

# Quantum in Pictures Lecture Series

Lecturer: Stefano Gogioso

Fri 30 June 2023 –Morning Lecture



INDIANA UNIVERSITY BLOOMINGTON

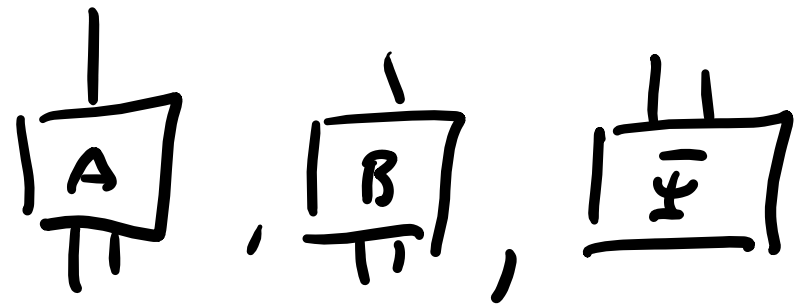
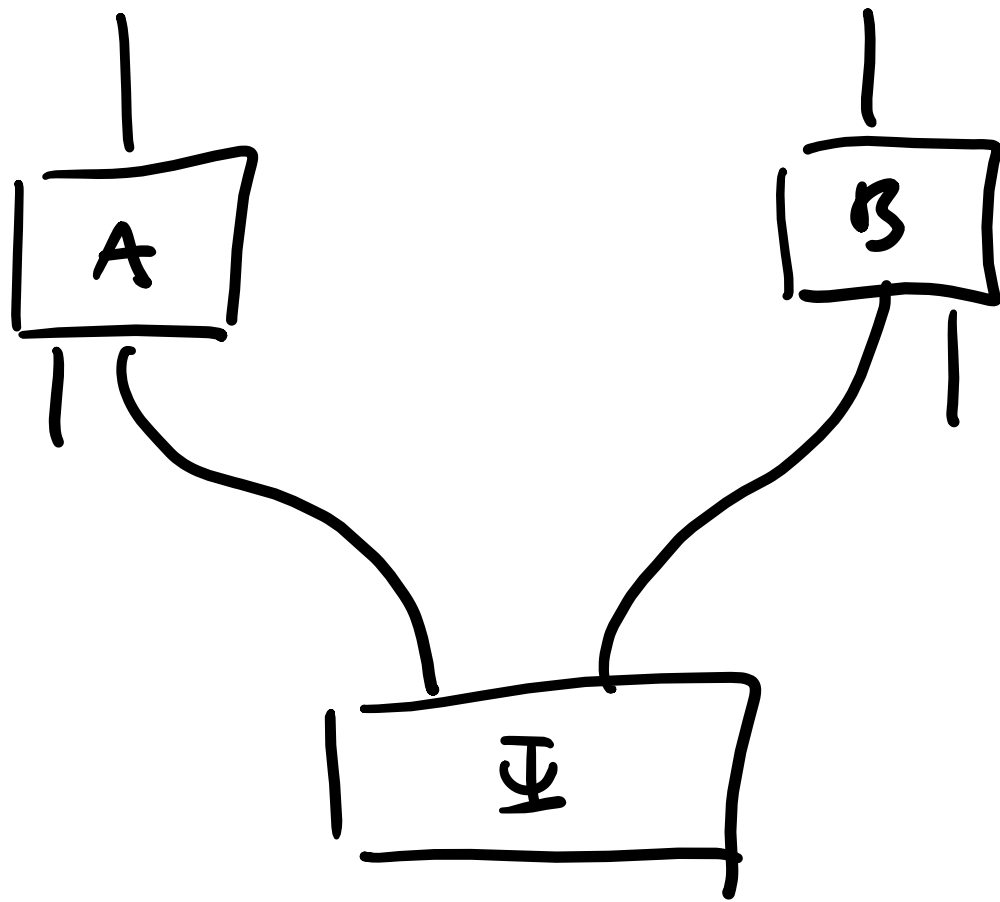
$$\begin{array}{|c|} \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array} \begin{array}{|c|} \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array} = \begin{array}{|c|} \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array} \dots \begin{array}{|c|} \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array}$$

$\rho$  is causal

e.g.

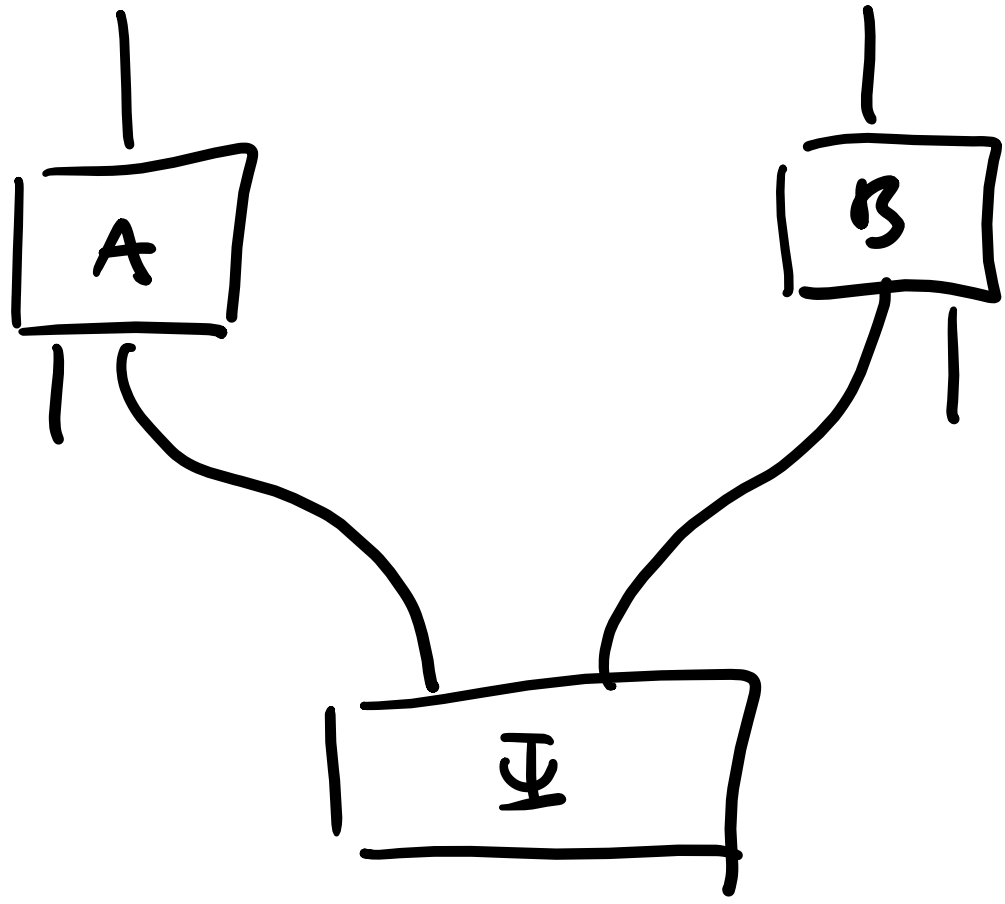
$$\begin{array}{|c|} \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array} \mathbb{C}^d = \sum_{j=0}^{d-1} \begin{array}{|c|} \hline j \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array} = \text{Tr}[-] \quad (|j\rangle)_{j=0}^{d-1} \text{ or } \mathcal{B}$$

