

INFERENCE RULES

Strengthening Rules

$$\frac{P \Rightarrow P' \bullet \{P'\} S \{Q'\} \bullet Q' \Rightarrow Q}{\{P\} S \{Q\}}$$

Assignment Rule

$$\frac{P \Rightarrow Q[v^t]}{\{P\} v := t \{Q\}} \quad Q[v^t] \text{ denotes substitution of term } t \text{ for free occurrences of variable } v \text{ in } Q$$

Compound Rule

$$\frac{\{P\} S_1 \{Q\} \bullet \{Q\} S_2 \{R\}}{\{P\} \text{ begin } S_1 ; S_2 \text{ end } \{R\}}$$

Conditional Rule

$$\frac{\{P \wedge B\} S_1 \{Q\} \bullet \{P \wedge \neg B\} S_2 \{Q\}}{\{P\} \text{ if } B \text{ then } S_1 \text{ else } S_2 \{Q\}}$$

While Rule

$$\frac{\{P \wedge B\} S \{P\}}{\{P\} \text{ while } B \text{ do } S \{P \wedge \neg B\}}$$

Useful While Rule

$$\frac{P \Rightarrow I \bullet \{I \wedge B\} S \{I\} \bullet (I \wedge \neg B) \Rightarrow Q}{\{P\} \text{ while } B \text{ inv } I \text{ do } S \{Q\}}$$

Assignment Elimination (Right)

$$\frac{\{P \wedge (v = t)\} S \{Q\}}{\{P\} \text{ begin } v := t ; S \text{ end } \{Q\}} \quad \text{if } v \text{ does not occur free in } P \text{ or } t$$

Assignment Elimination (Left)

$$\frac{\{P\} S \{Q[v^t]\}}{\{P\} \text{ begin } S ; v := t \text{ end } \{Q\}}$$

Loop Initialization

$$\frac{\{P\} S_1 \{I\} \bullet \{I\} \text{ while } B \text{ inv } I \text{ do } S_2 \{Q\}}{\{P\} \text{ begin } S_1 ; \text{ while } B \text{ inv } I \text{ do } S_2 \{Q\}}$$

Conditional Elimination

$$\frac{\{P\} S_1 \{B \Rightarrow Q \wedge \neg B \Rightarrow Q'\} \bullet \{Q\} S_2 \{R\} \bullet \{Q'\} S_3 \{R\}}{\{P\} \text{ begin } S_1 ; \text{ if } B \text{ then } S_2 \text{ else } S_3 \{R\}}$$