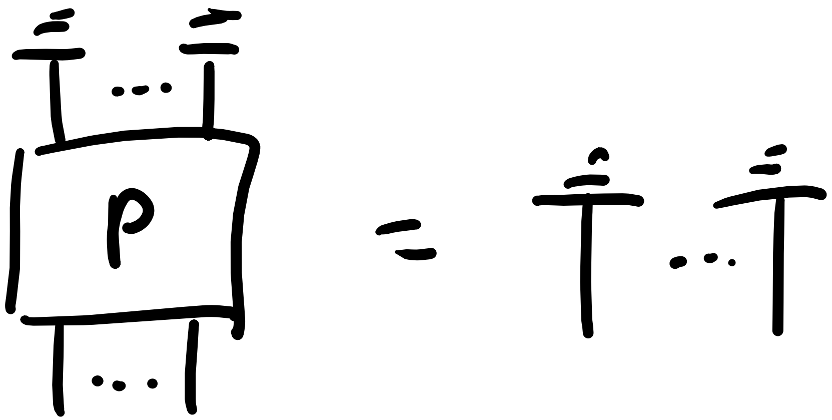
Quantum in Pictures Lecture Series

Lecturer: Stefano Gogioso

Fri 30 June 2023 –Morning Lecture

INDIANA UNIVERSITY BLOOMINGTON



P is causal

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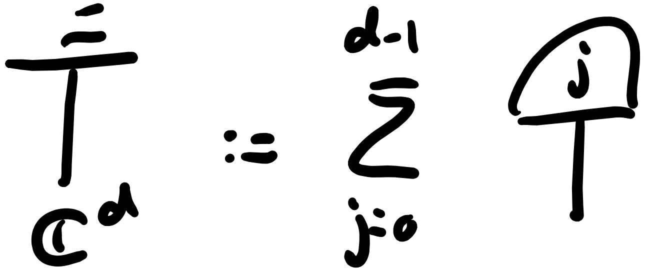
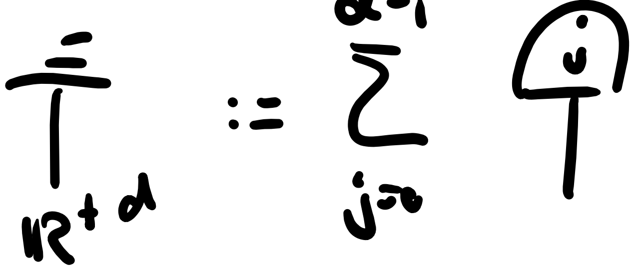
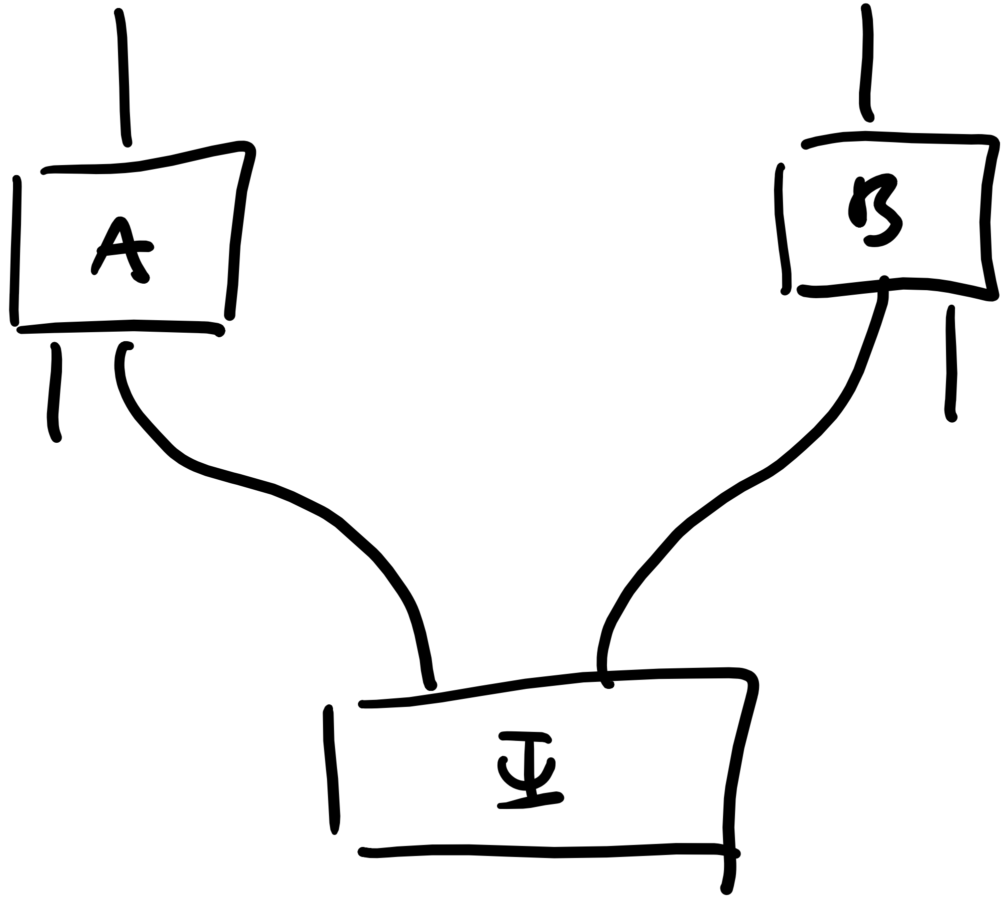
e.g.

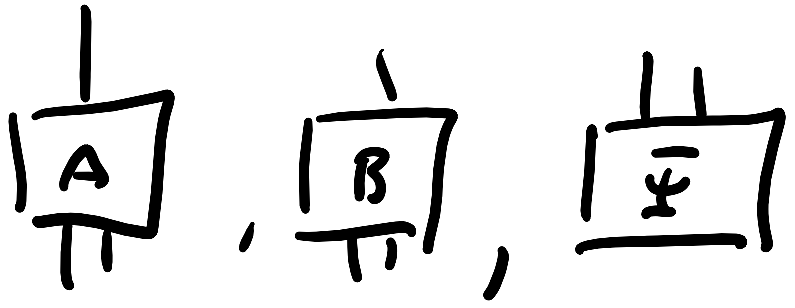


$= \mathrm{Tr}[\,\_\,] \qquad (|j\rangle)_{j=0}^{d-1}$ ONB



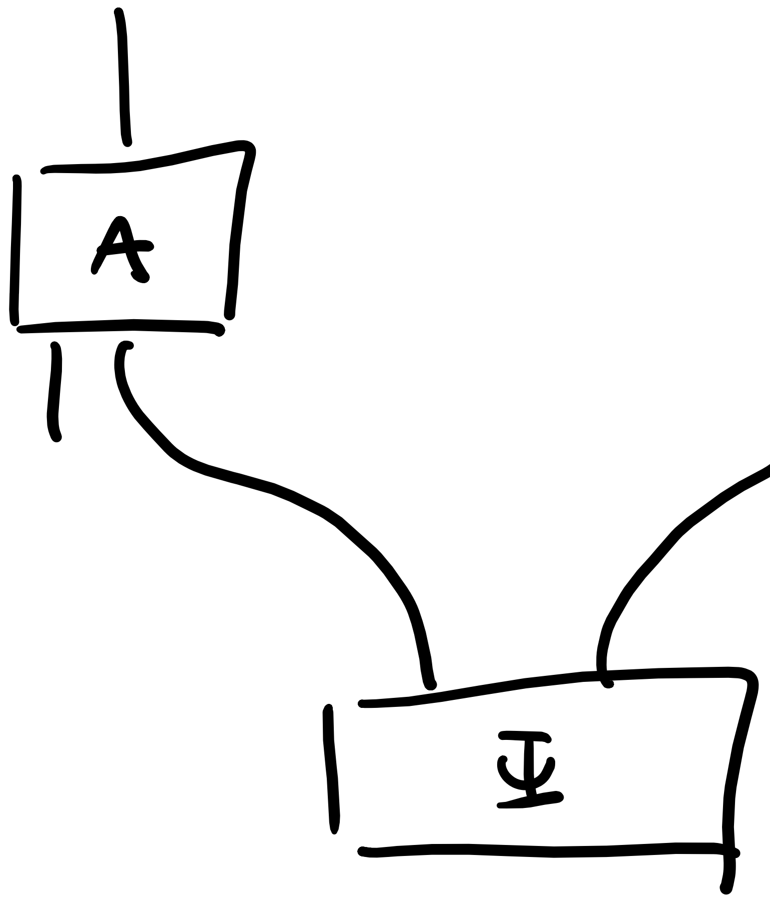
$j = 0, \dots, d-1$ (deterministic states)

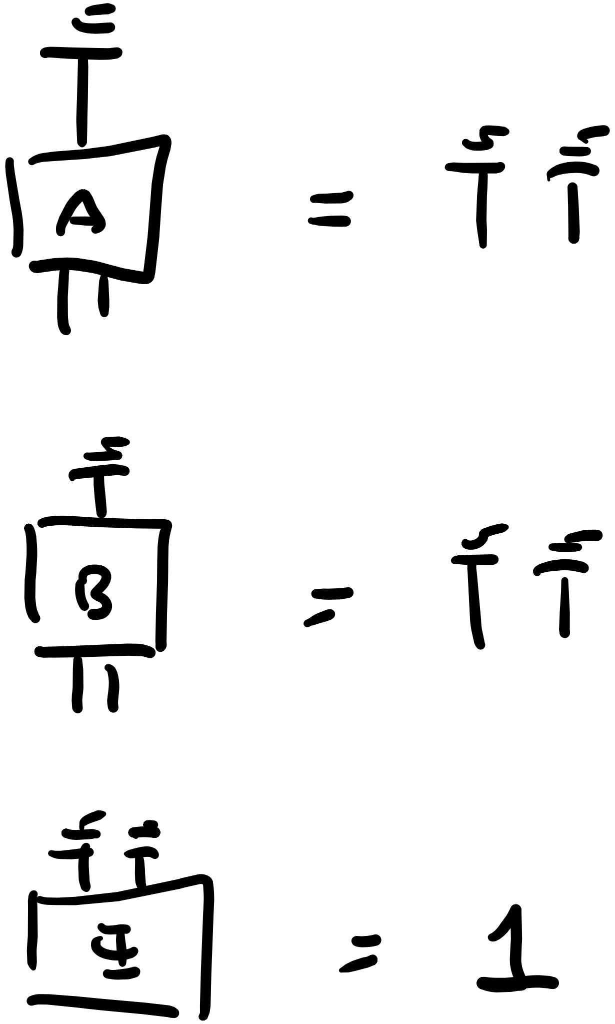


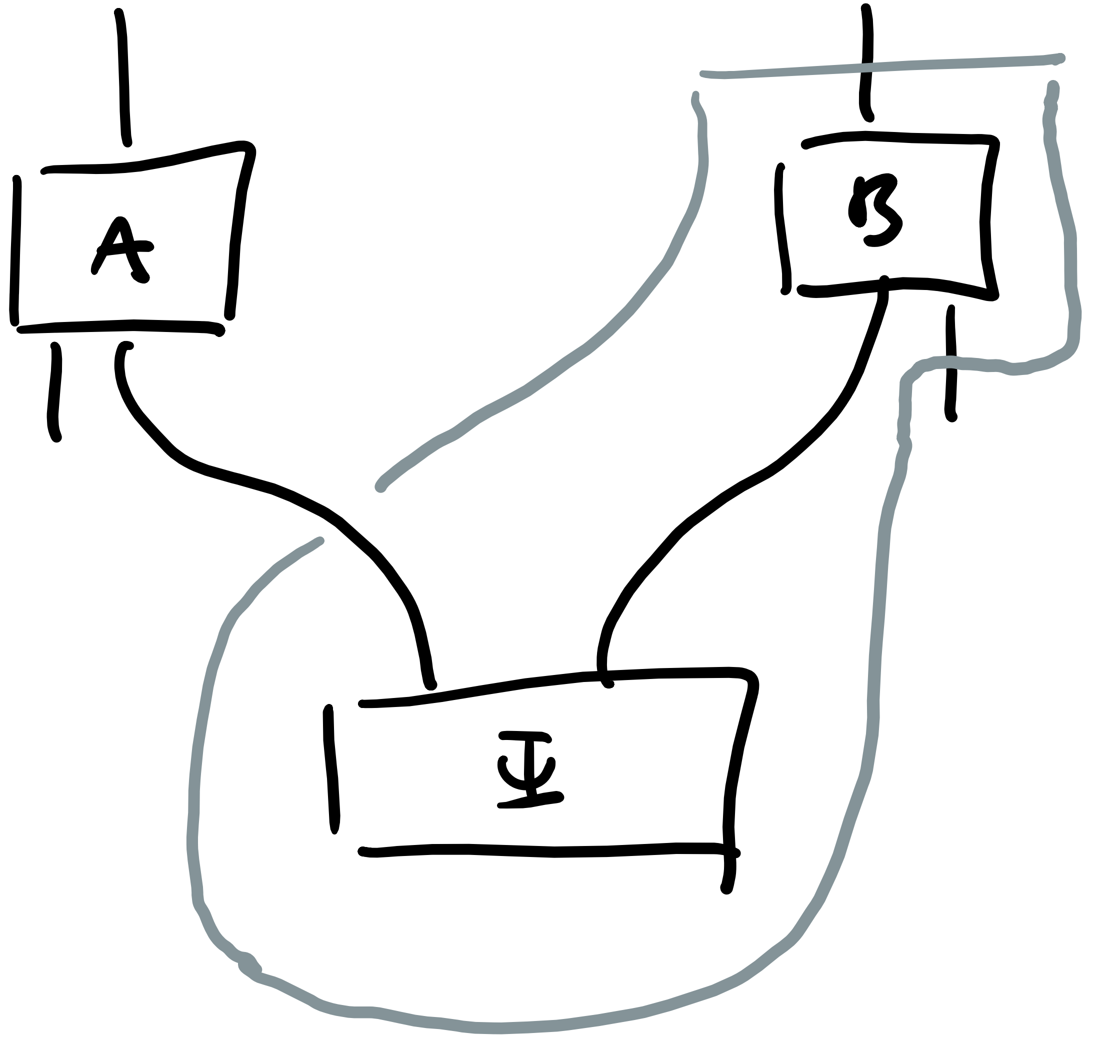


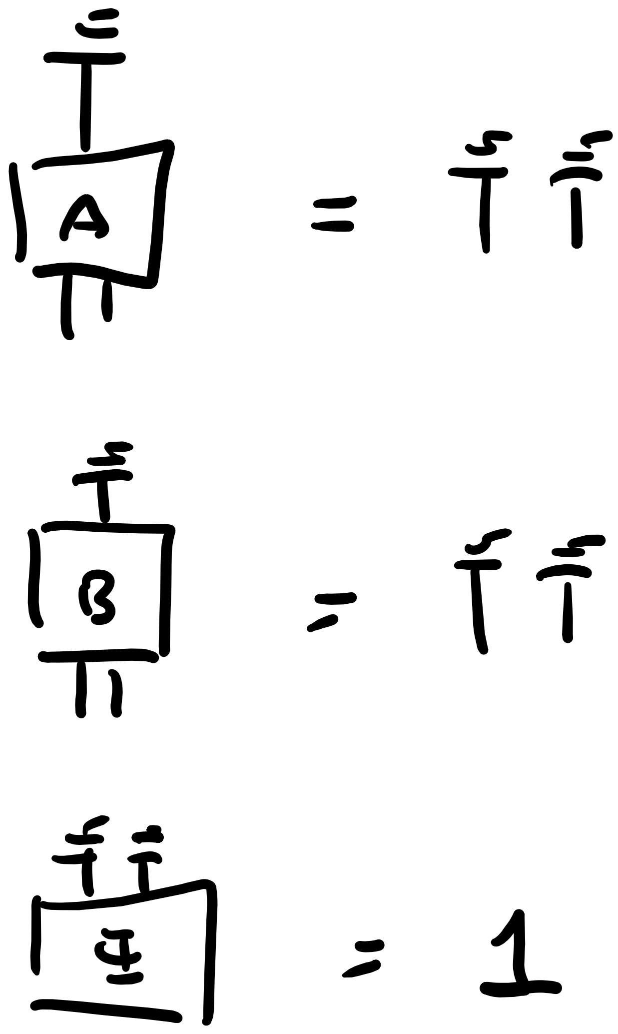
are all causal
(sure-boxes , "deterministic")

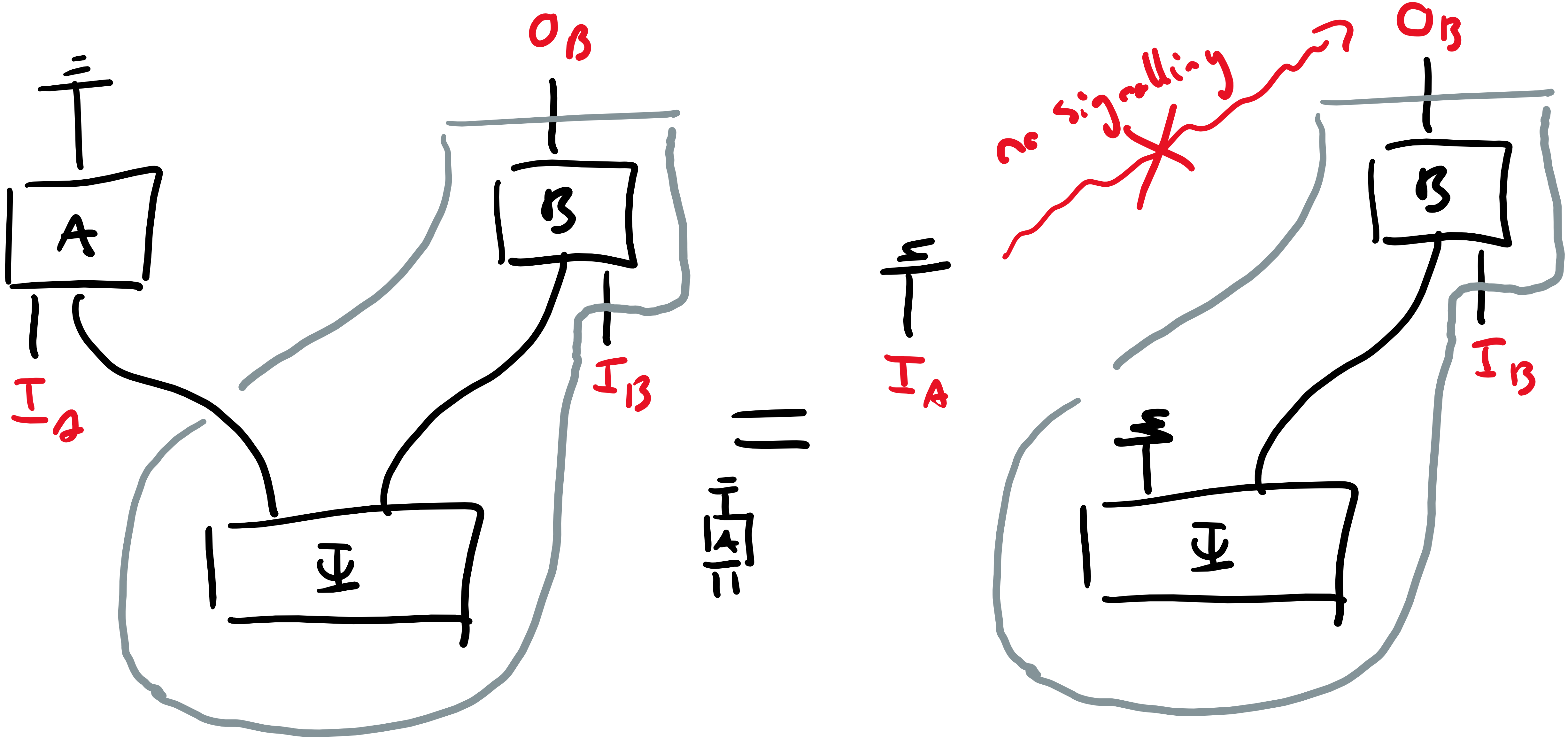
Diagrams forming a partial order (no cycles)