$$\begin{array}{|c|} \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array} \mathbb{R}^{+d} = \sum_{i=0}^{d-1} \begin{array}{|c|} \hline i \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array} \quad j = 0, \dots, d-1 \quad (\text{deterministic states})$$

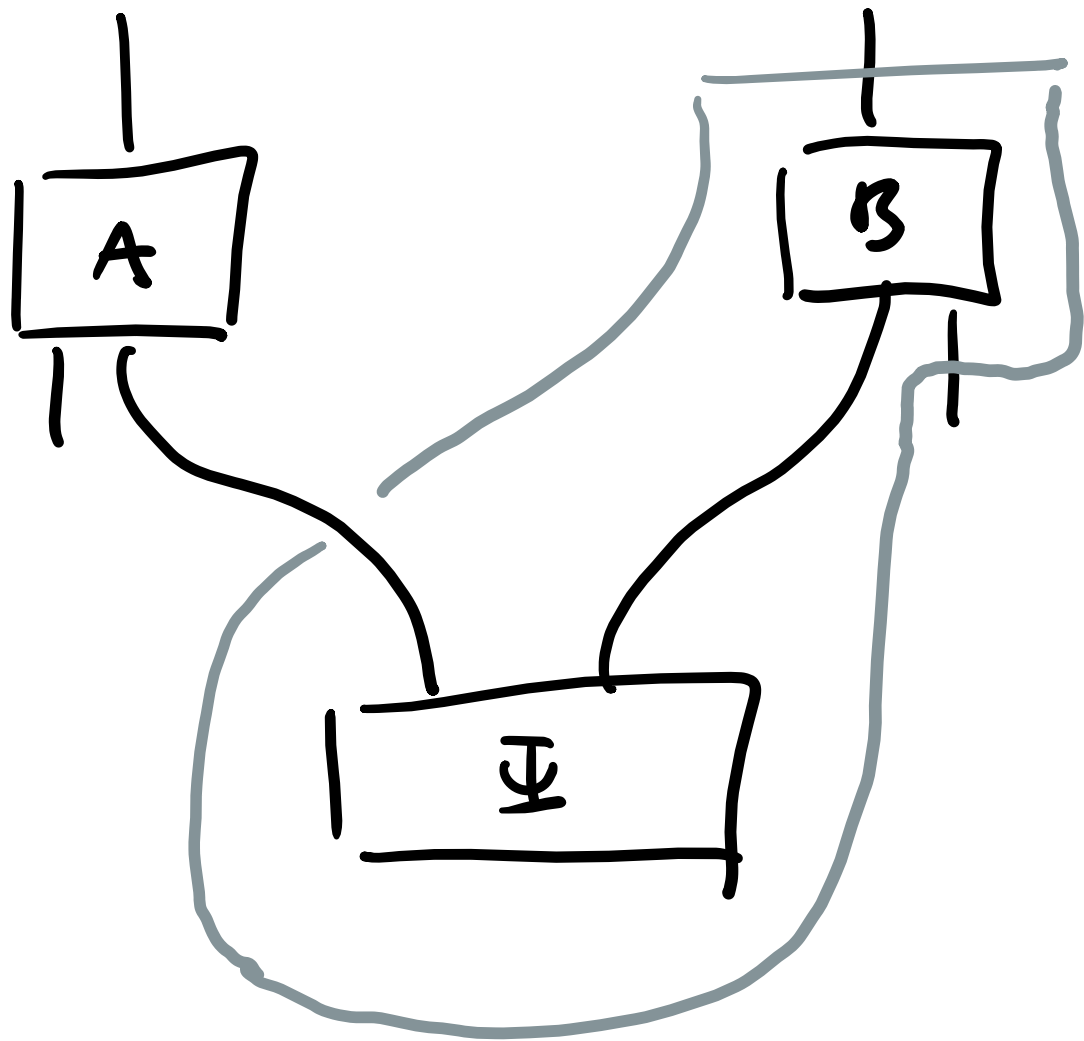


are all causal  
(sure-things, "deterministic")

