

$$
\begin{array}{rcl}
\textsc{Pre} & \equiv & x = A \wedge y = B \\
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\textsc{Else} & \equiv & \texttt{begin } x := x - 1 \texttt{ ; } z := z + y \texttt{ end} \\
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\textsc{Loop} & \equiv & \texttt{while } \textsc{Test} \texttt{ inv } \{ \textsc{Inv} \} \texttt{ do } \textsc{Body}
\end{array}
$$

Goals:

(1) $\{\textsc{Pre}\}$ `begin` $z :=$ `1 ;` $\textsc{Loop}$ `end` $\{\textsc{Post}\}$

(2) $\{\textsc{Pre}\}\ z := 1\ \{\textsc{Inv}\}$

(3) $\{\textsc{Inv}\}\ \textsc{Loop}\ \{\textsc{Post}\}$

(4) $\{\textsc{Pre}\} \Rightarrow \textsc{Inv}\begin{bmatrix}1\\ z\end{bmatrix}$

(5) $\{\textsc{Inv} \wedge x \neq 0\}\ \textsc{Body}\ \{\textsc{Inv}\}$

(6) $\textsc{Inv} \wedge x = 0 \Rightarrow \textsc{Post}$

(7) $\{\textsc{Inv} \wedge x \neq 0 \wedge even?(x)\}\ \textsc{Then}\ \{\textsc{Post}\}$

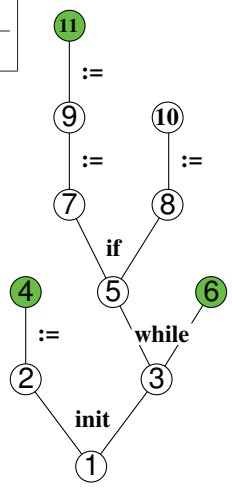
(8) $\{\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x)\}\ \textsc{Else}\ \{\textsc{Post}\}$

(9) $\{\textsc{Inv} \wedge x \neq 0 \wedge even?(x)\}\ x := \frac{1}{2}x\ \{\textsc{Post}\begin{bmatrix}2y\\ y\end{bmatrix}\}$

→ (10) $\{\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x)\}\ x := x - 1\ \{\textsc{Post}\begin{bmatrix}z+y\\ z\end{bmatrix}\}$

(11) $\textsc{Inv} \wedge x \neq 0 \wedge even?(x) \Rightarrow \textsc{Post}\begin{bmatrix}2y\\ y\end{bmatrix}\begin{bmatrix}\frac{1}{2}x\\ x\end{bmatrix}$

$$\frac{P \Rightarrow Q\begin{bmatrix}T\\ v\end{bmatrix}}{\{P\}\ v := T\ \{Q\}}$$



Program M – Synthesis of Verification Conditions

```
{ Pre}
begin
z := 0;
while Test inv { Inv } do Body
end
{ Post}
```

$$
\begin{array}{rcl}
\textsc{Pre} & \equiv & x = A \wedge y = B \\
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\end{array}
$$

Goals:

(1) $\{\textsc{Pre}\}$ `begin` z `:=` 1 `;` $\textsc{Loop}$ `end` $\{\textsc{Post}\}$

(2) $\{\textsc{Pre}\}$ z `:=` 1 $\{\textsc{Inv}\}$

(3) $\{\textsc{Inv}\}$ $\textsc{Loop}$ $\{\textsc{Post}\}$

(4) $\{\textsc{Pre}\} \Rightarrow \textsc{Inv}\begin{bmatrix}1\\ z\end{bmatrix}$

(5) $\{\textsc{Inv} \wedge x \neq 0\}$ $\textsc{Body}$ $\{\textsc{Inv}\}$

(6) $\textsc{Inv} \wedge x = 0 \Rightarrow \textsc{Post}$

(7) $\{\textsc{Inv} \wedge x \neq 0 \wedge even?(x)\}$ $\textsc{Then}$ $\{\textsc{Post}\}$

(8) $\{\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x)\}$ $\textsc{Else}$ $\{\textsc{Post}\}$

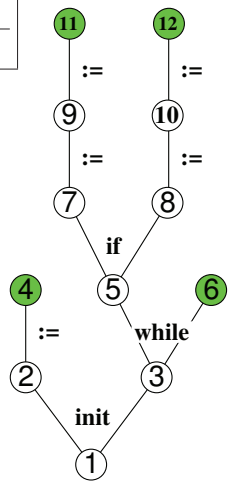
(9) $\{\textsc{Inv} \wedge x \neq 0 \wedge even?(x)\}$ x `:=` $\frac{1}{2}x$ $\{\textsc{Post}\begin{bmatrix}2y\\ y\end{bmatrix}\}$