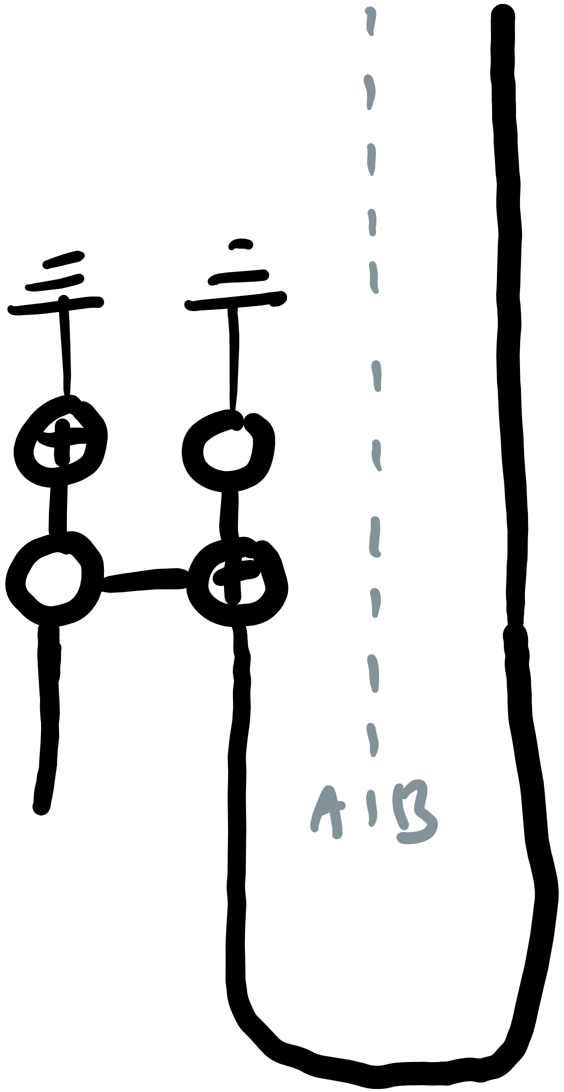

A
B
Φ

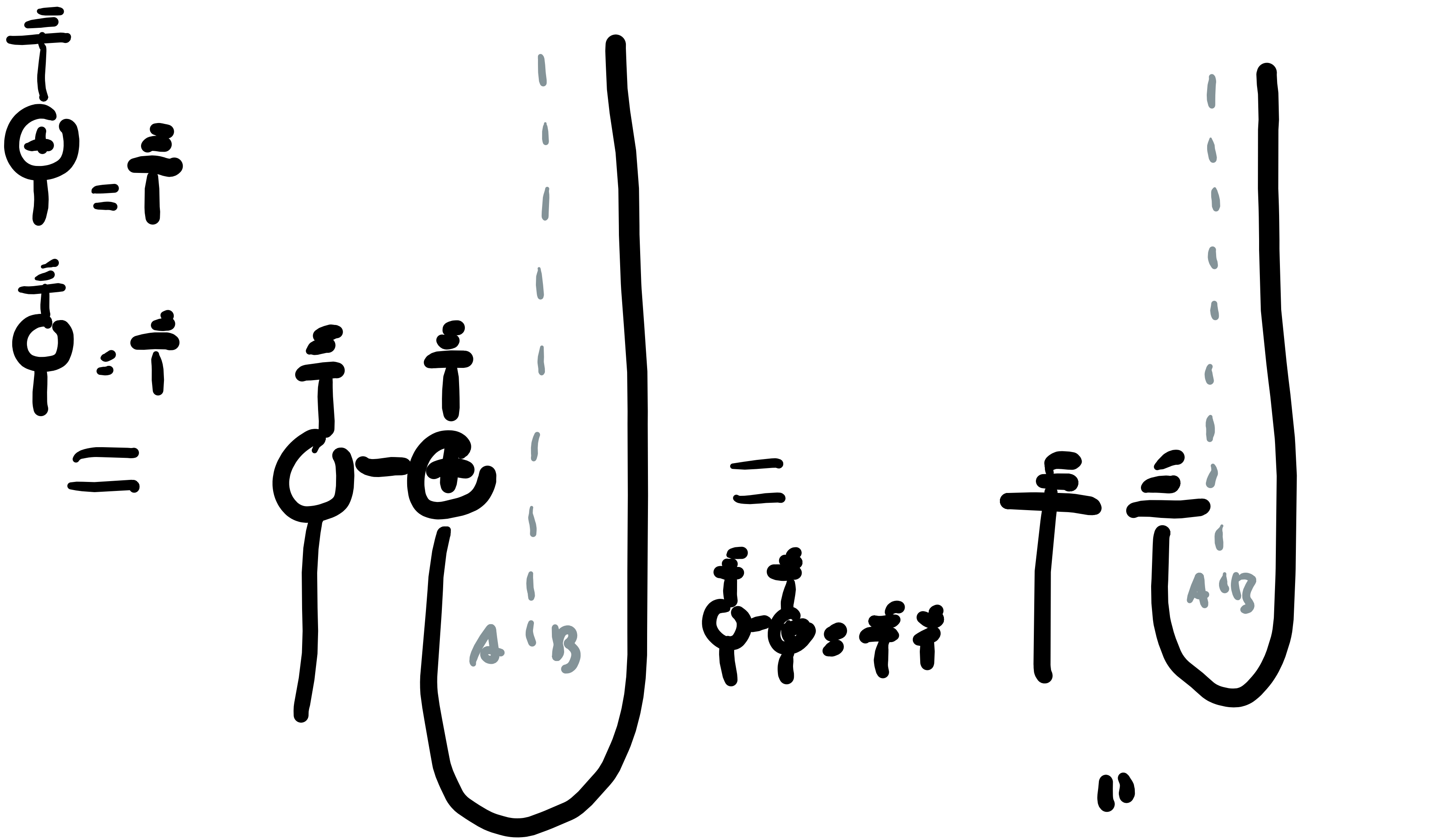

A
=
B
=
Φ
= 1

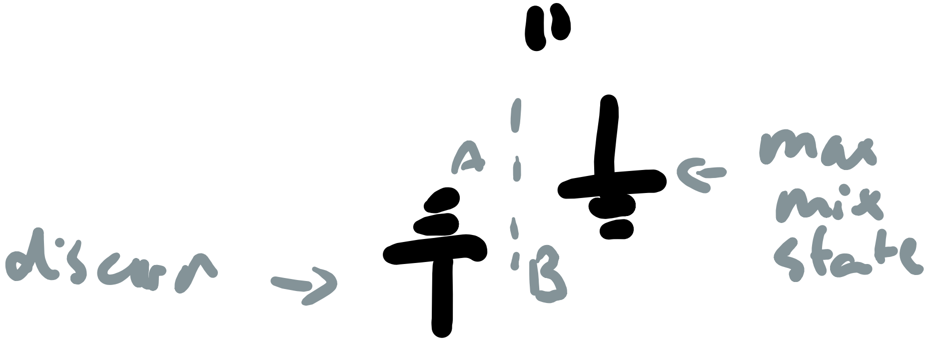

A
B
Ψ


A
=
B
=
Ψ
= 1


A
B
Ψ
O_B
I_A
I_B
=
no signalling

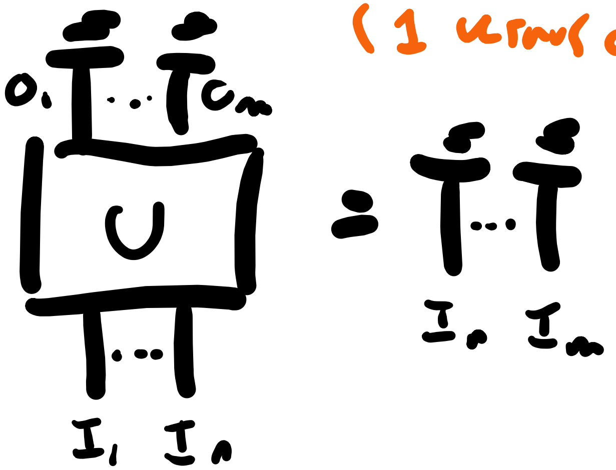






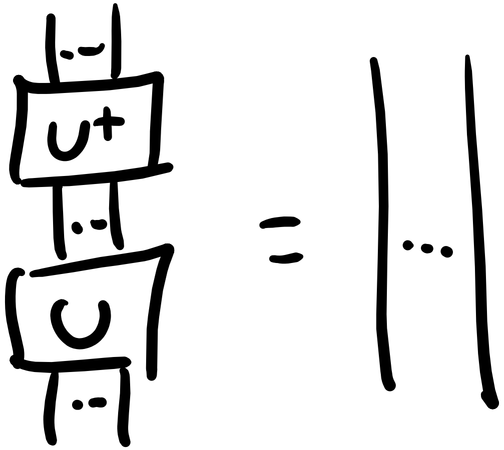
Local state at Bob's without comms from Alice

CP maps which are CPTP
(1 Kraus op)

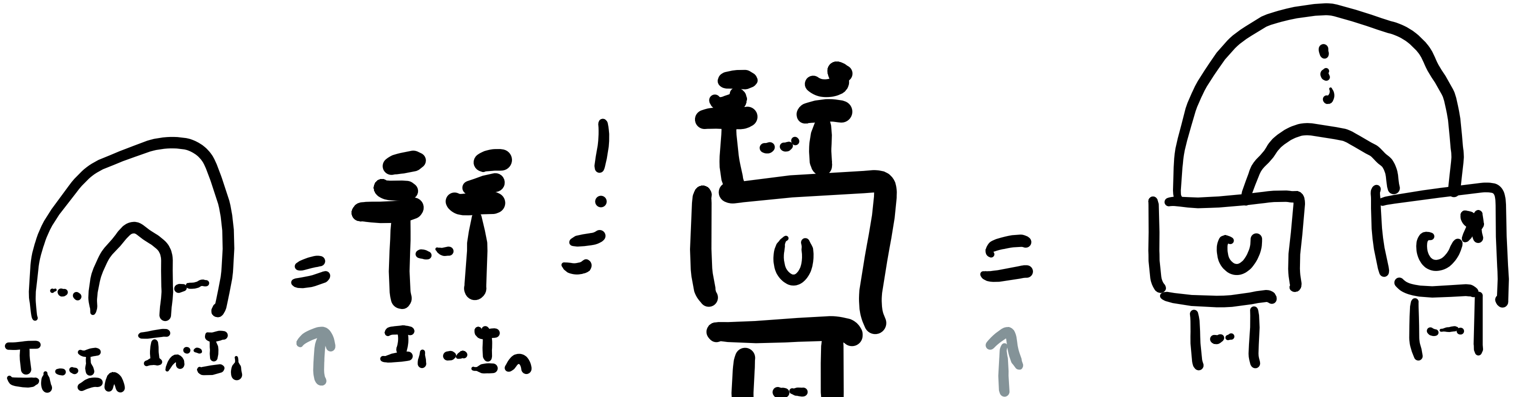


$\iff$ U isometry

(Kraus op is an isometry)

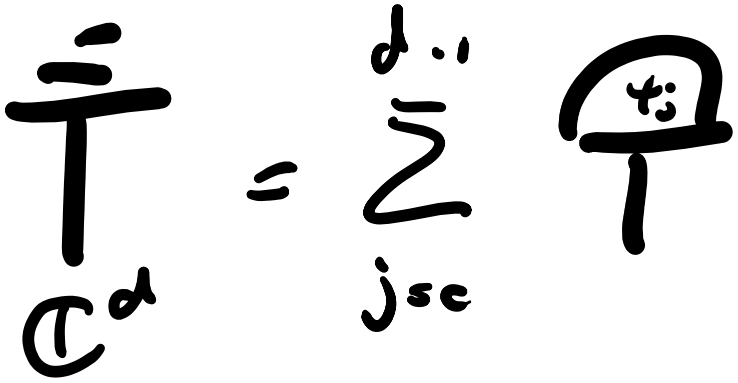


Proof:



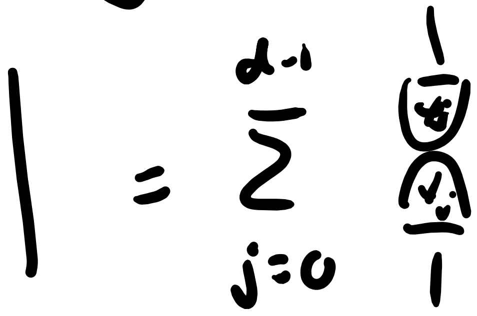
undouble undouble

□

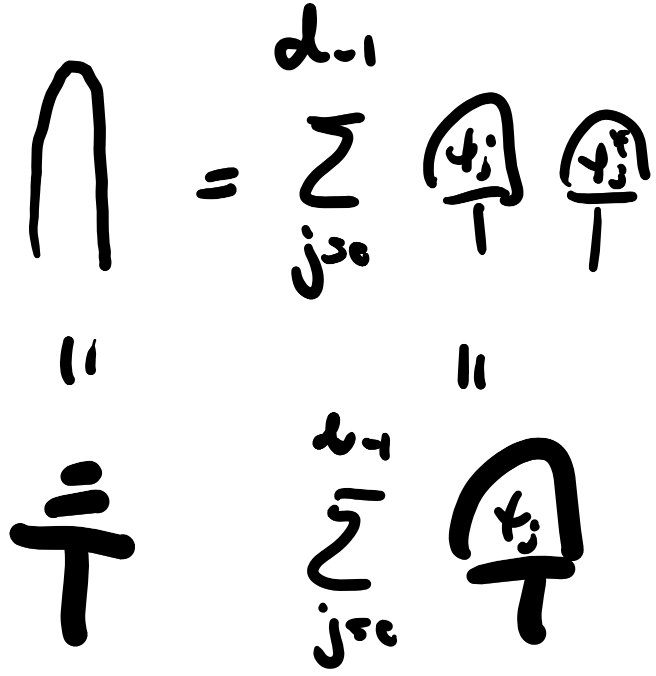


$\forall \left(|\psi_j\rangle\right)_{j=0}^{d-1}$ ONB

Proof:

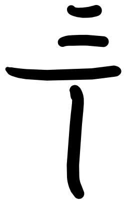


bend wire $\Longrightarrow$

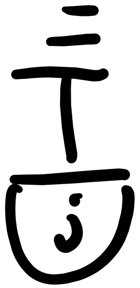


$id = \sum_j |\psi_j\rangle\langle\psi_j|$

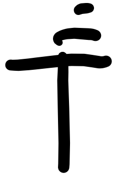
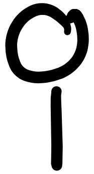
$\square$

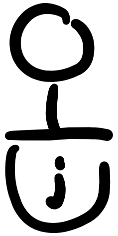


$\mathbb{R}^{+d}$ s.t.



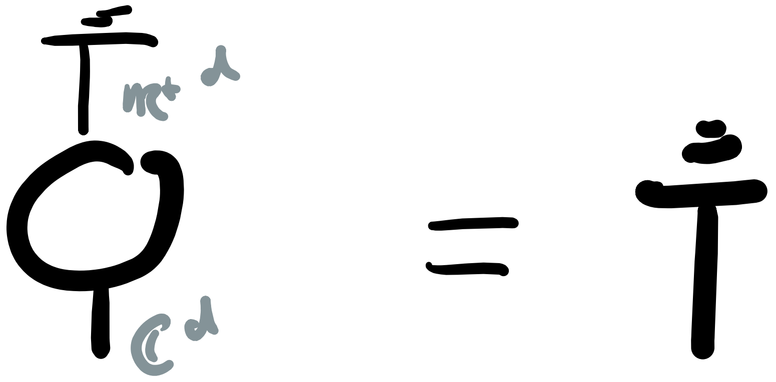
$= 1 \quad \forall\ j = 0, \dots, d-1$

 $:=$ 



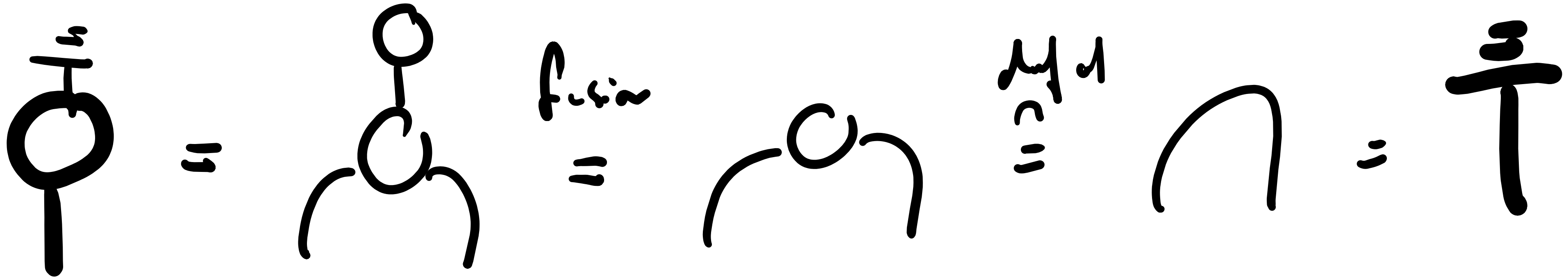
$= 1 \quad \forall\ j = 0, \dots, d-1$

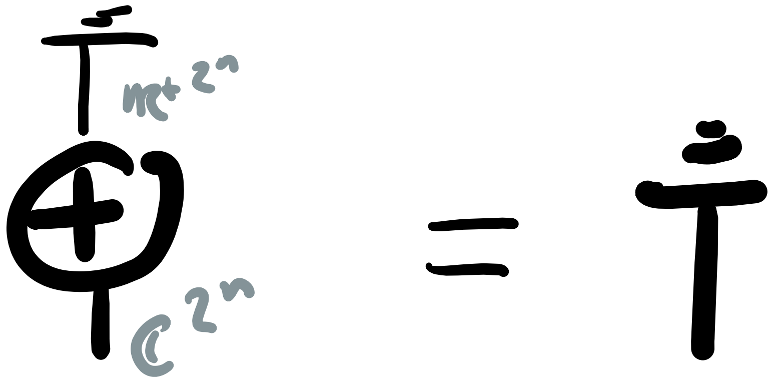
$$\langle + | j \rangle = 1 \quad \forall j$$



2 bar.1

Proof:

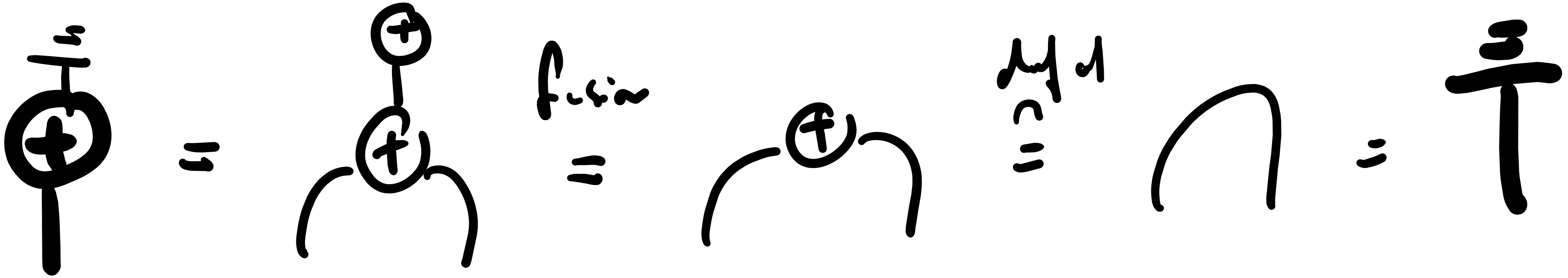




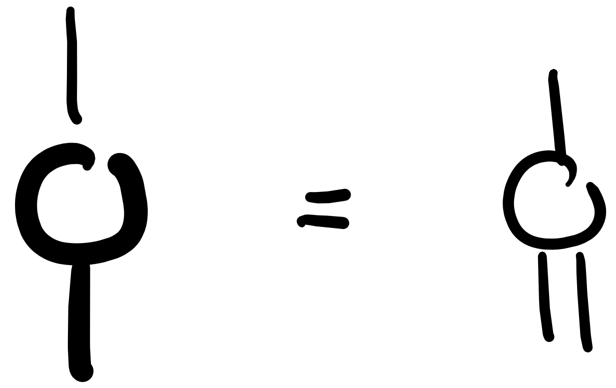
× basis

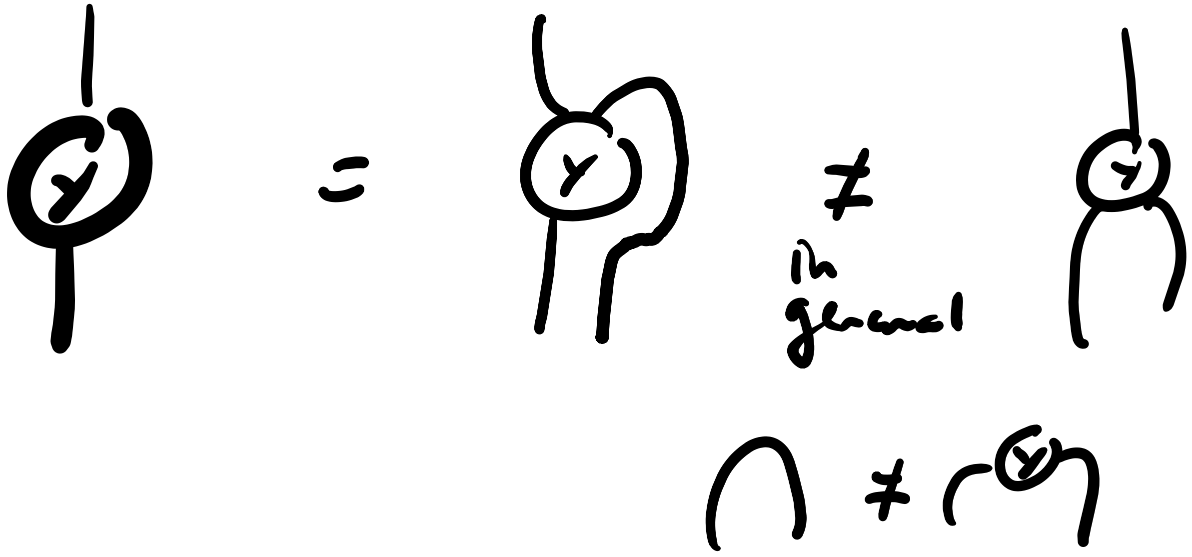
(for $\mathbb{C}[\mathbb{Z}_2^n]$)

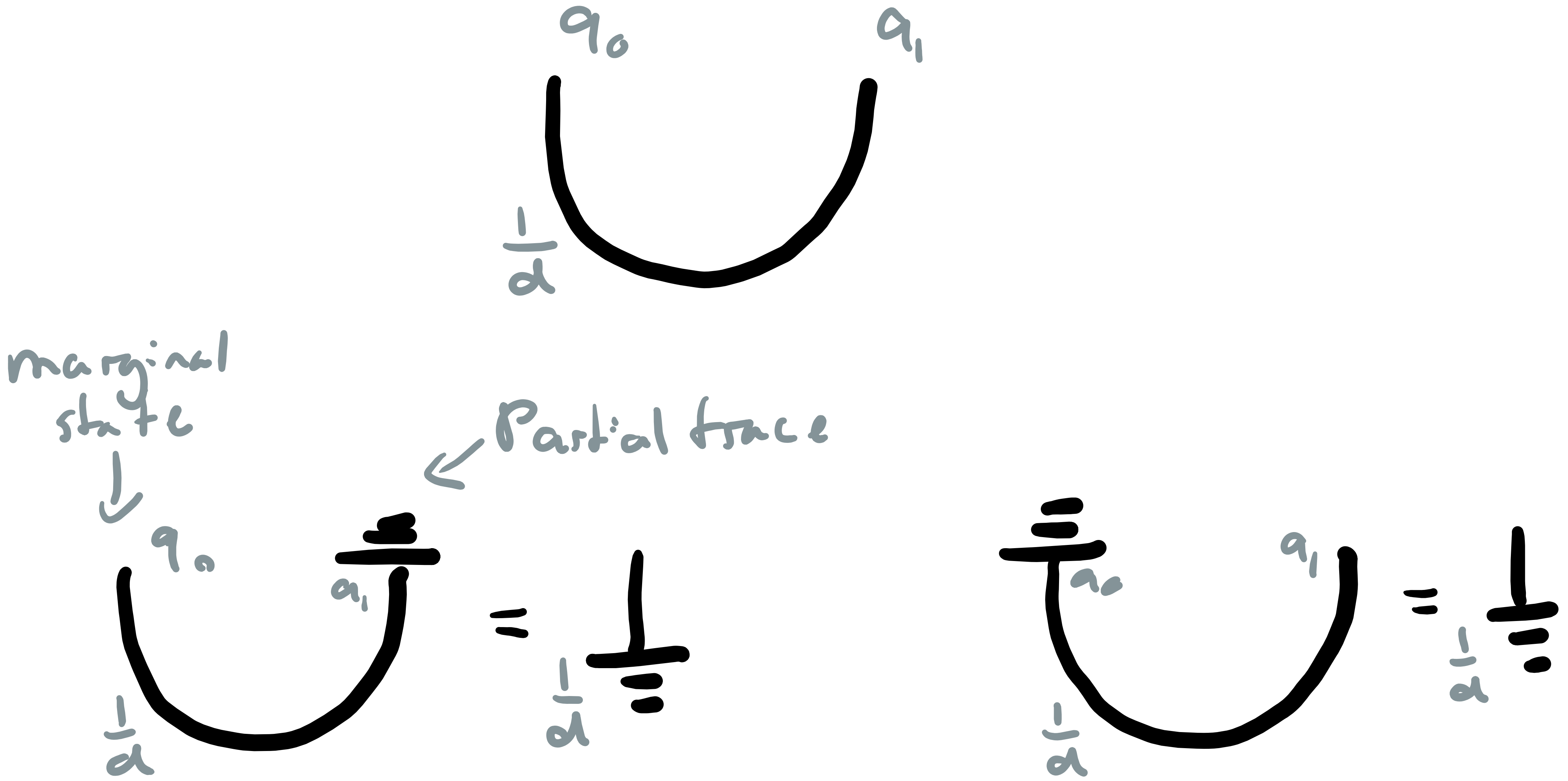
Proof:

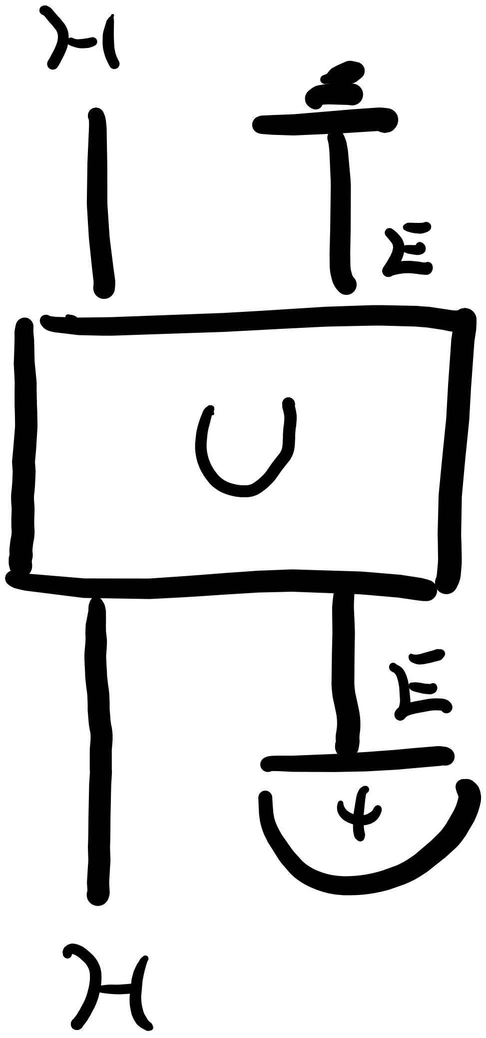


CAVEAT






q0
q1
1/d
marginal state
Partial trace
q0
q1
1/d
= 1/d
q1
q0
1/d
= 1/d



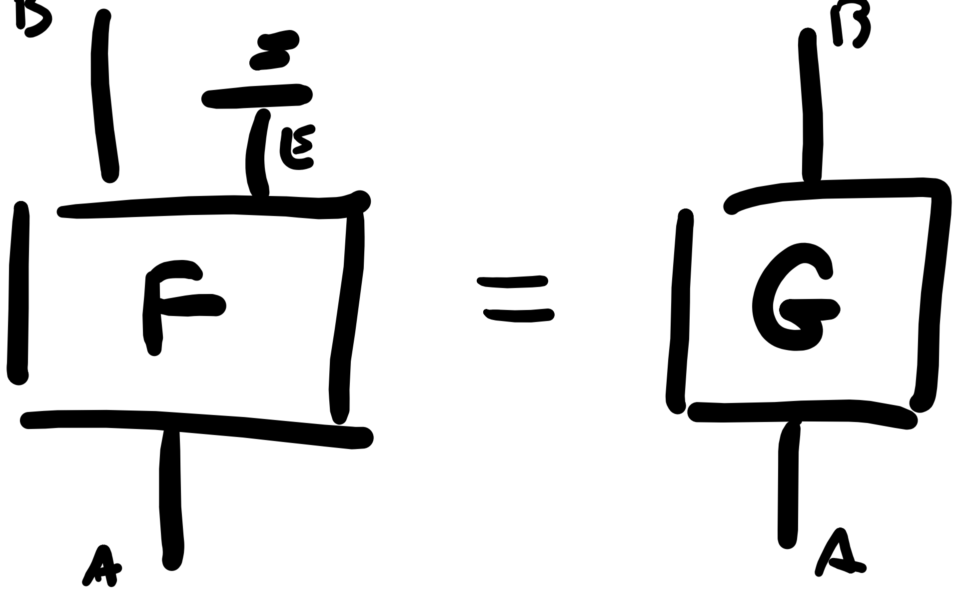
U unitary

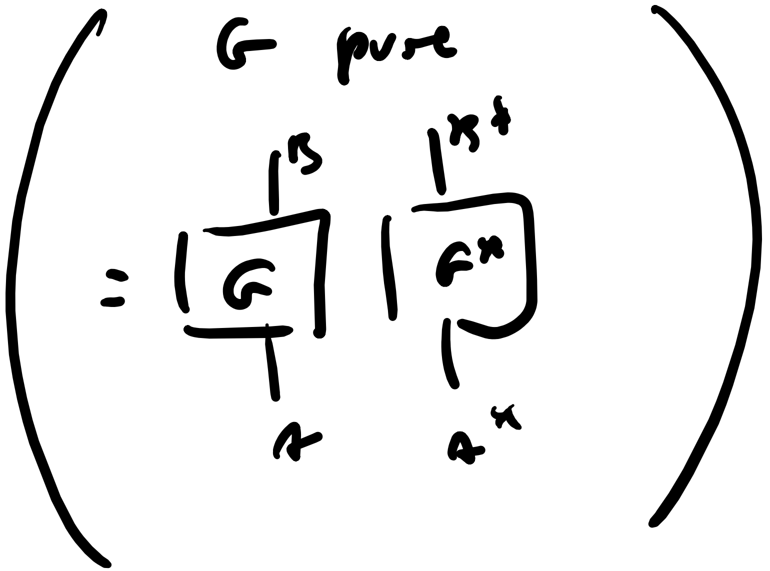
$|\psi\rangle$ pure state

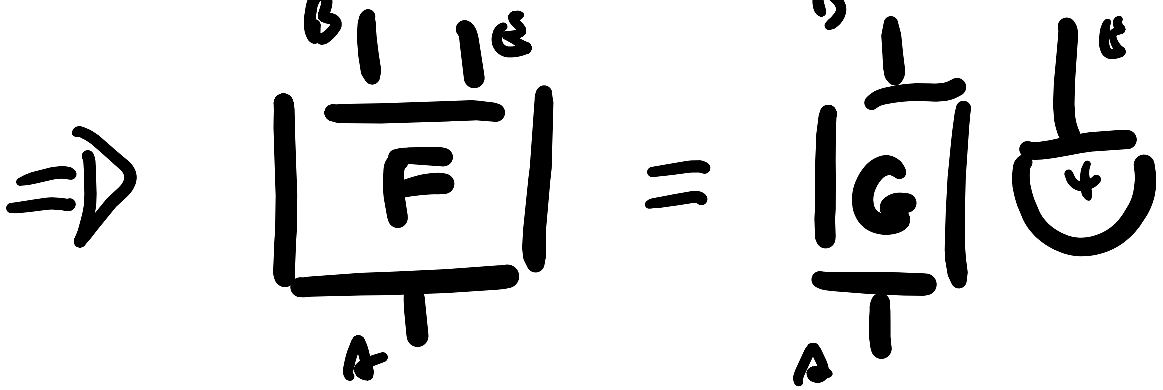
CPTP map

(dissipation ?
ignorance ?)