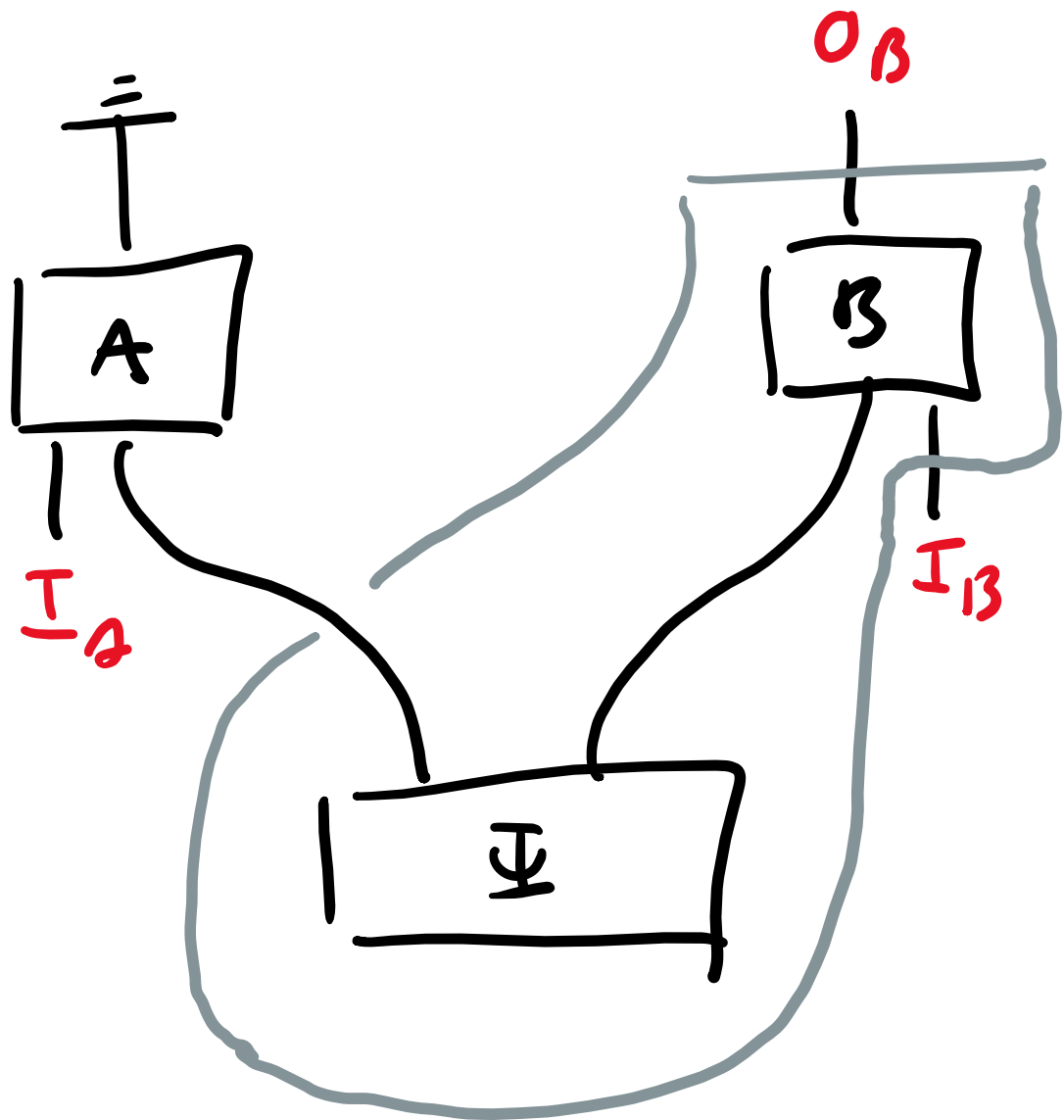
Diagrams forming a partial order (no cycles)



$$\begin{array}{l}
 \begin{array}{c} \text{---} \\ | \\ \boxed{A} \\ | \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \\
 \begin{array}{c} \text{---} \\ | \\ \boxed{B} \\ | \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \\
 \begin{array}{c} \text{---} \\ | \\ \boxed{C} \\ | \\ \text{---} \end{array} = 1
 \end{array}$$