(10) $\{\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x)\}$ x `:=` $x - 1$ $\{\textsc{Post}\begin{bmatrix}z+y\\ z\end{bmatrix}\}$

(11) $\textsc{Inv} \wedge x \neq 0 \wedge even?(x) \Rightarrow \textsc{Post}\begin{bmatrix}2y\\ y\end{bmatrix}\begin{bmatrix}\frac{1}{2}x\\ x\end{bmatrix}$

(12) $\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x) \Rightarrow \textsc{Post}\begin{bmatrix}z+y\\ z\end{bmatrix}\begin{bmatrix}x-1\\ x\end{bmatrix}$

$$\frac{P \Rightarrow Q\begin{bmatrix}T\\ v\end{bmatrix}}{\{P\}\ v := T\ \{Q\}}$$



Program M – Synthesis of Verification Conditions

```
{ Pre}
begin
z := 0;
while Test inv { Inv } do Body
end
{ Post}
```

$$
\begin{array}{rcl}
\textsc{Pre} & \equiv & x = A \wedge y = B \\
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\textsc{Test} & \equiv & x \neq 0 \\
\textsc{Then} & \equiv & \texttt{begin } x := \tfrac{1}{2}x \texttt{ ; } y := 2y \texttt{ end} \\
\textsc{Else} & \equiv & \texttt{begin } x := x - 1 \texttt{ ; } z := z + y \texttt{ end} \\
\textsc{Body} & \equiv & \texttt{if } even?(x) \texttt{ then } \textsc{Then} \texttt{ else } \textsc{Else} \\
\textsc{Loop} & \equiv & \texttt{while } \textsc{Test} \texttt{ inv } \{ \textsc{Inv} \} \texttt{ do } \textsc{Body}
\end{array}
$$

Goals:

(1) $\{\textsc{Pre}\}$ `begin` z `:=` 1 `;` $\textsc{Loop}$ `end` $\{\textsc{Post}\}$

(2) $\{\textsc{Pre}\}$ z `:=` 1 $\{\textsc{Inv}\}$

(3) $\{\textsc{Inv}\}$ $\textsc{Loop}$ $\{\textsc{Post}\}$

(4) $\{\textsc{Pre}\} \Rightarrow \textsc{Inv}\begin{bmatrix}1\\ z\end{bmatrix}$

(5) $\{\textsc{Inv} \wedge x \neq 0\}$ $\textsc{Body}$ $\{\textsc{Inv}\}$

(6) $\textsc{Inv} \wedge x = 0 \Rightarrow \textsc{Post}$

(7) $\{\textsc{Inv} \wedge x \neq 0 \wedge even?(x)\}$ $\textsc{Then}$ $\{\textsc{Post}\}$

(8) $\{\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x)\}$ $\textsc{Else}$ $\{\textsc{Post}\}$

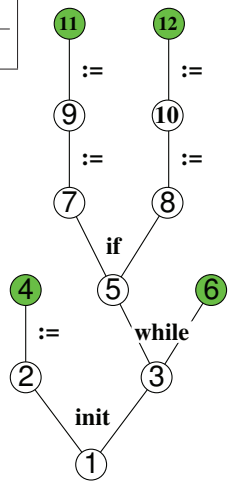
(9) $\{\textsc{Inv} \wedge x \neq 0 \wedge even?(x)\}$ x `:=` $\frac{1}{2}x$ $\{\textsc{Post}\begin{bmatrix}2y\\ y\end{bmatrix}\}$

(10) $\{\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x)\}$ x `:=` $x - 1$ $\{\textsc{Post}\begin{bmatrix}z+y\\ z\end{bmatrix}\}$

(11) $\textsc{Inv} \wedge x \neq 0 \wedge even?(x) \Rightarrow \textsc{Post}\begin{bmatrix}2y\\ y\end{bmatrix}\begin{bmatrix}\frac{1}{2}x\\ x\end{bmatrix}$

(12) $\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x) \Rightarrow \textsc{Post}\begin{bmatrix}z+y\\ z\end{bmatrix}\begin{bmatrix}x-1\\ x\end{bmatrix}$

$$\frac{P \Rightarrow Q\begin{bmatrix}T\\ v\end{bmatrix}}{\{P\}\ v := T\ \{Q\}}$$