Purification Principle

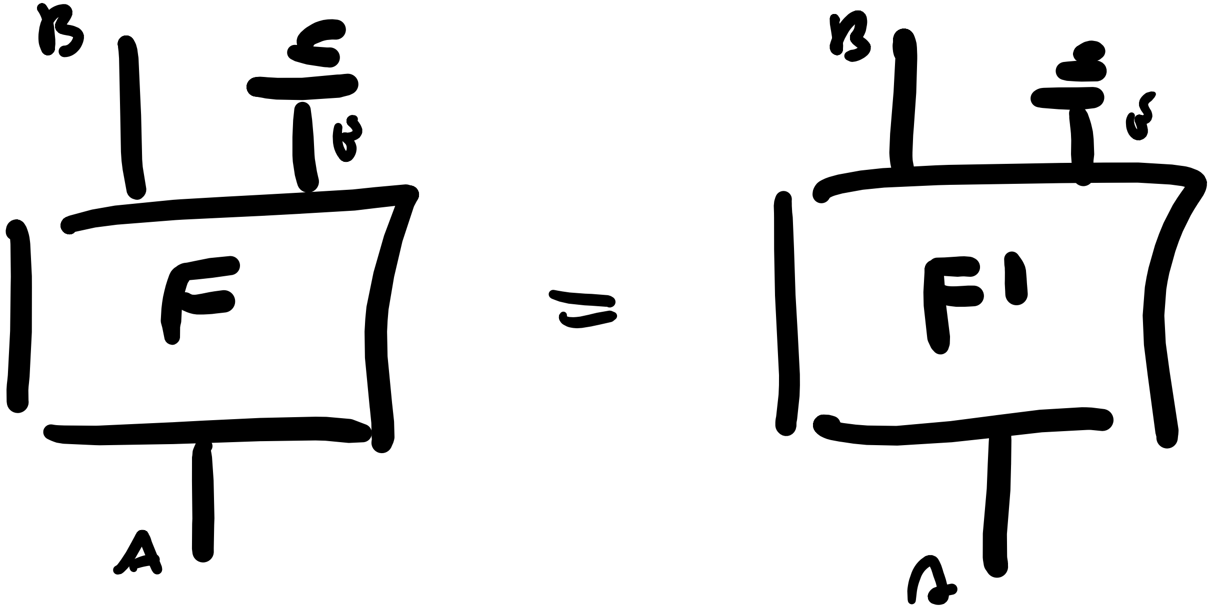




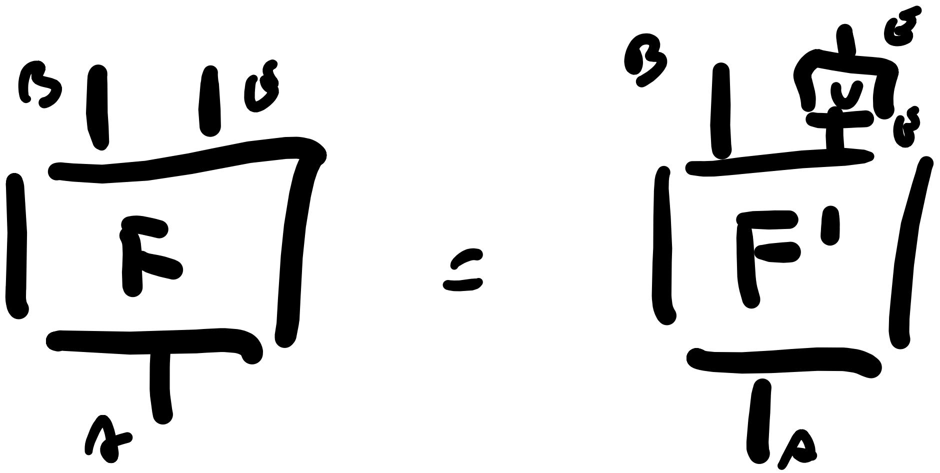


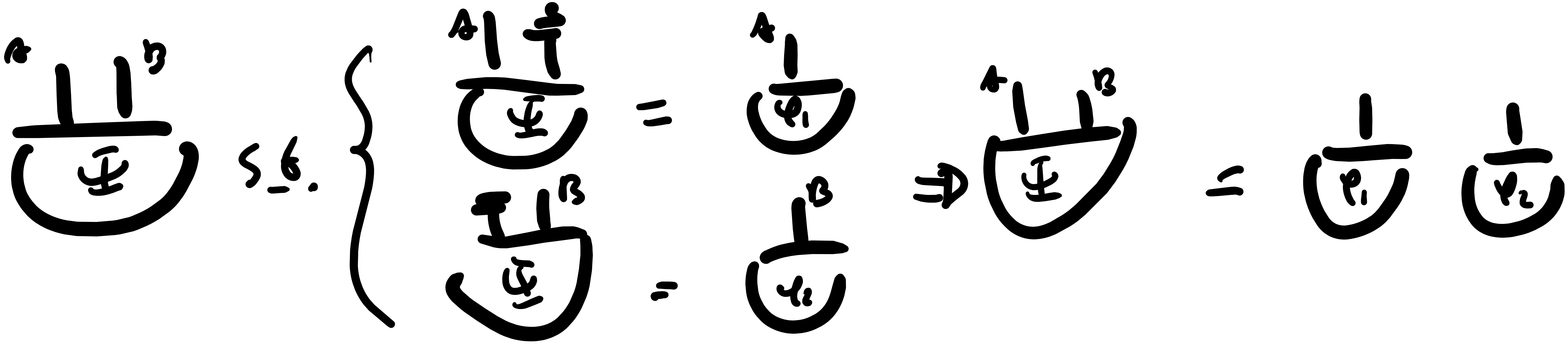
← any pure state $|\psi\rangle$

EUP Essentially Unique Purification Principle

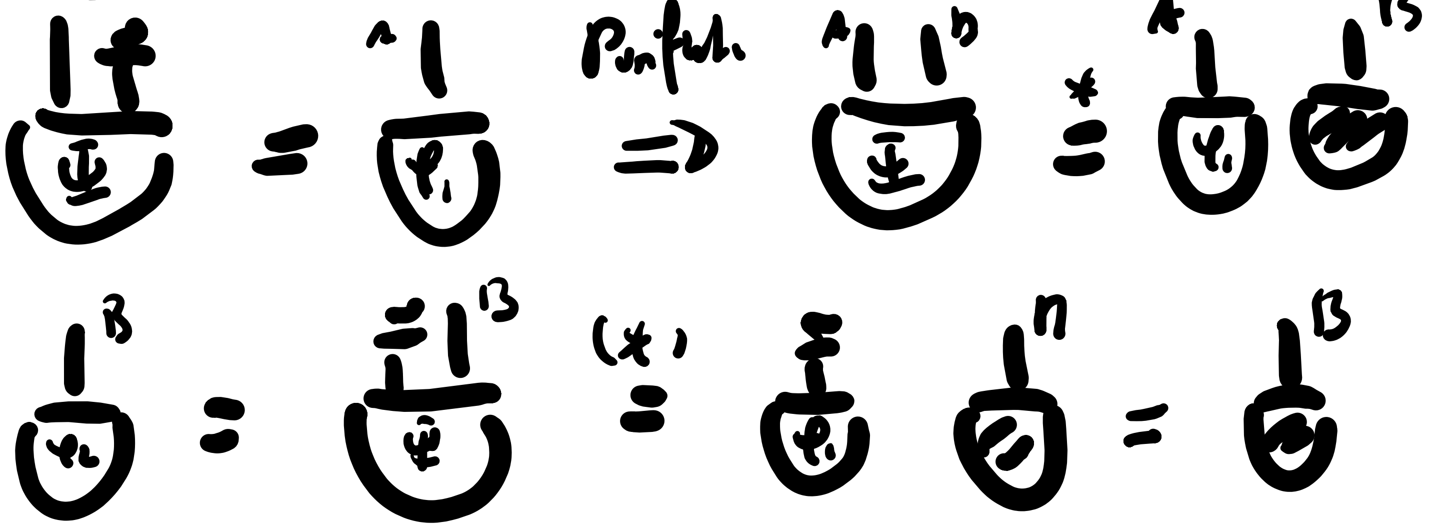


$\Rightarrow \exists\, U_{\text{unitary}}$



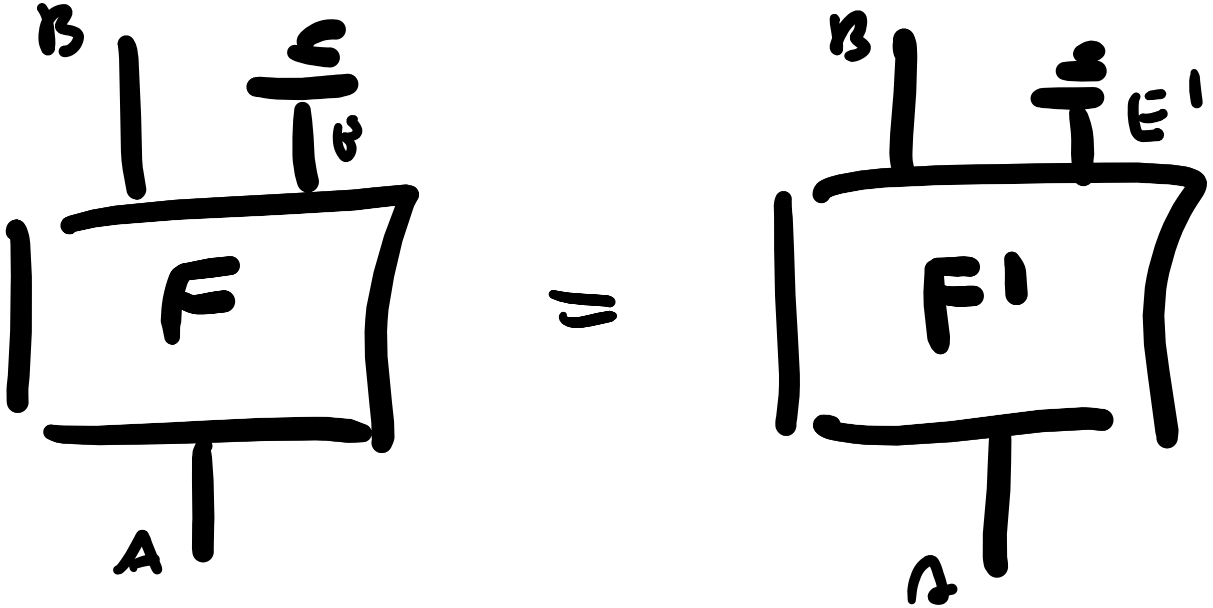


Proof:

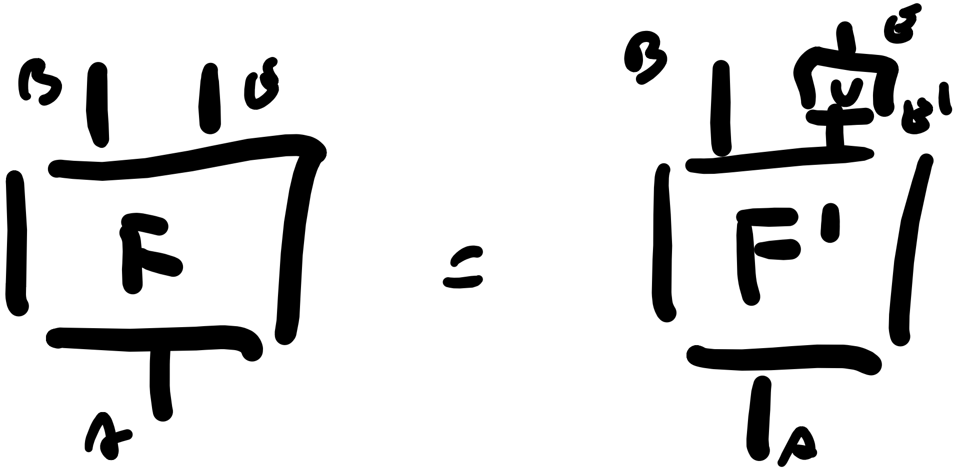


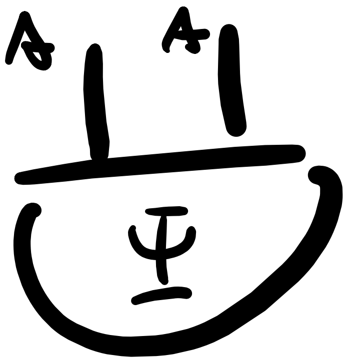
□

EUP Essentially Unique Purification Principle

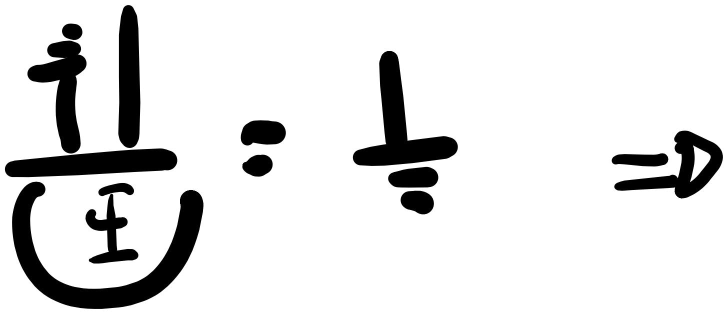


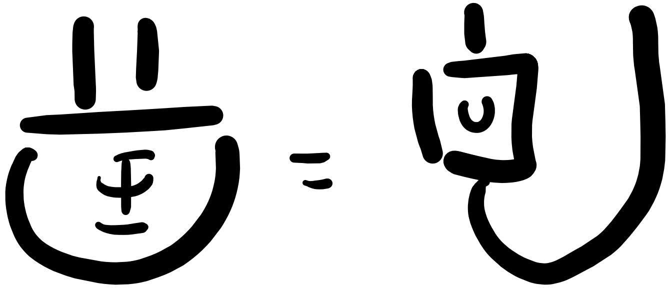
$\Rightarrow \exists$ U isometry





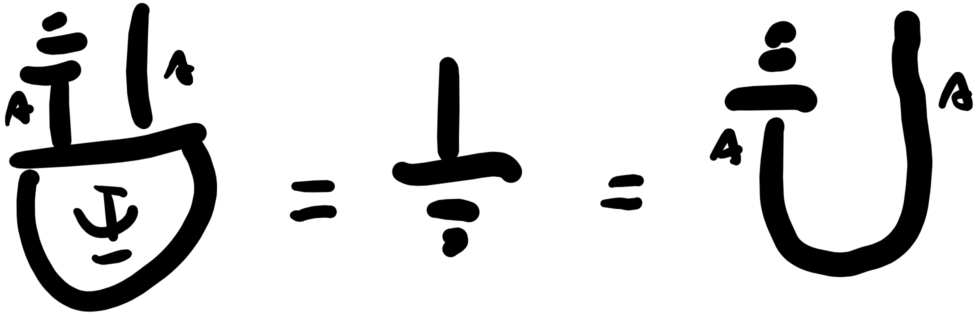
s.t.

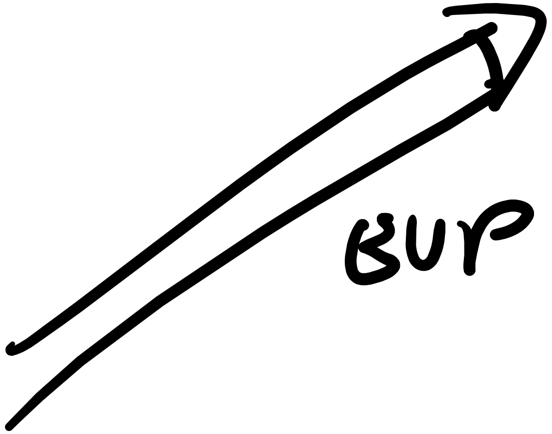




for some U unitary

Proof:





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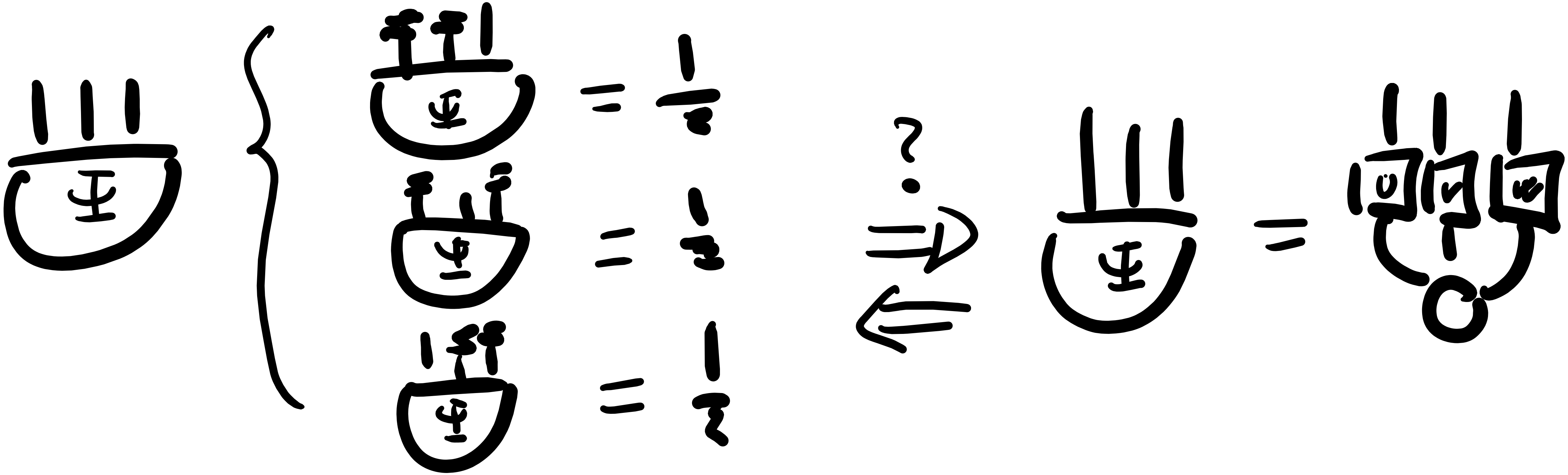
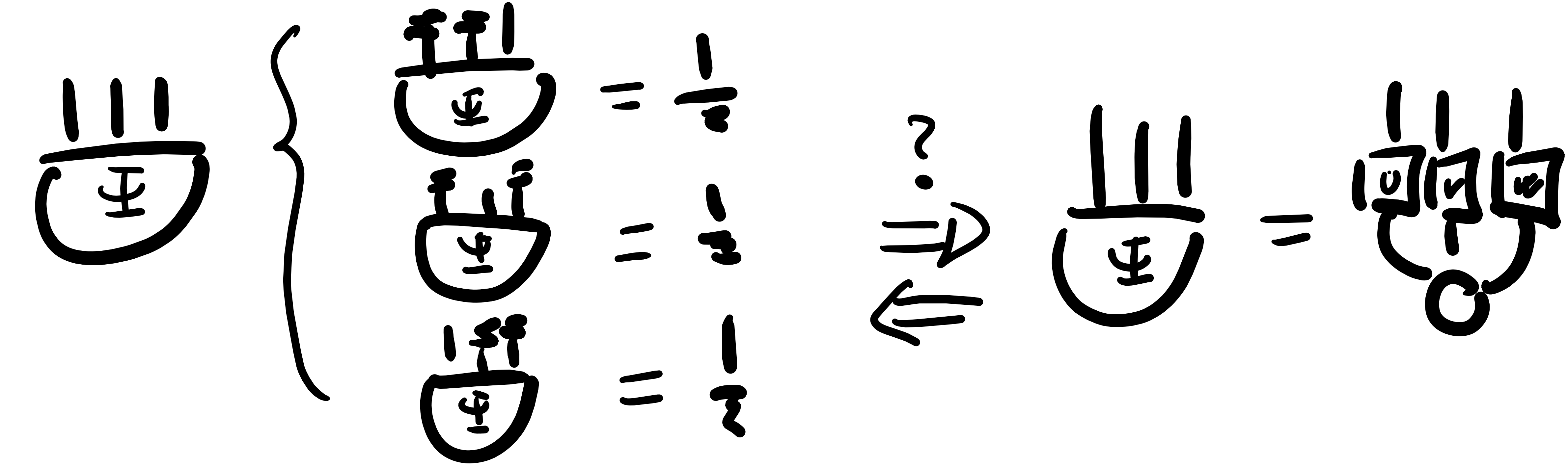
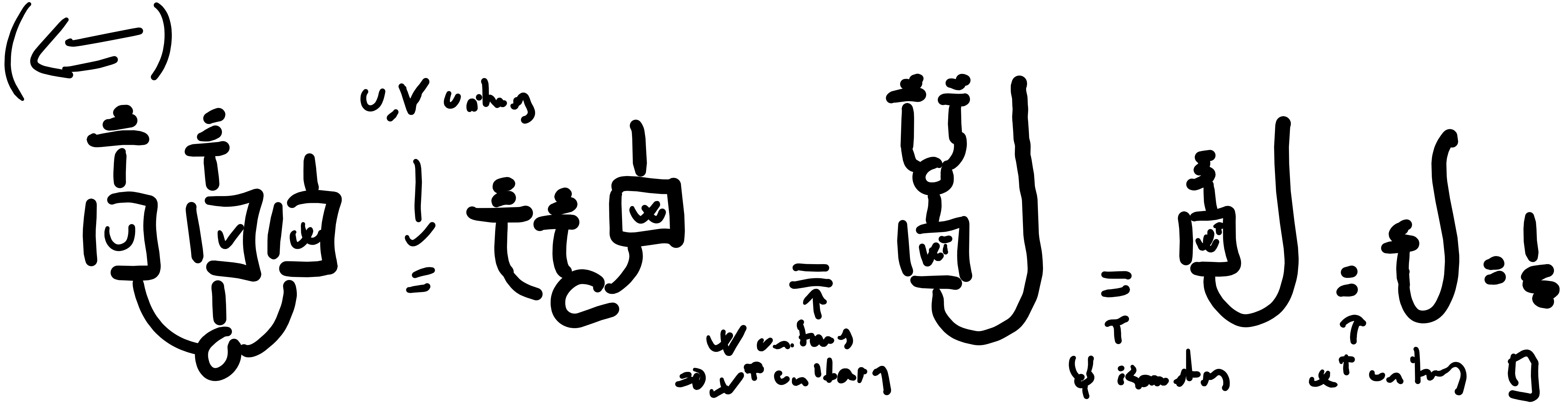
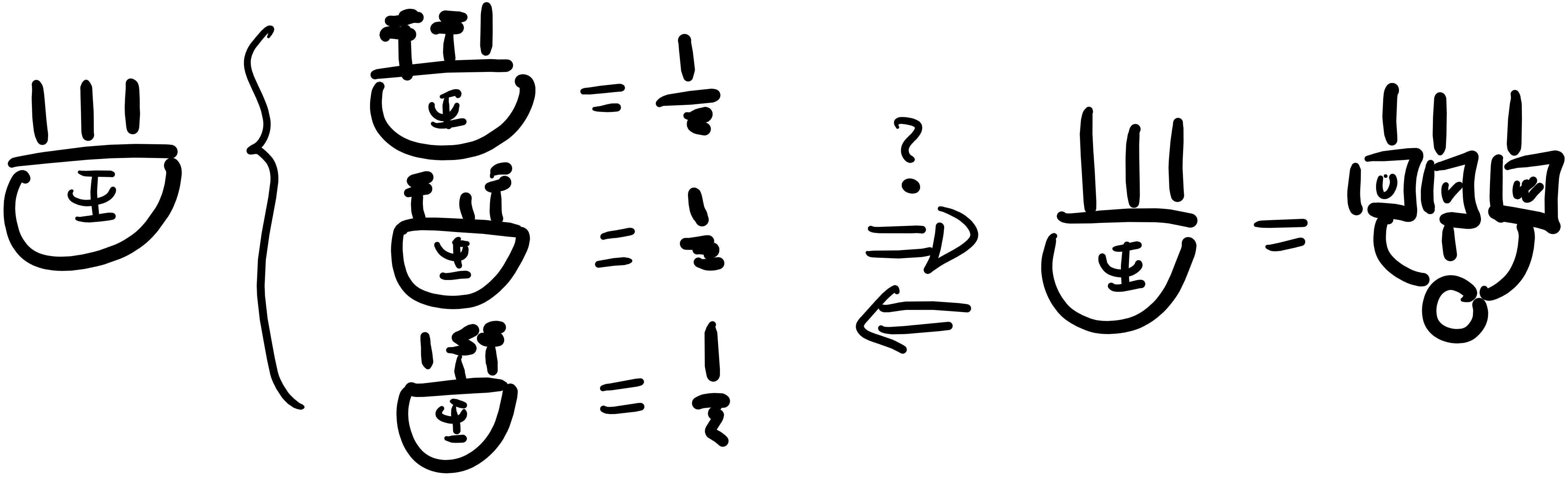
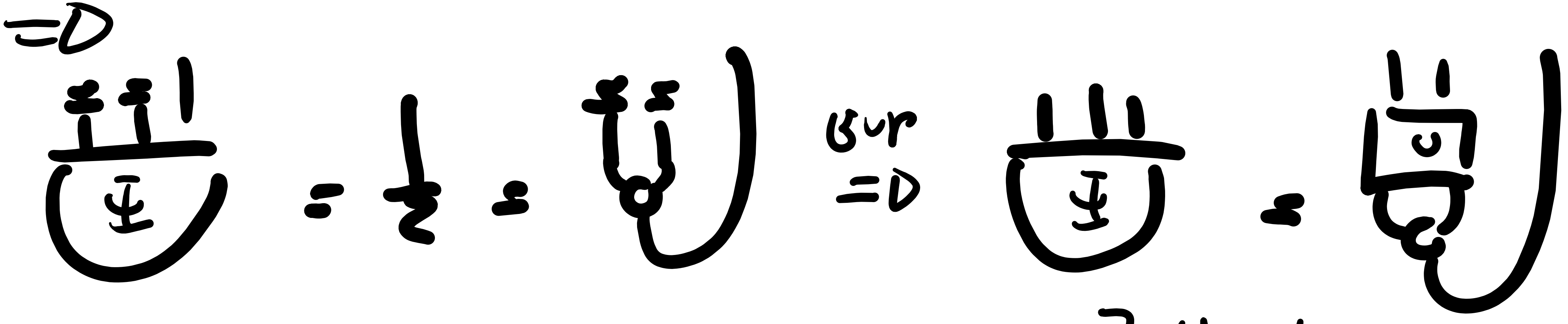