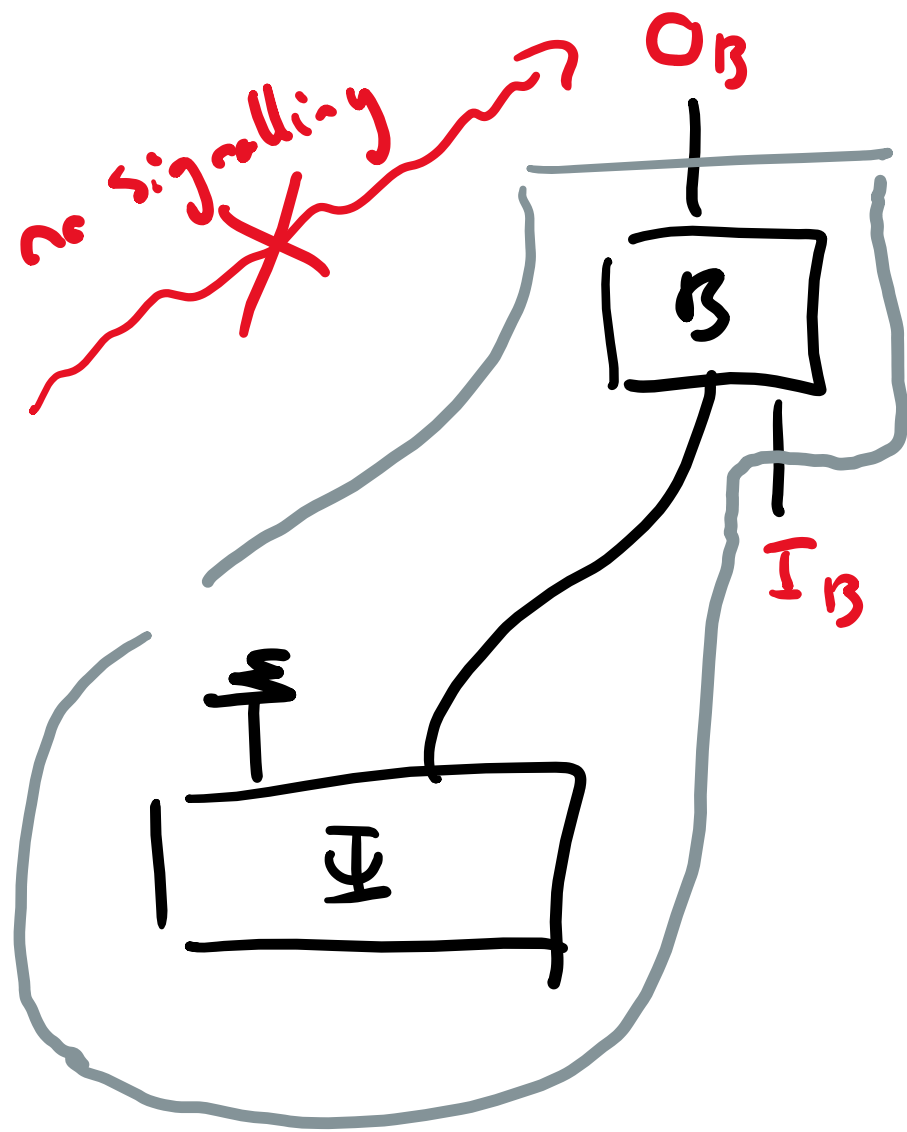


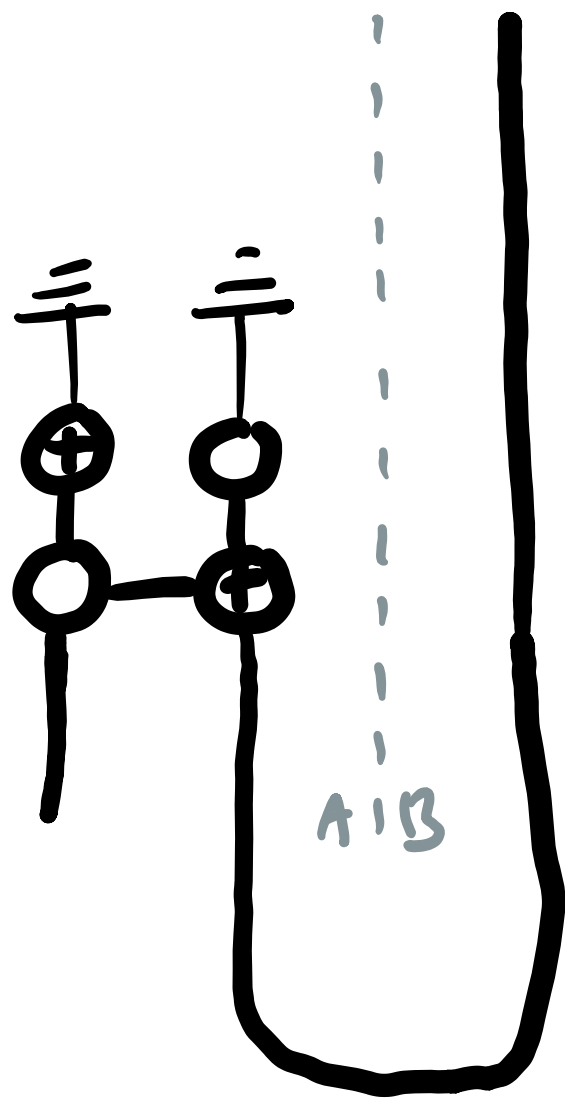
$$\begin{aligned}
 & \begin{array}{c} \text{---} \\ | \\ \boxed{A} \\ | \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \\
 & \begin{array}{c} \text{---} \\ | \\ \boxed{B} \\ | \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \\
 & \begin{array}{c} \text{---} \\ | \\ \boxed{C} \\ | \\ \text{---} \end{array} = 1
 \end{aligned}$$



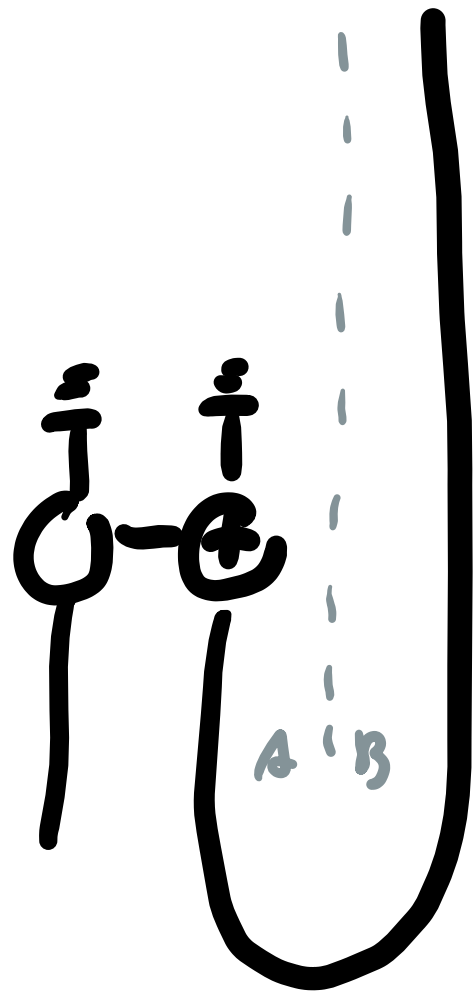
$I_A$

$I_A$





$$\begin{aligned} & \text{H} \text{---} \text{CNOT} \text{---} \text{H} \\ & \text{H} \text{---} \text{CNOT} \text{---} \text{H} \\ & = \\ & \text{H} \text{---} \text{CNOT} \text{---} \text{H} \\ & = \end{aligned}$$



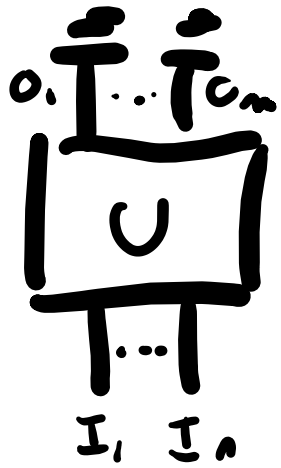
$$= \text{H} \text{---} \text{CNOT} \text{---} \text{H}$$



Local state at Bob's  
without comms from Alice

discor  $\rightarrow$   $\frac{1}{2}$   $\leftarrow$  max mix state

CP maps which are CPTP  
(1 Kraus op)

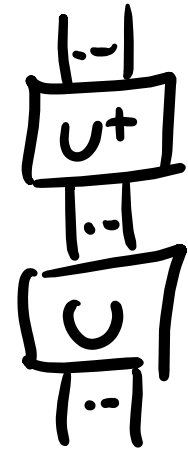


$$= \begin{matrix} \bullet & \bullet \\ | & | \\ \dots & \dots \\ \bullet & \bullet \\ I_1 & I_n \end{matrix}$$

$\Leftrightarrow$

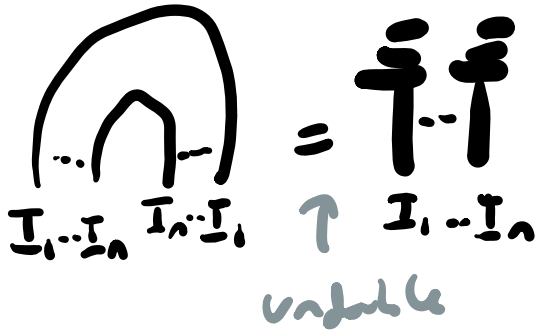
U isometry

(Kraus op is an isometry)