Program M – Translation of Verification Conditions

```
{ Pre}
begin
z := 0;
while Test inv { Inv } do Body
end
{ Post}
```

$$
\begin{array}{rcl}
\textsc{Pre} & \equiv & x = A \wedge y = B \\
\textsc{Inv} & \equiv & z + xy = AB \\
\textsc{Post} & \equiv & z = AB \\
\textsc{Test} & \equiv & x \neq 0 \\
\textsc{Then} & \equiv & \texttt{begin } x := \frac{1}{2}x \texttt{ ; } y := 2y \texttt{ end} \\
\textsc{Else} & \equiv & \texttt{begin } x := x - 1 \texttt{ ; } z := z + y \texttt{ end} \\
\textsc{Body} & \equiv & \texttt{if } even?(x) \texttt{ then } \textsc{Then} \texttt{ else } \textsc{Else} \\
\textsc{Loop} & \equiv & \texttt{while } \textsc{Test} \texttt{ inv } \{ \textsc{Inv} \} \texttt{ do } \textsc{Body}
\end{array}
$$

Goals:

(1) $\{\textsc{Pre}\}$ `begin` z `:=` 1 `;` $\textsc{Loop}$ `end` $\{\textsc{Post}\}$

(2) $\{\textsc{Pre}\}$ z `:=` 1 $\{\textsc{Inv}\}$

(3) $\{\textsc{Inv}\}$ $\textsc{Loop}$ $\{\textsc{Post}\}$

(4) $\{\textsc{Pre}\} \Rightarrow \textsc{Inv}\begin{bmatrix}1\\ z\end{bmatrix}$

(5) $\{\textsc{Inv} \wedge x \neq 0\}$ $\textsc{Body}$ $\{\textsc{Inv}\}$

(6) $\textsc{Inv} \wedge x = 0 \Rightarrow \textsc{Post}$

(7) $\{\textsc{Inv} \wedge x \neq 0 \wedge even?(x)\}$ $\textsc{Then}$ $\{\textsc{Post}\}$

(8) $\{\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x)\}$ $\textsc{Else}$ $\{\textsc{Post}\}$

(9) $\{\textsc{Inv} \wedge x \neq 0 \wedge even?(x)\}$ x `:=` $\frac{1}{2}x$ $\{\textsc{Post}\begin{bmatrix}2y\\ y\end{bmatrix}\}$

(10) $\{\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x)\}$ x `:=` $x - 1$ $\{\textsc{Post}\begin{bmatrix}z+y\\ z\end{bmatrix}\}$

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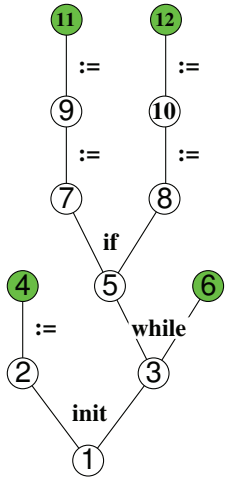

Verification Conditions:

(4) $\{\textsc{Pre}\} \Rightarrow \textsc{Inv}\begin{bmatrix}1\\ z\end{bmatrix}$

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Goals:

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(3) $\{\textsc{Inv}\}$ $\textsc{Loop}$ $\{\textsc{Post}\}$

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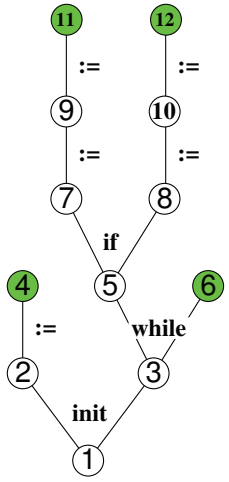
(12) $\textsc{Inv} \wedge x \neq 0 \wedge \neg even?(x) \Rightarrow \textsc{Post}\begin{bmatrix}z+y\\ z\end{bmatrix}\begin{bmatrix}x-1\\ x\end{bmatrix}$



Verification Conditions:

(4) $x = A$ and $y = B$ imply $0 + xy = AB$

(6) $z + xy = AB$ and $x = 0$ imply $z = AB$

(11) $z + xy = AB$ and $x \neq 0$ and $even?(x)$ imply $z + \left(\frac{1}{2}x\right)(2y) = AB$

(12) $z + xy = AB$ and $x \neq 0$ and $odd?(x)$ imply $(z + y) + (x - 1)y = AB$