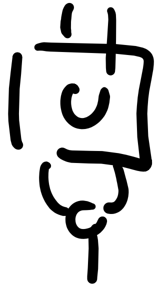


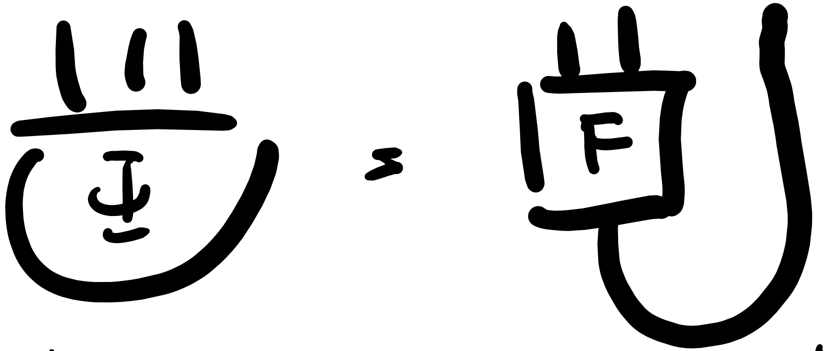

U, V unitary
W unitary
⇒ W^T unitary
ψ isometry
W^T unitary
□





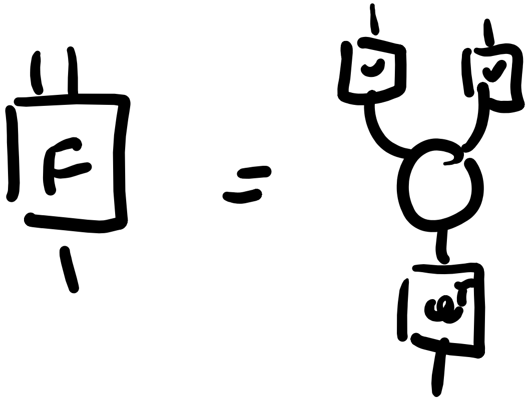
∃ U unitary

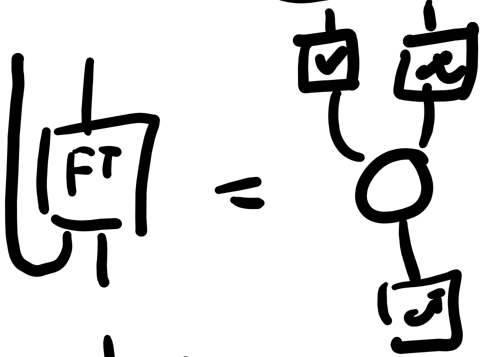
 isometry

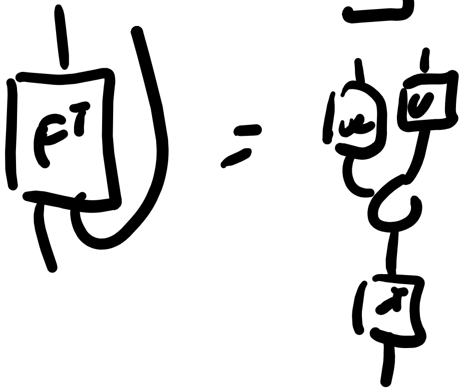


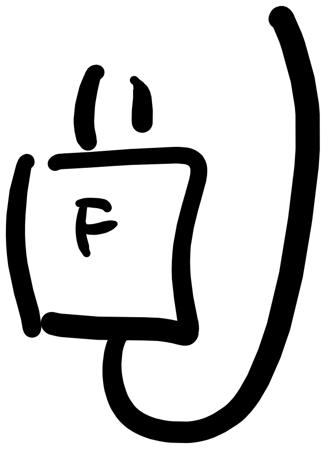
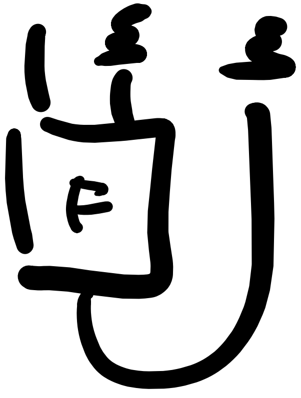
F isometry

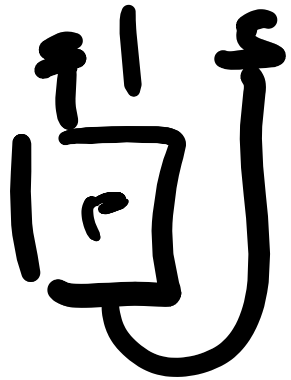
e.g.



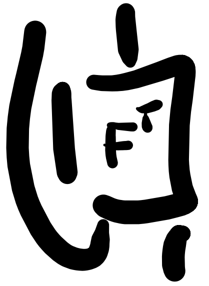
 isometry

 isometry

 st.  $^{(1)}$ s

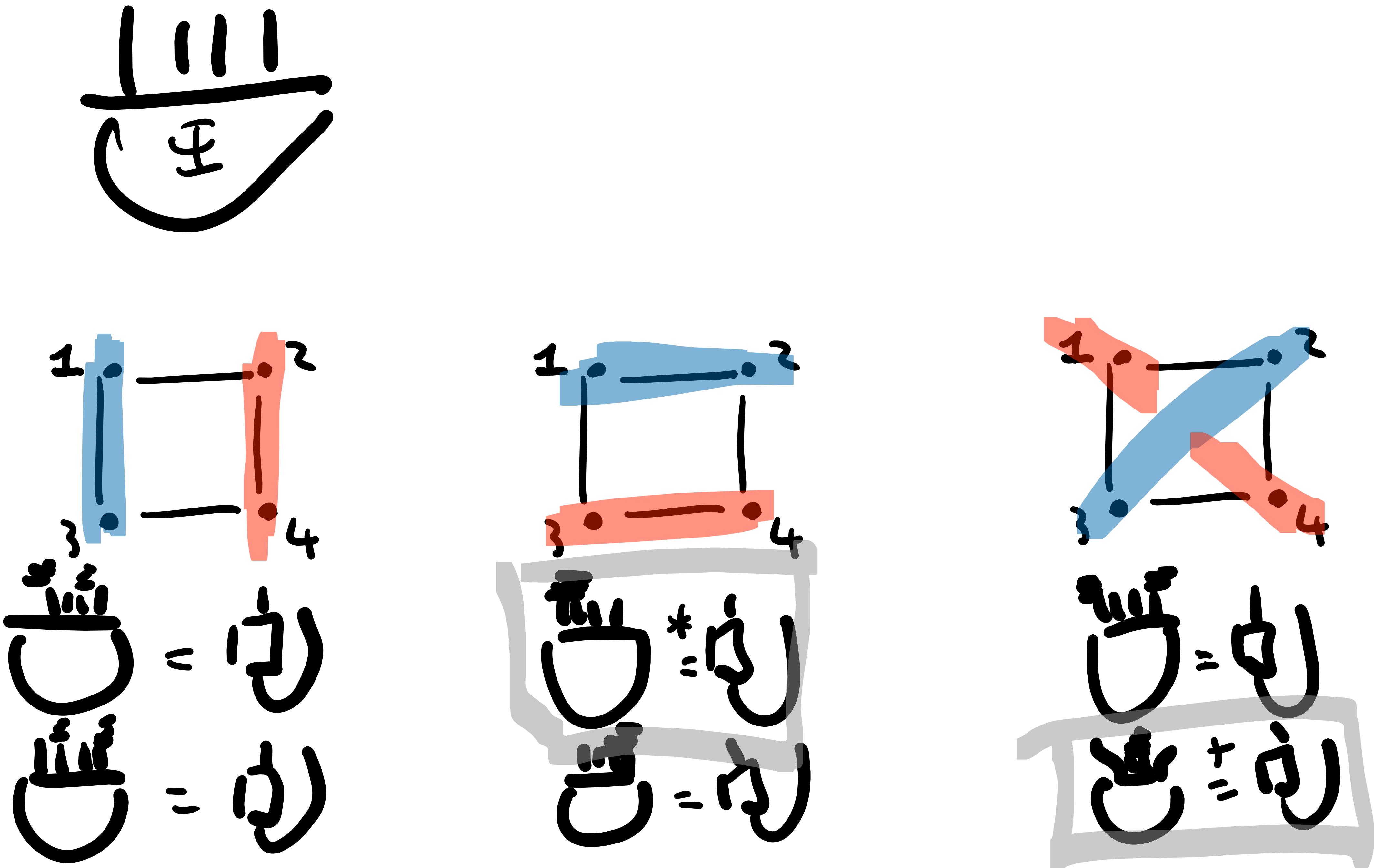
and  $^{(2)}$ s

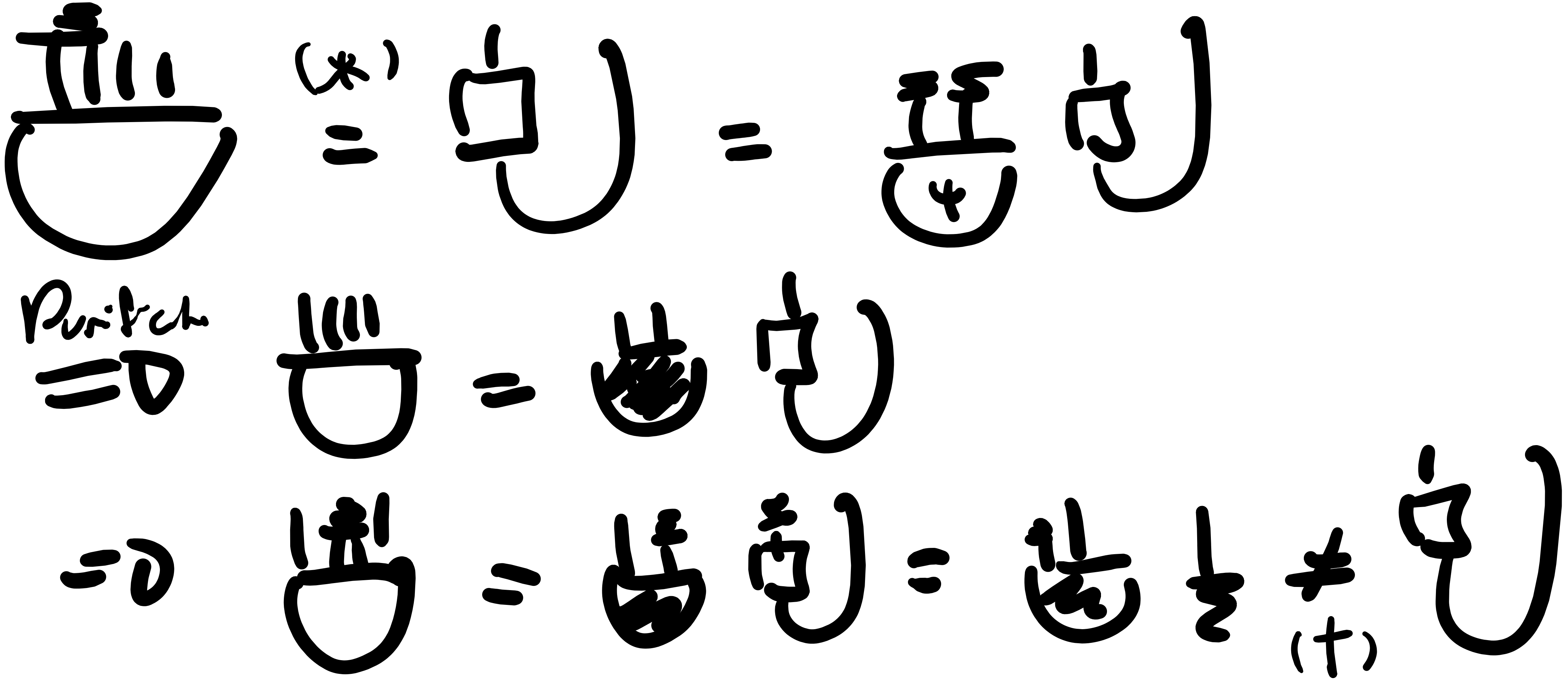
BUP $\Rightarrow$ $\begin{cases} (1) \\ (2) \end{cases}$

 isometry

 isometry

 isometry


1
2
3
4
1
2
3
4
1
2
3
4

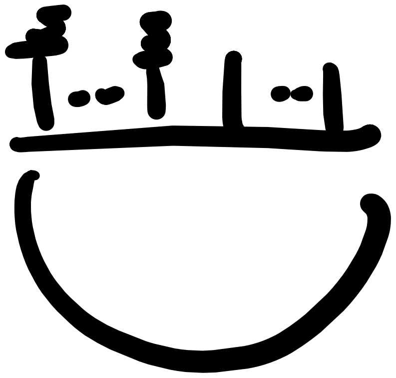

(*)
Pwitch
(†)

□

$AME(n, m)$

number of parties

dimension of space



maximally entangled

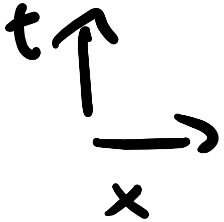
$$\sum_{\mu} \sum_{j} \mathrm{Tr}[\mu_j \rho]^2 = 0$$