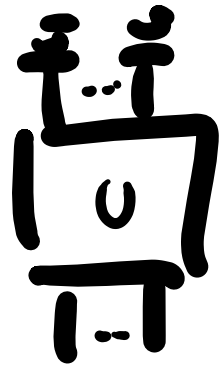


$$= \begin{matrix} | & | \\ \dots & \dots \\ | & | \end{matrix}$$

Proof:

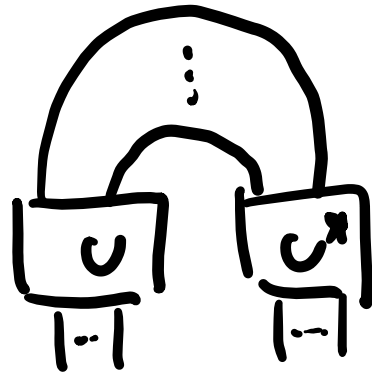


$=$



$=$

variable



$$\begin{matrix} | & | \\ \dots & \dots \\ | & | \\ I_1 & I_n \end{matrix} = \dots$$





$$\mathbb{R}^d = \sum_{j=0}^{d-1} \text{span} \{ | \psi_j \rangle \} \quad \forall (| \psi_j \rangle)_{j=0}^{d-1} \text{ ONB}$$

Proof:

$$| \psi \rangle = \sum_{j=0}^{d-1} \langle \psi_j | \psi \rangle | \psi_j \rangle$$

band  
wire  
 $\Rightarrow$

$$| \psi \rangle = \sum_{j=0}^{d-1} \langle \psi_j | \psi \rangle | \psi_j \rangle = \sum_{j=0}^{d-1} \langle \psi_j | \psi \rangle | \psi_j \rangle$$

$$| \psi \rangle = \sum_j \langle \psi_j | \psi \rangle | \psi_j \rangle$$

□

$$\prod_{\mathbb{R}^{+d}} \stackrel{\text{st}}{=} \prod_j = 1 \quad \forall j=0, \dots, d-1$$

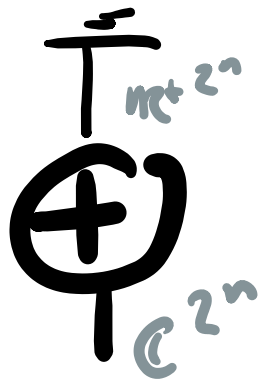
$$\prod_j = 0 \quad \frac{0}{j} = 1 \quad \forall j=0, \dots, d-1$$

$$\langle + | j \rangle = 1 \quad \forall j$$

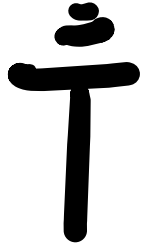
$$\text{⌐}_{\mathbb{C}^d}^{\mathbb{R}^d} = \text{⌐} \quad 2 \text{ bar. } \mathbb{R}$$

Proof:

$$\text{⌐}_{\mathbb{C}^d}^{\mathbb{R}^d} = \text{⌐} \overset{\text{fusion}}{=} \text{⌐} \overset{\text{fusion}}{=} \text{⌐} = \text{⌐}$$



=

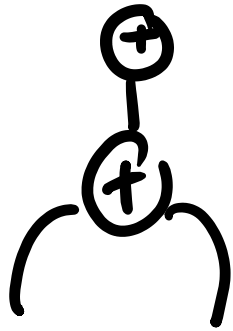


X bar.  
(for  $\mathbb{C}[\mathbb{Z}^{2n}]$ )

Proof:



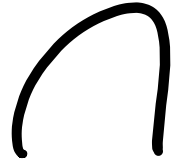
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fusion  
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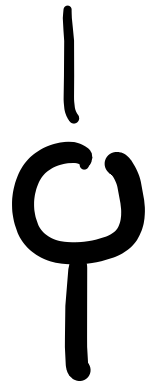


fusion  
=



=





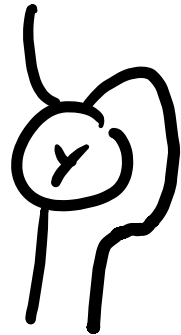
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CALCULAI



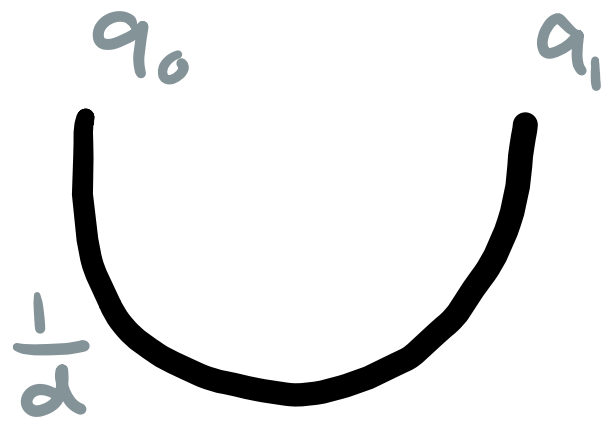
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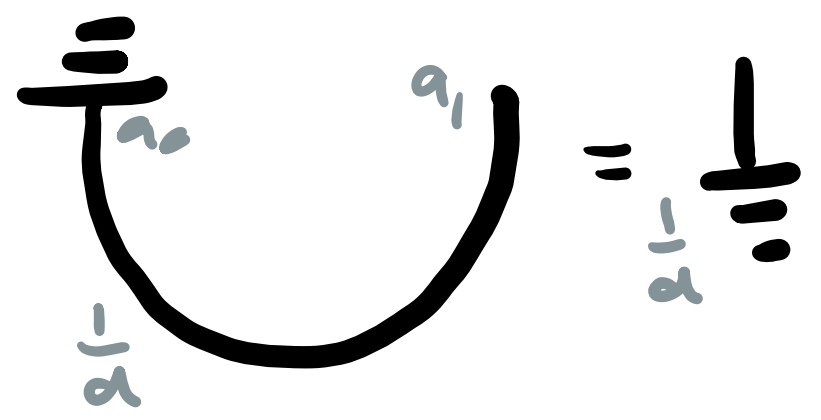
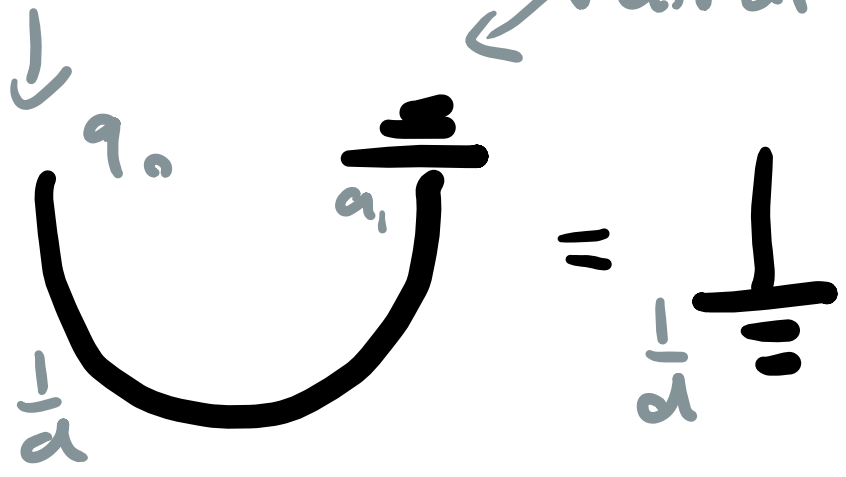


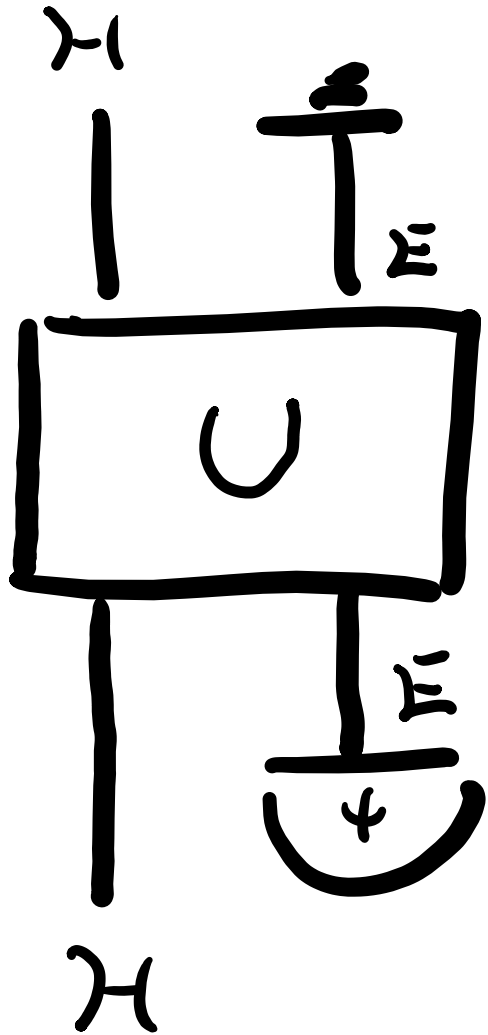
$\cap \neq \cup$



marginal state

Partial trace



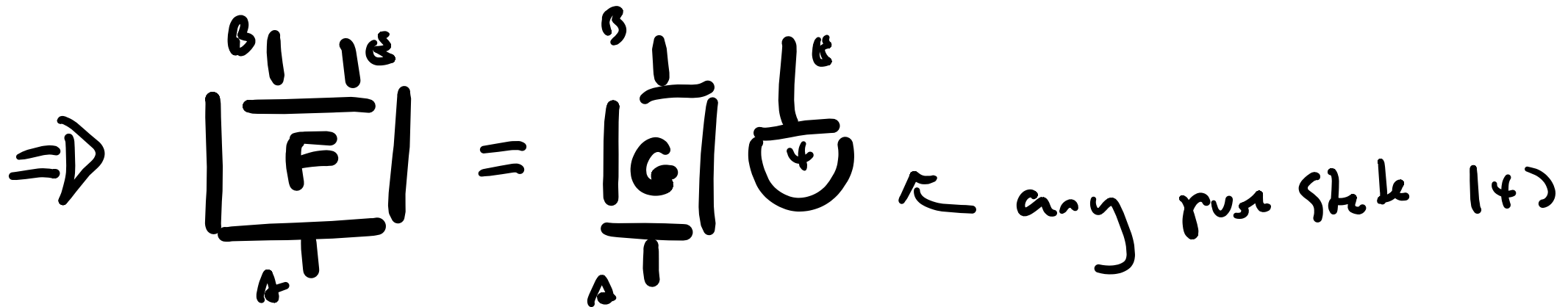
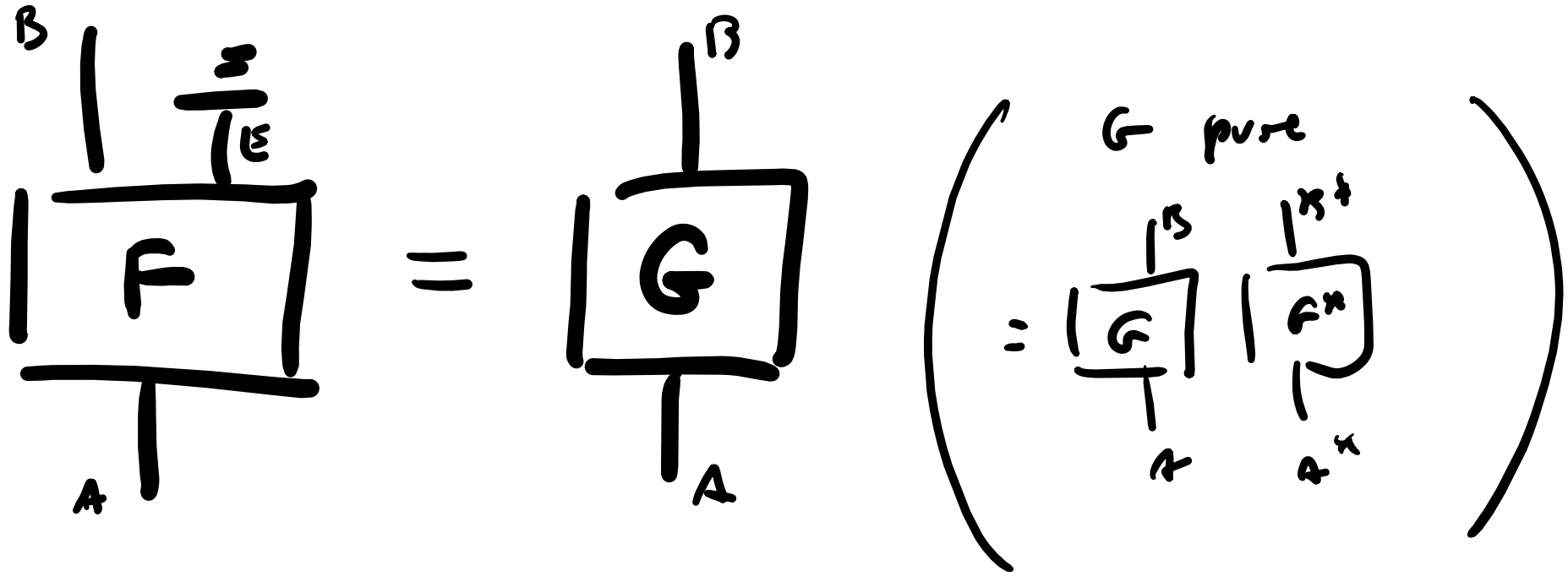