$\Longleftrightarrow$ $\left\{ \right.$

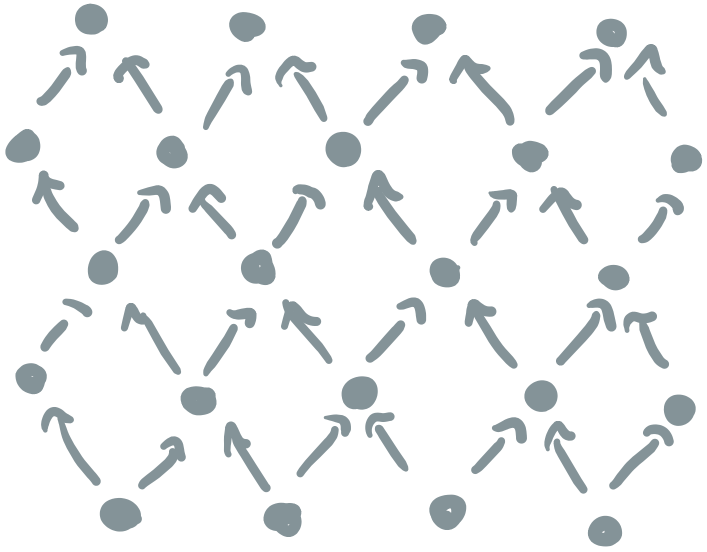
$= \frac{1}{2}$

$\vdots$

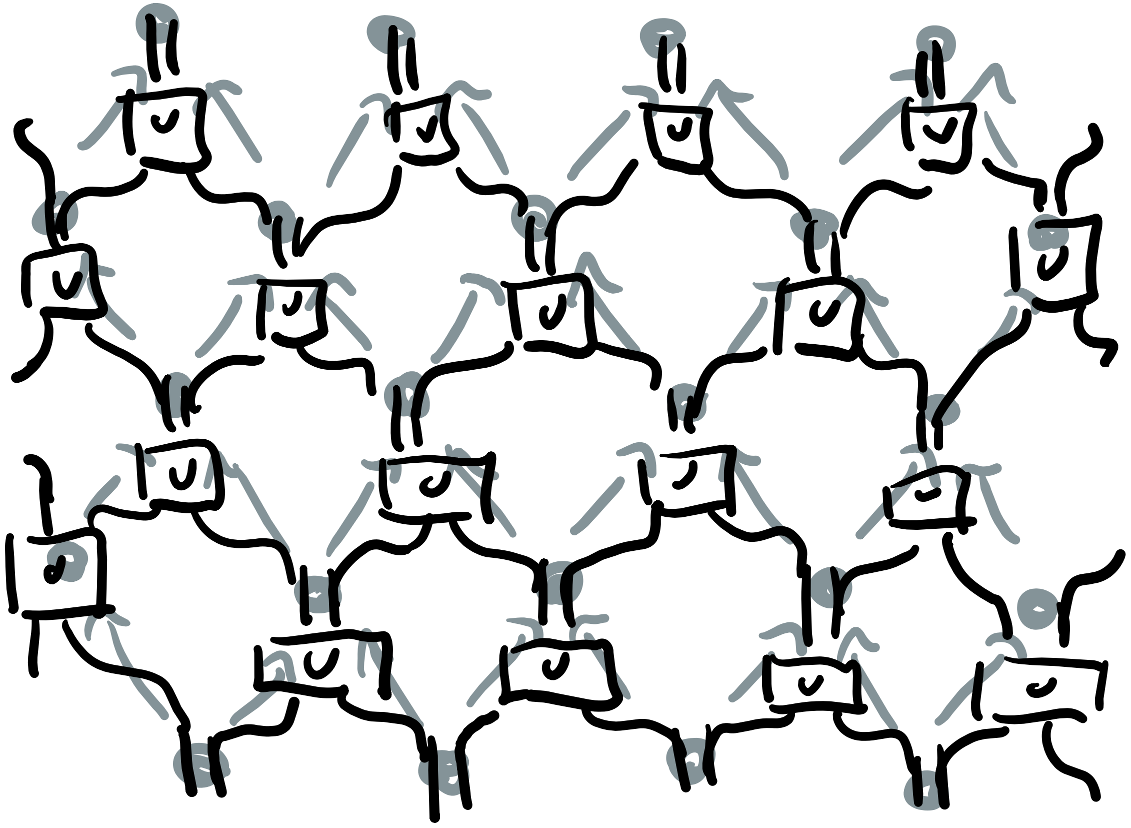
$= \frac{1}{2}$



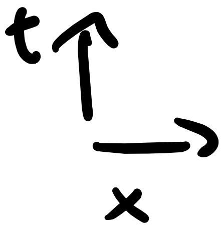
(1+1)-Minkowski



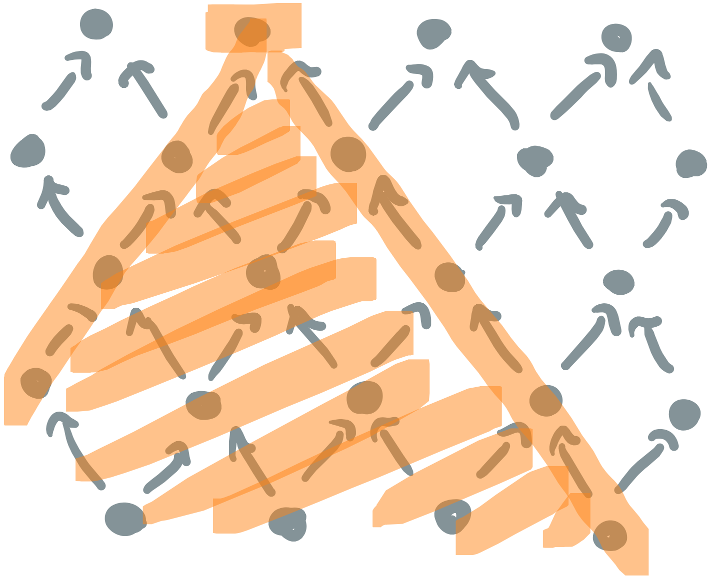
Hasse diagram for causal order



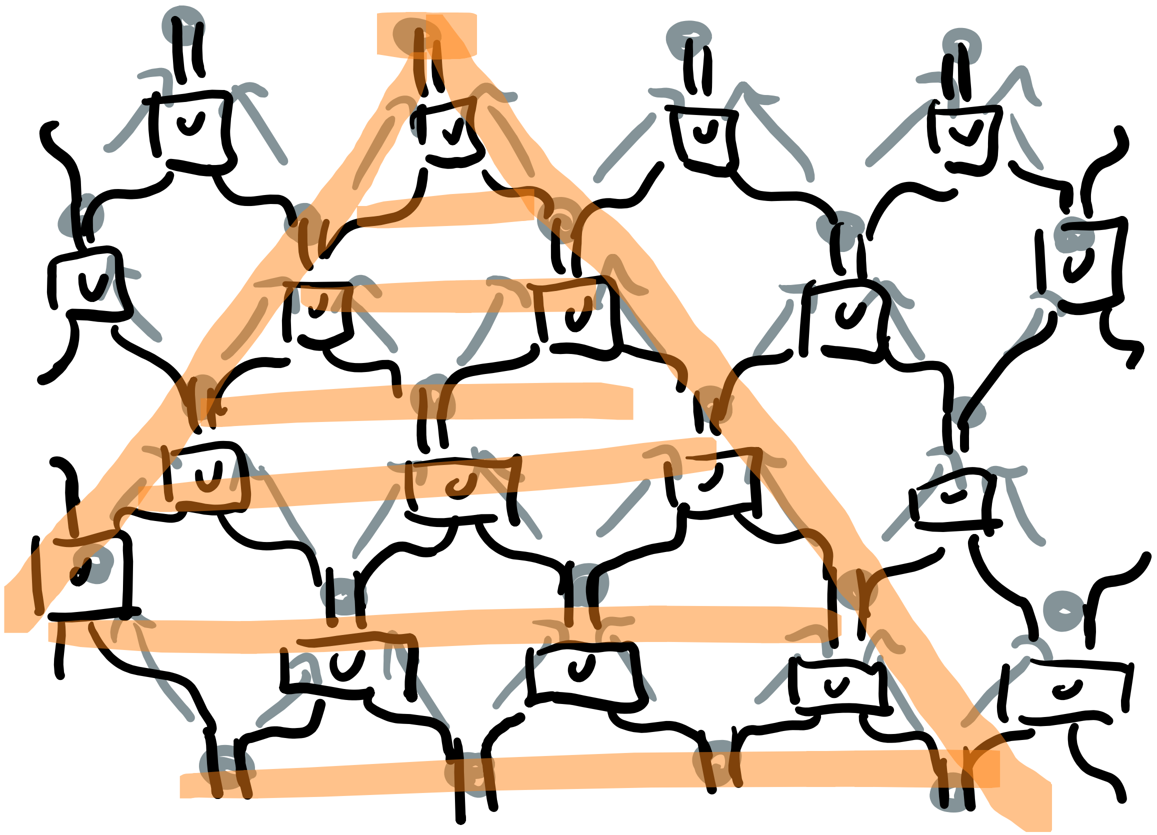
$H \otimes H$



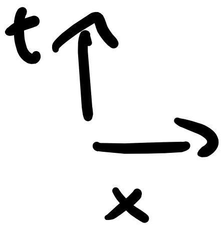
(1+1)-Minkowski



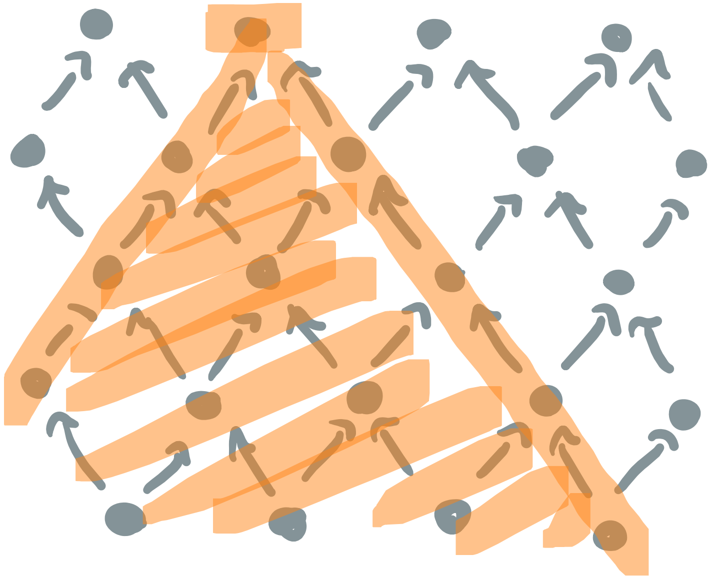
Hasse diagram for causal order



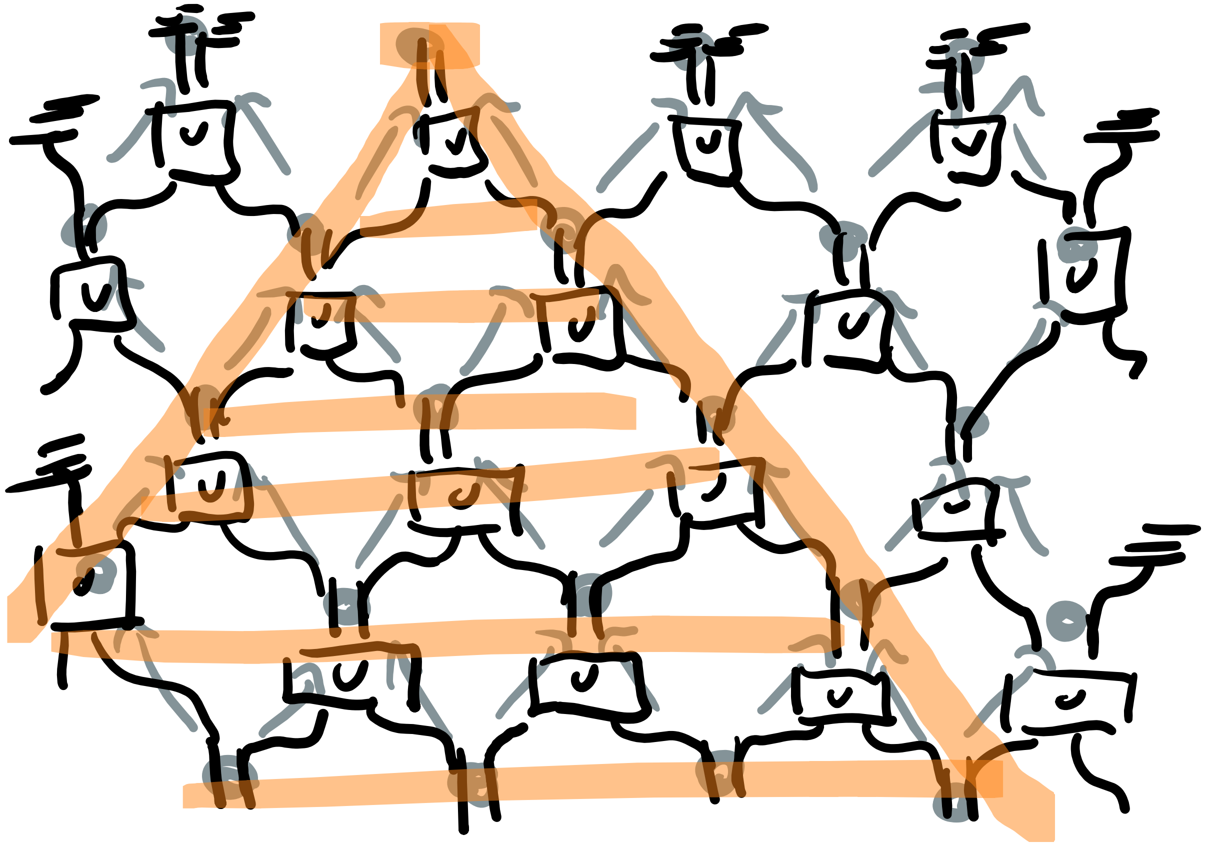
$H \otimes H$



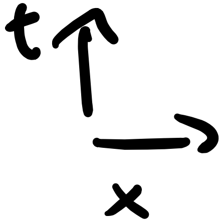
(1+1)-Minkowski



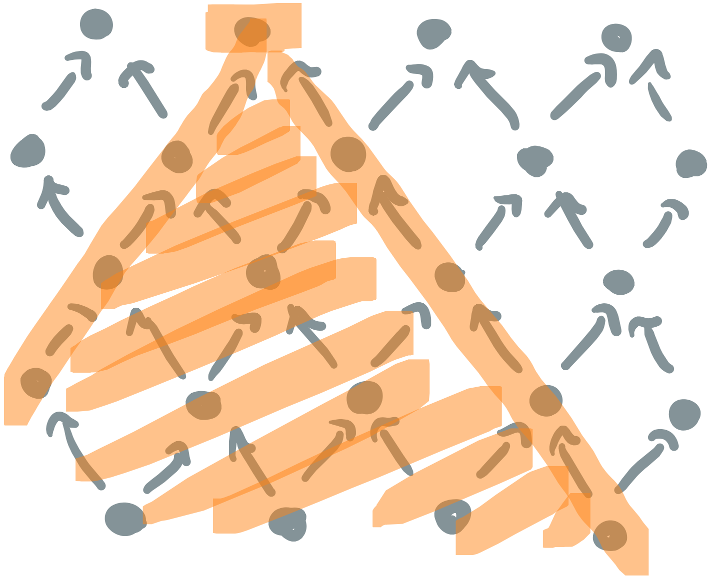
Hasse diagram for causal order



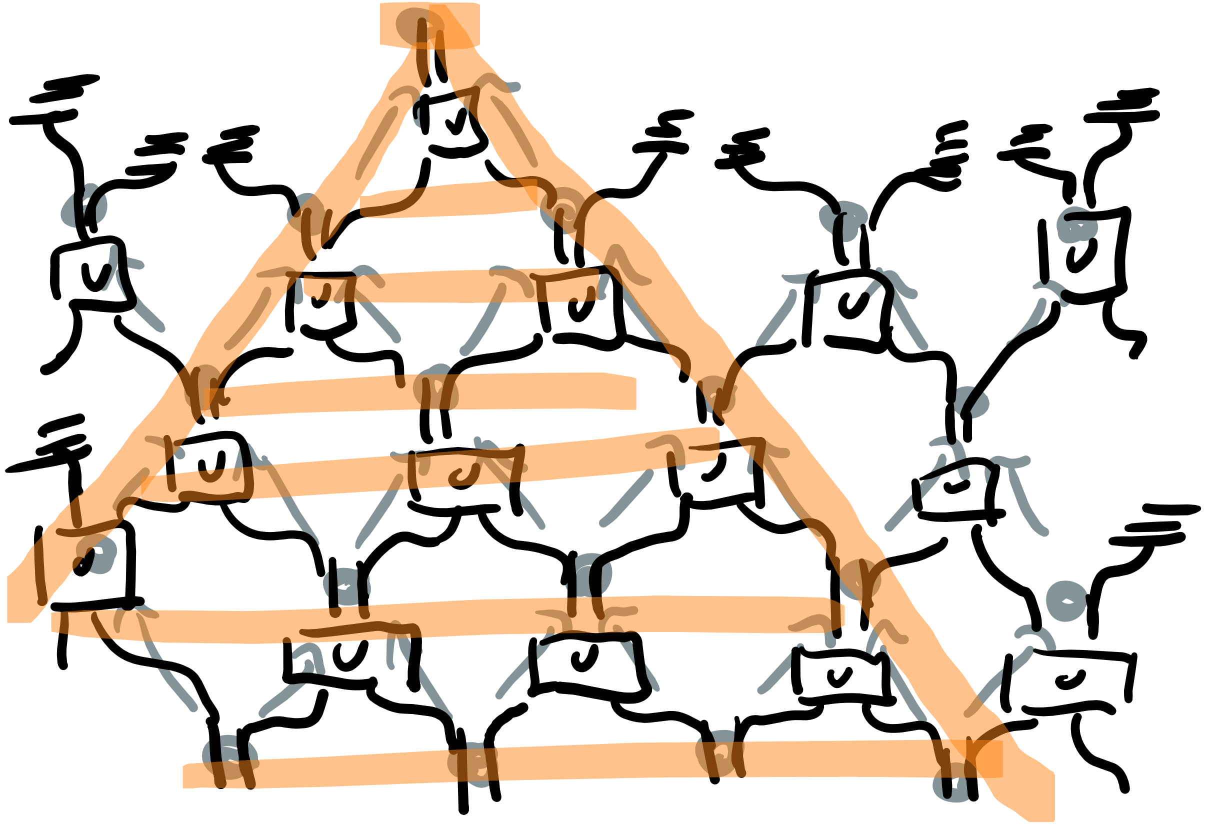
H⊗H



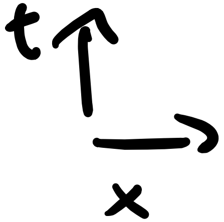
(1+1)-Minkowski



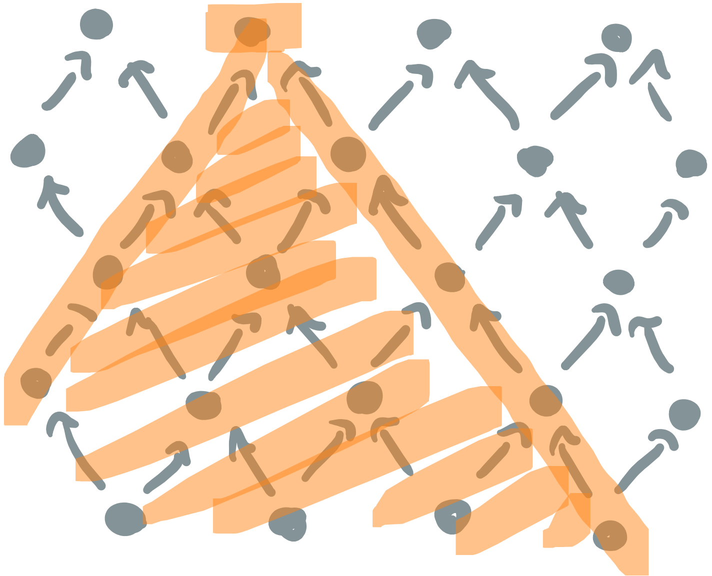
Hasse diagram for causal order



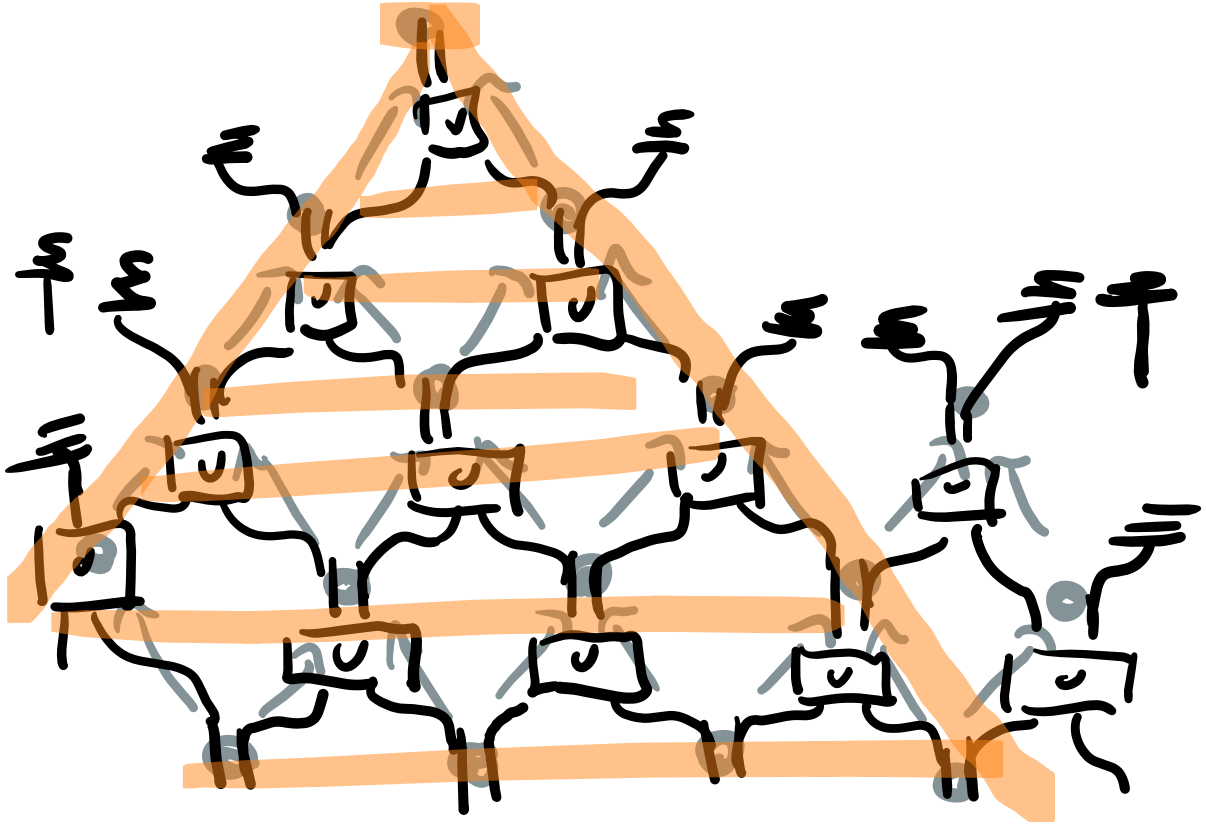
$H \otimes H$



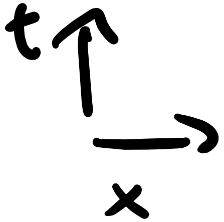
(1+1)-Minkowski



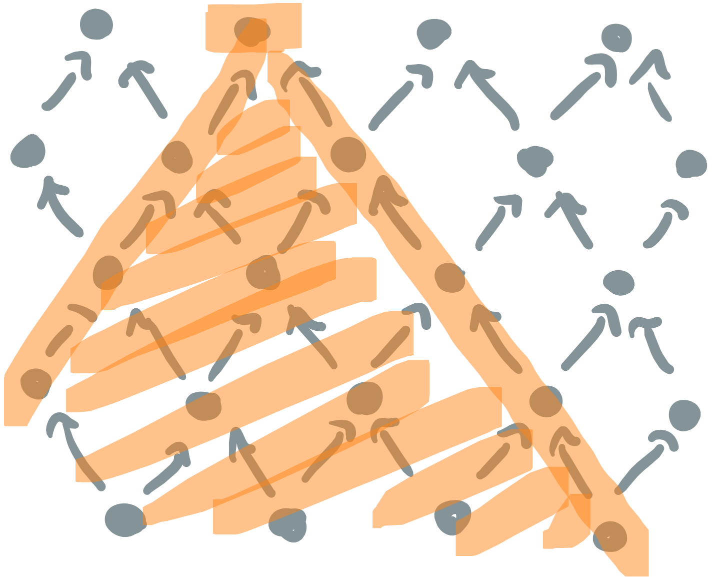
Hasse diagram for causal order



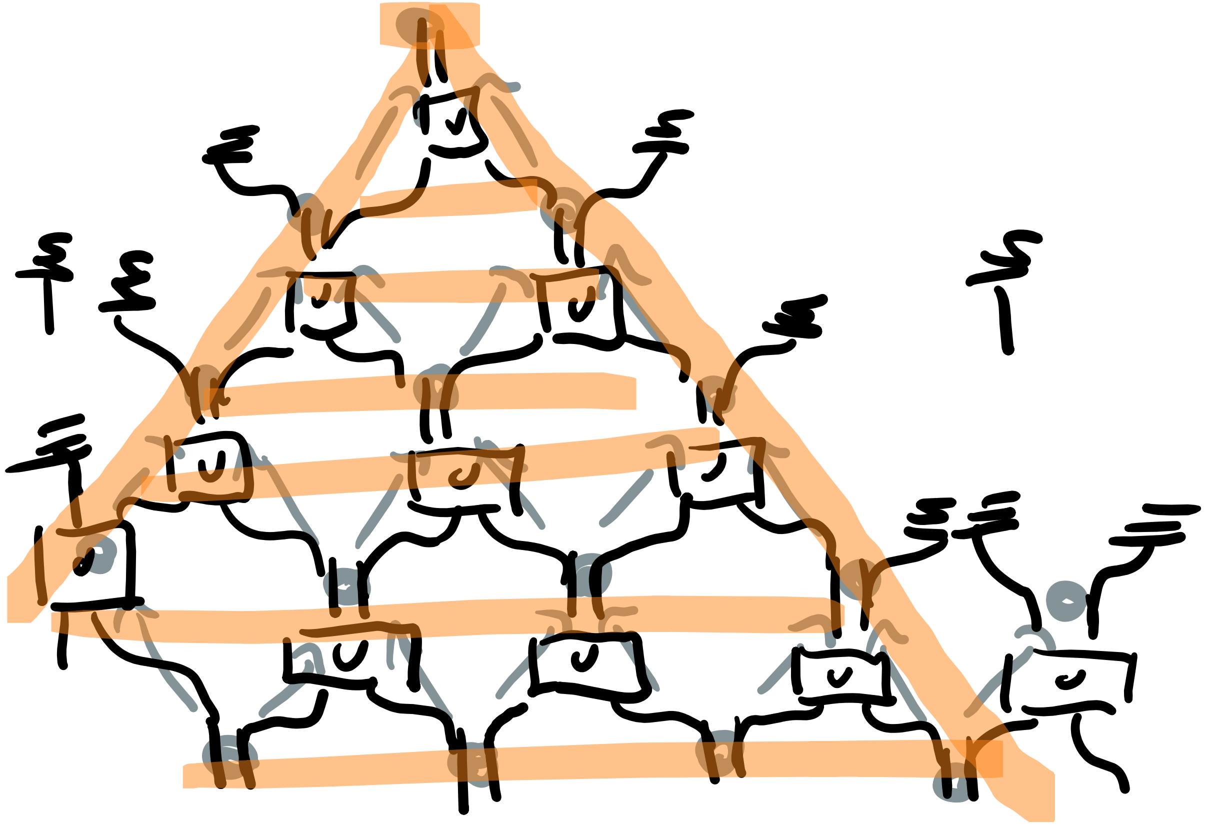
$H \otimes H$



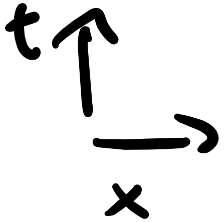
(1+1)-Minkowski



Hasse diagram for causal order



$H \otimes H$



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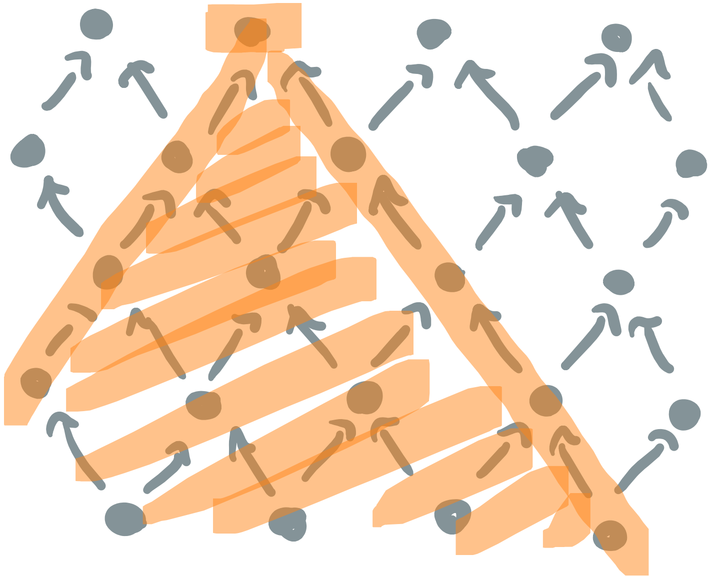
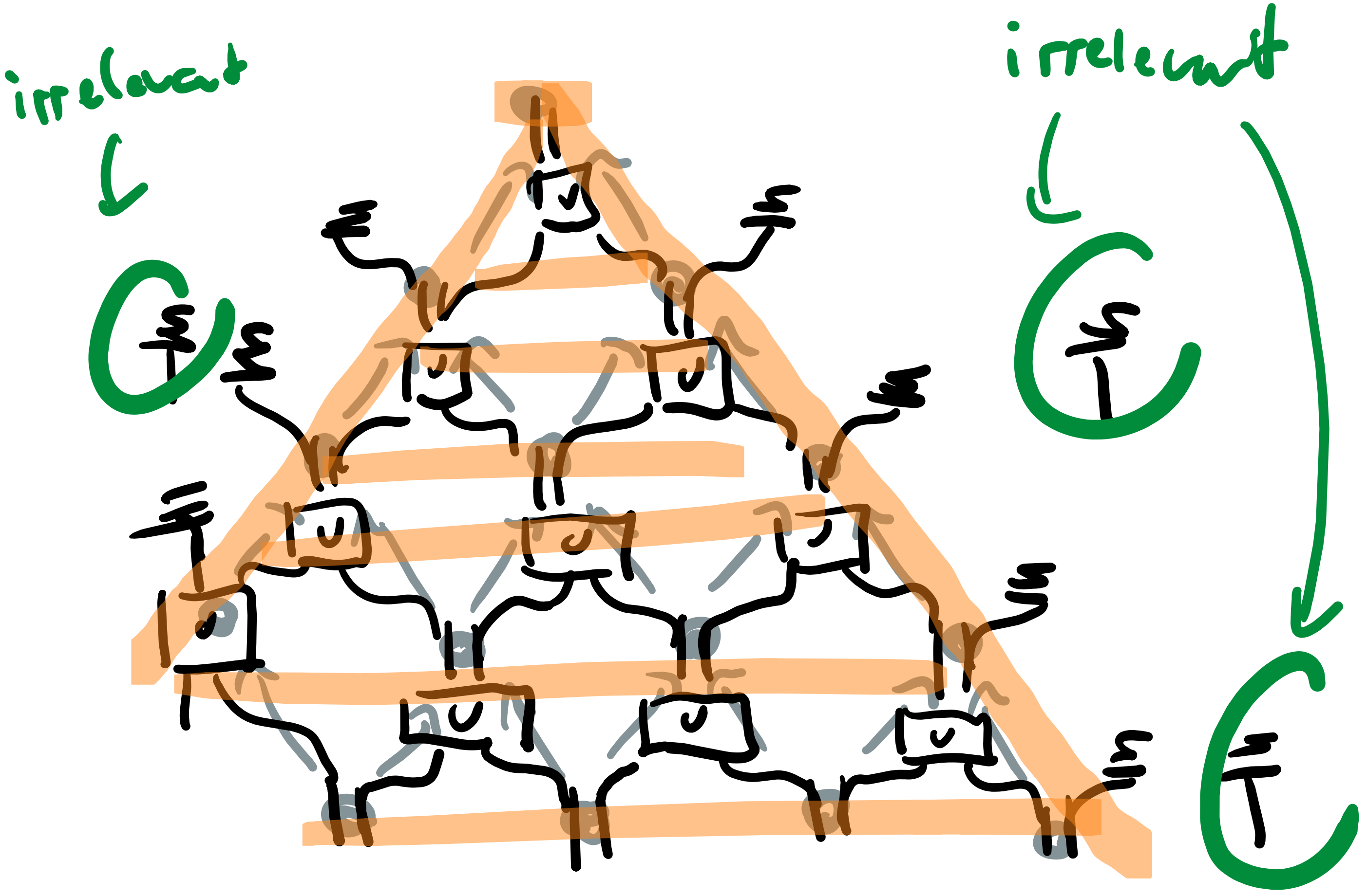
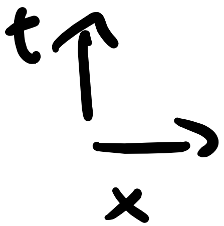


Hasse diagram for causal order



$H \otimes H$



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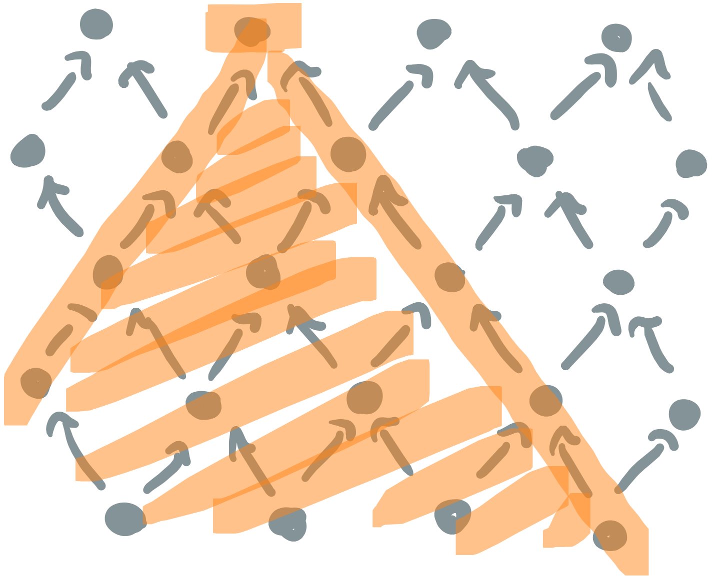
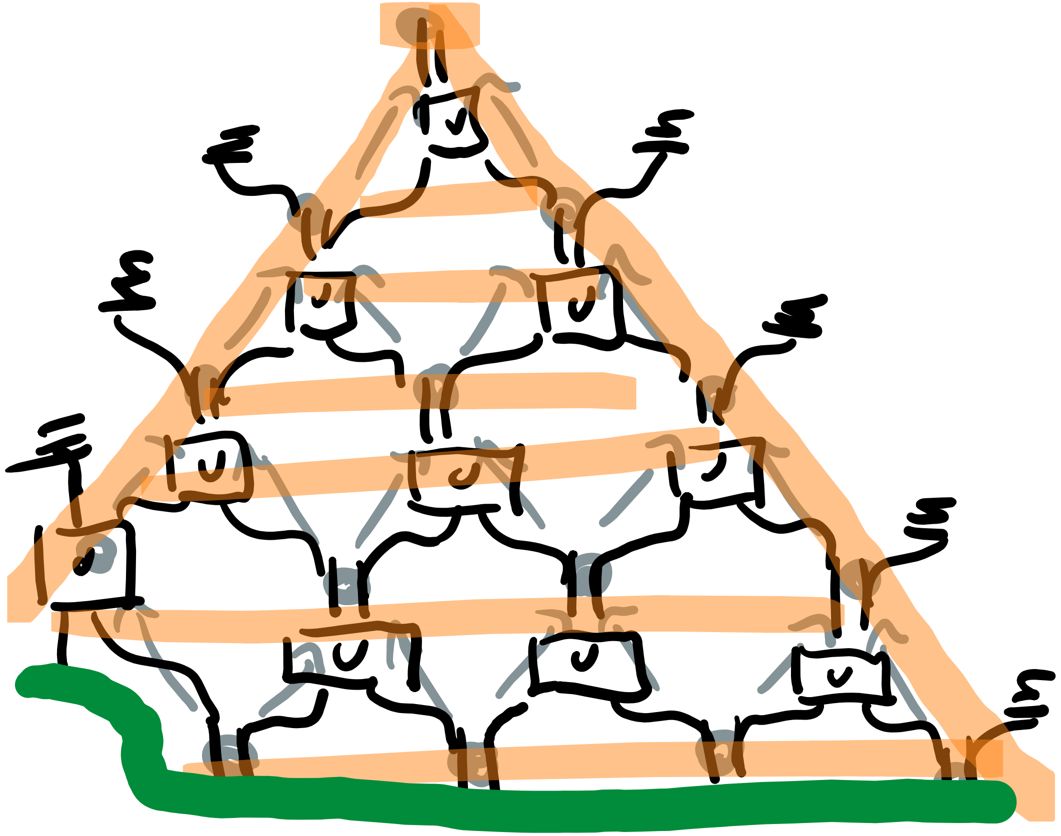
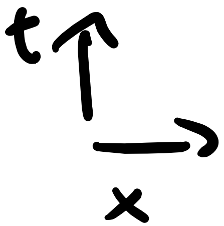


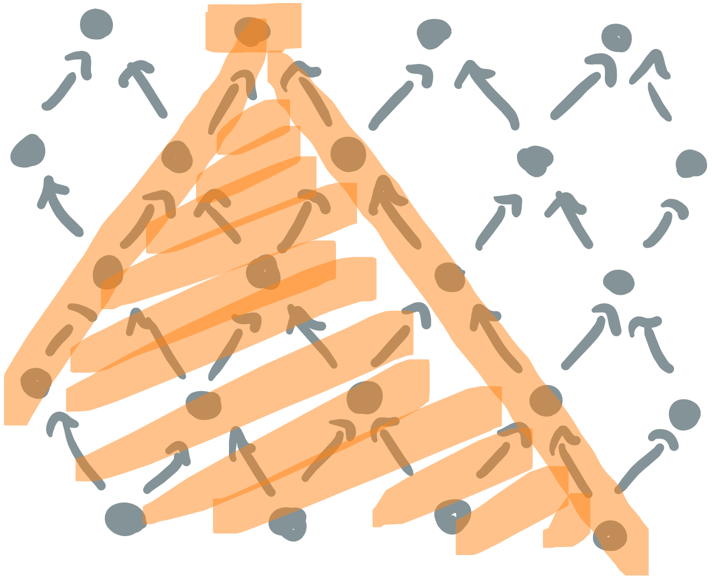
Hasse diagram for causal order



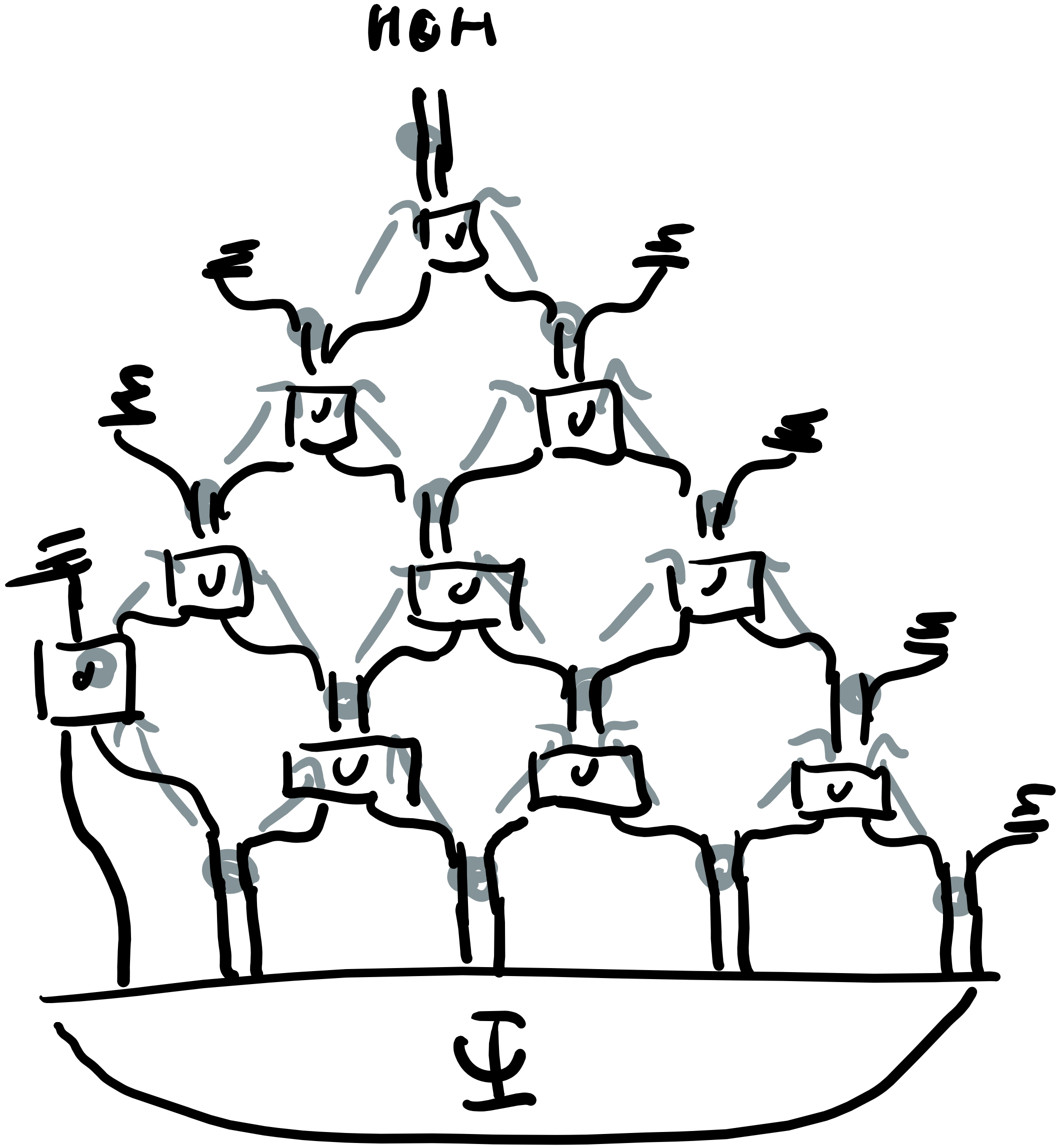
$H \otimes H$

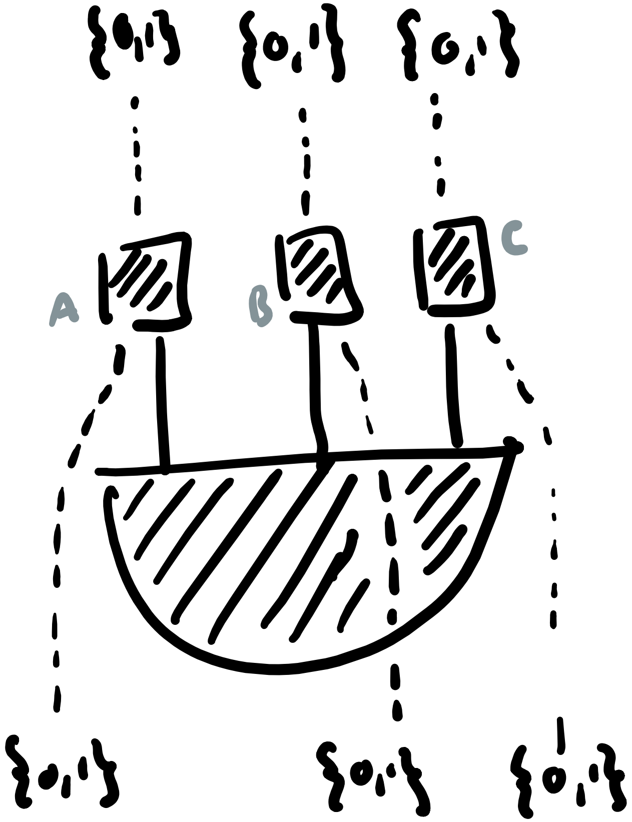


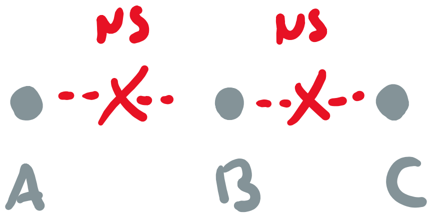
(1+1)-Minkowski

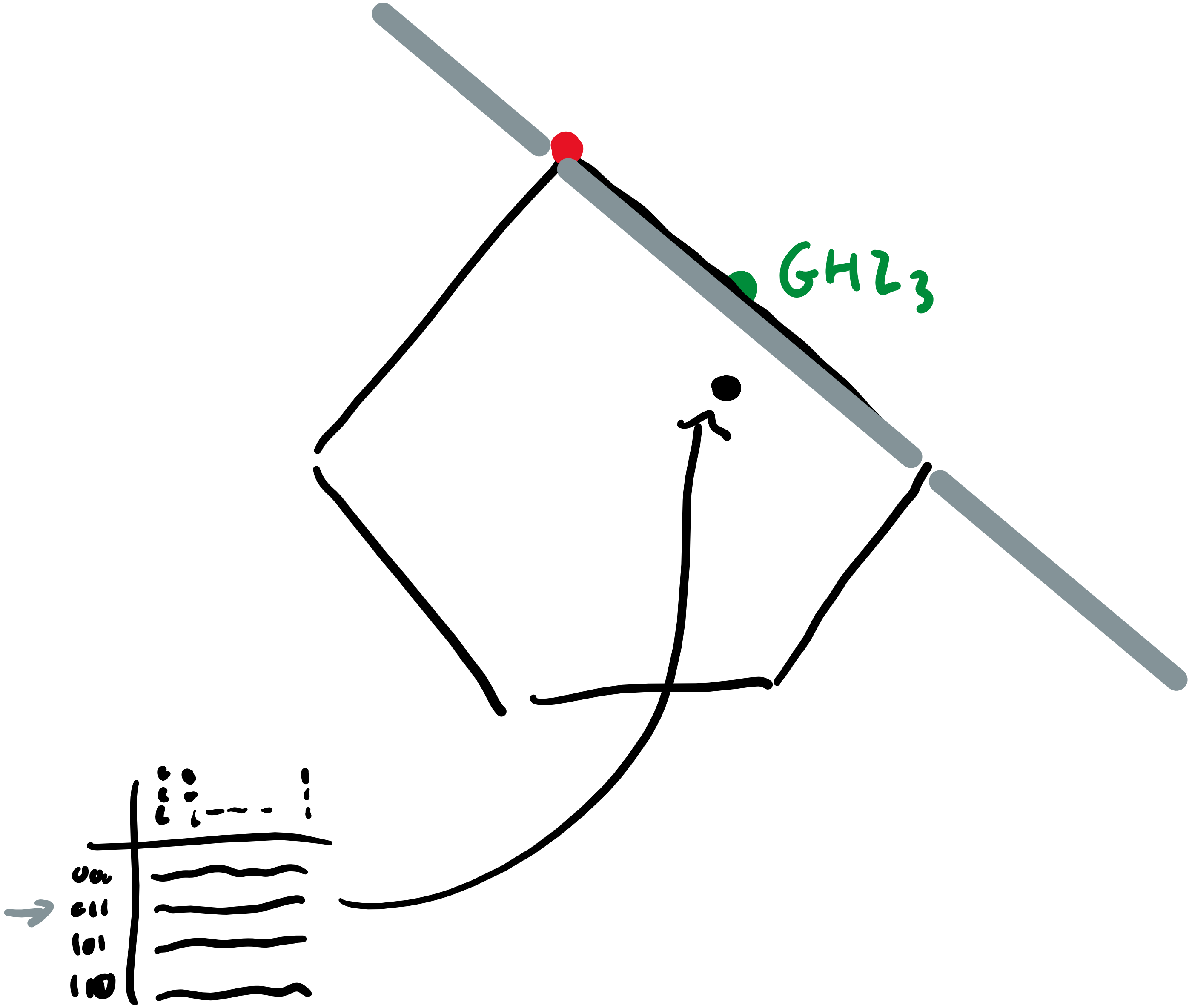


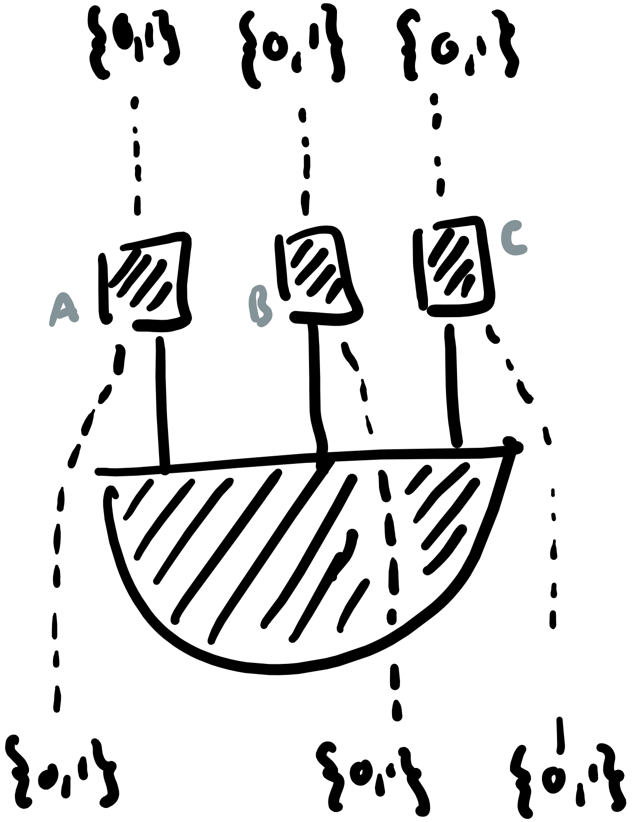
Hasse diagram for causal order

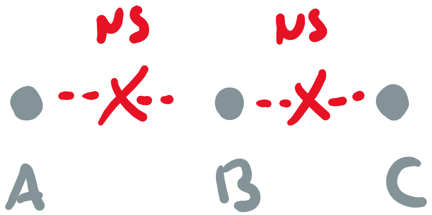



{0,1}
{0,1}
{0,1}
A
B
C
{0,1}
{0,1}
{0,1}


NS
NS
A
B
C


GHZ3
000
011
101
110





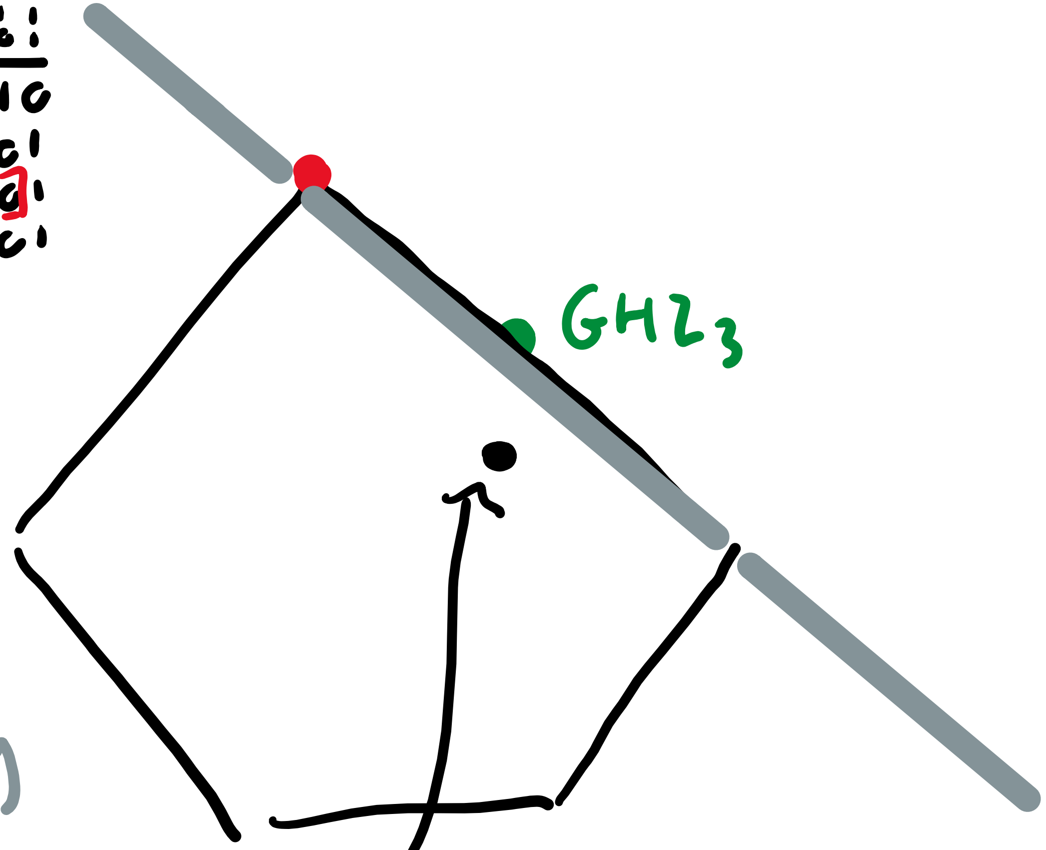
	000	001	010	011	100	101	110	111
→ 000	1	0	0	$\boxed{1}$	0	1	1	0
011	0	1	1	0	1	0	0	1
→ 101	0	1	1	0	1	0	$\boxed{0}$	1
110	0	1	1	0	1	0	0	1

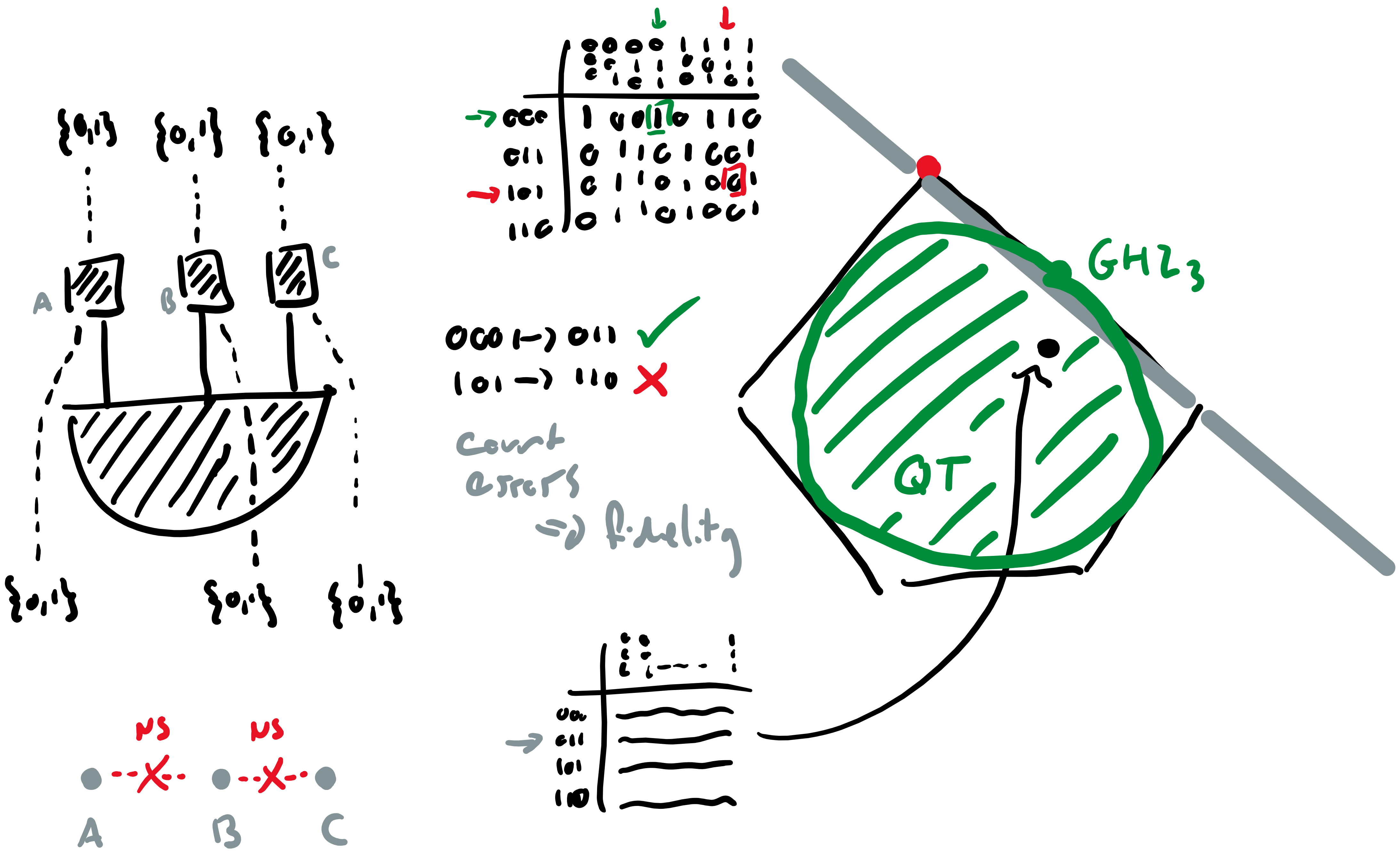
000 ↦ 011 ✓

101 → 110 ✗

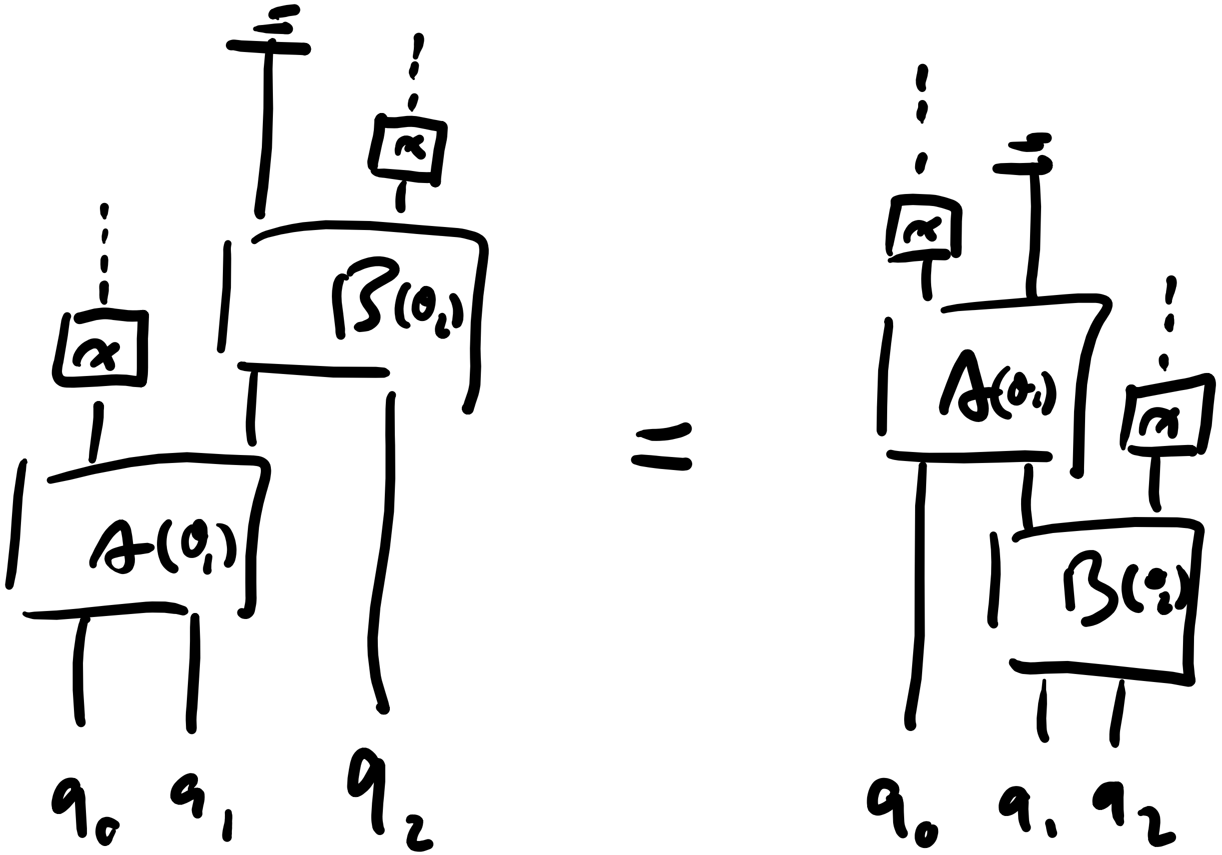
count errors

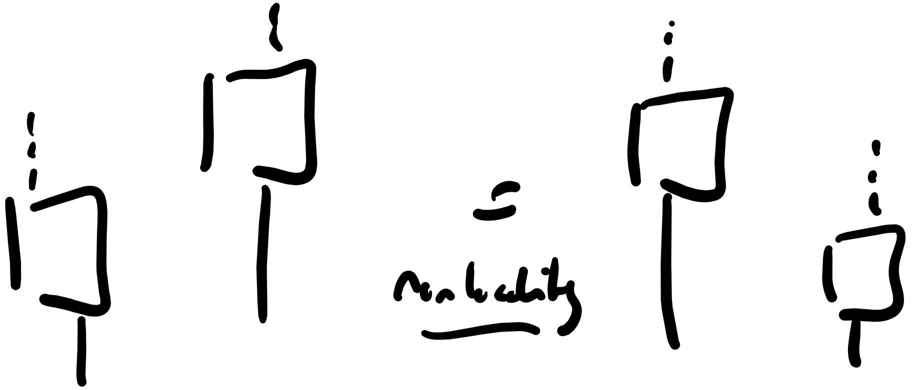
⇒ fidelity





{0,1} {0,1} {0,1}
A B C
{0,1} {0,1} {0,1}
NS NS
A B C
000 ↦ 011
101 → 110
Count errors
⇒ fidelity
GHZ3
QT





$A(O_1)$, $B(O_2)$

simultaneously performed

$\Downarrow$

contextuality