U unitary  
 147 pure state

CPTP map

(dissipation?  
 ignorance?)

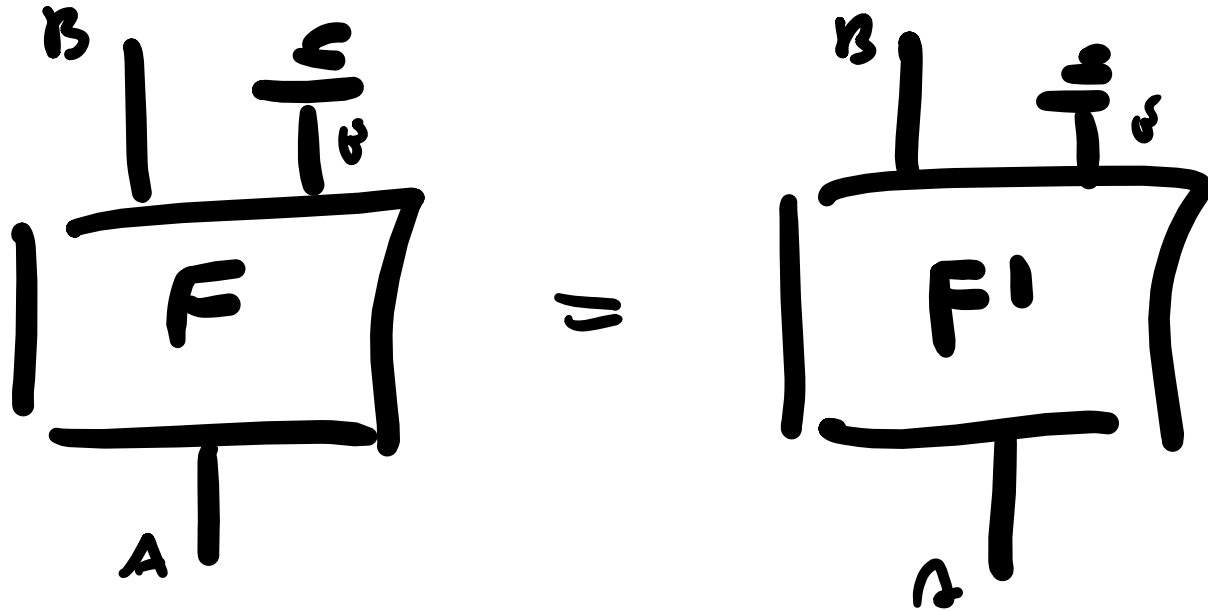
# Purification Principle



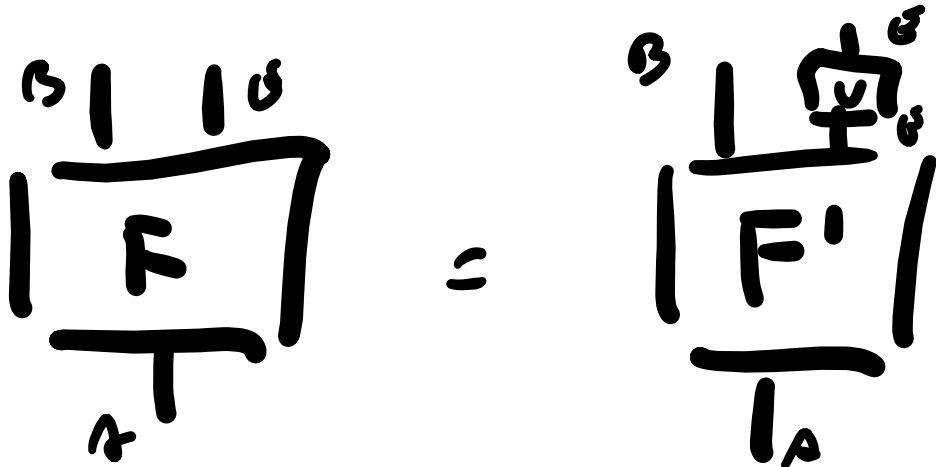


BUP

# Essentially Unique Purification Principle



$\Rightarrow \exists U_{\text{unitary}}$



S.B.

$$\begin{array}{c}
 \begin{array}{c} \alpha \\ \text{---} \\ \psi \end{array} \\
 \cup \\
 \begin{array}{c} \beta \\ \text{---} \\ \psi \end{array}
 \end{array}
 \left\{ \begin{array}{l}
 \begin{array}{c} \alpha \\ \text{---} \\ \psi_1 \end{array} \\
 \cup \\
 \begin{array}{c} \beta \\ \text{---} \\ \psi_2 \end{array}
 \end{array}
 = \begin{array}{c} \alpha \\ \text{---} \\ \psi_1 \end{array}
 \Rightarrow \begin{array}{c} \alpha \\ \text{---} \\ \psi \end{array} = \begin{array}{c} \psi_1 \\ \text{---} \\ \psi \end{array} \cup \begin{array}{c} \psi_2 \\ \text{---} \\ \psi \end{array}$$

Prüf:

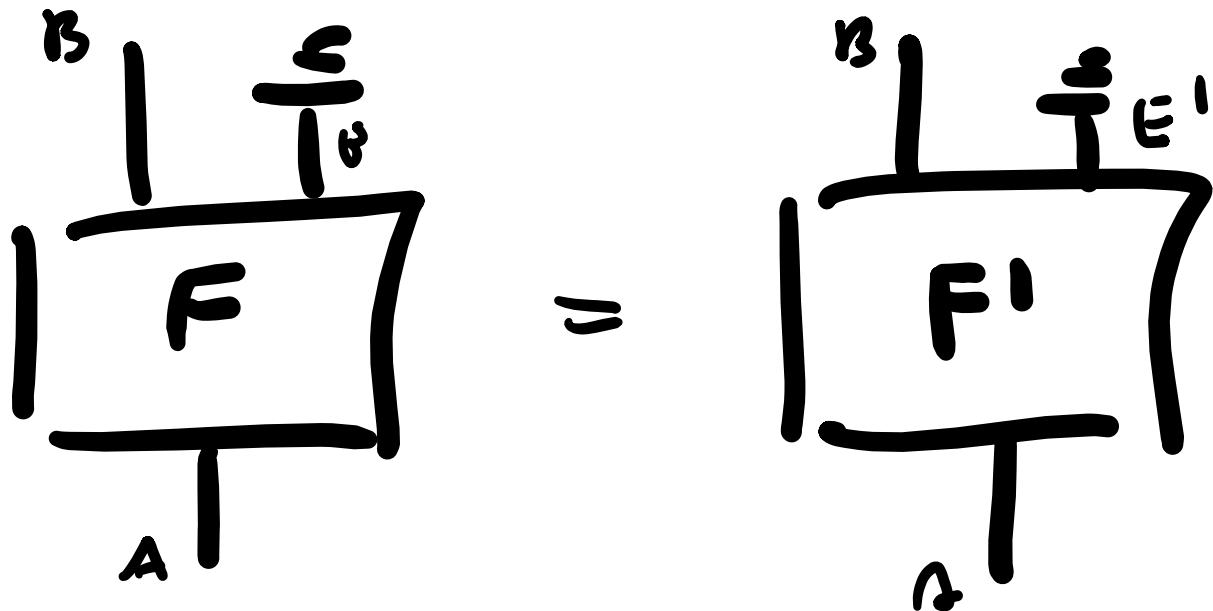
$$\begin{array}{c} \alpha \\ \text{---} \\ \psi \end{array} = \begin{array}{c} \alpha \\ \text{---} \\ \psi_1 \end{array} \stackrel{\text{Prüf.}}{\Rightarrow} \begin{array}{c} \alpha \\ \text{---} \\ \psi \end{array} = * \begin{array}{c} \alpha \\ \text{---} \\ \psi_1 \end{array} \cup \begin{array}{c} \beta \\ \text{---} \\ \psi_2 \end{array}$$

$$\begin{array}{c} \beta \\ \text{---} \\ \psi_2 \end{array} = \begin{array}{c} \beta \\ \text{---} \\ \psi_2 \end{array} \stackrel{(*)}{=} \begin{array}{c} \beta \\ \text{---} \\ \psi_2 \end{array} \cup \begin{array}{c} \beta \\ \text{---} \\ \psi_2 \end{array} = \begin{array}{c} \beta \\ \text{---} \\ \psi_2 \end{array}$$

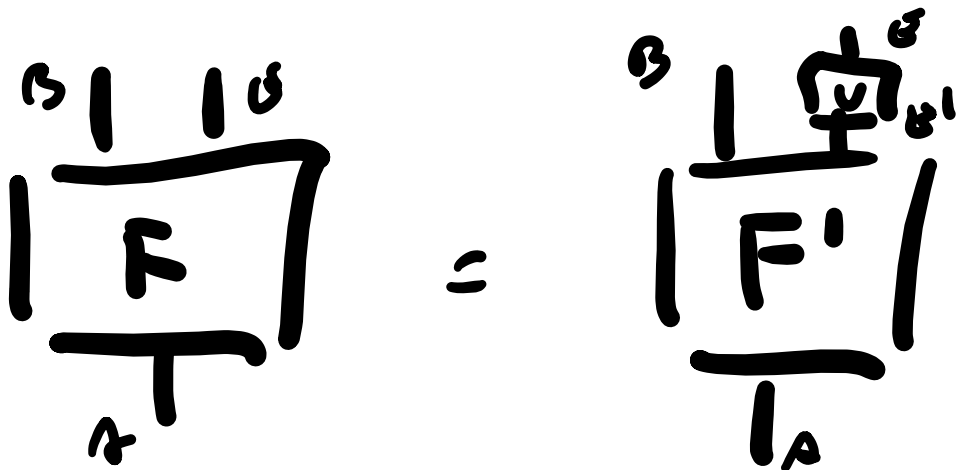
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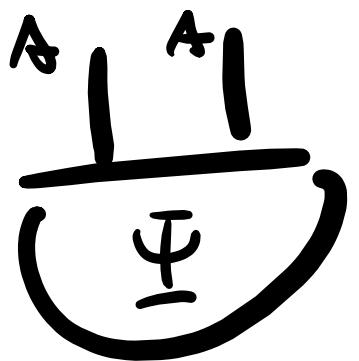
BUP

# Essentially Unique Purification Principle



$\Rightarrow \exists$  isometry  $U$



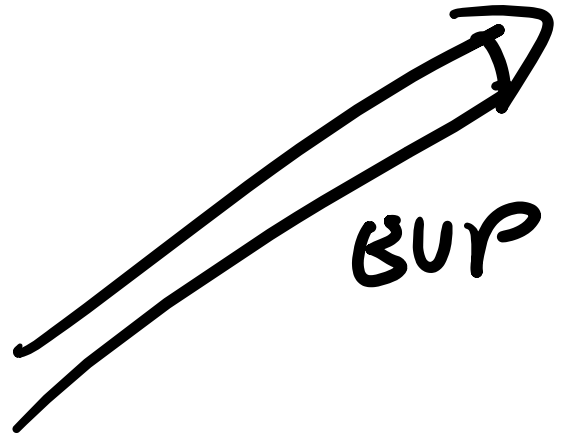


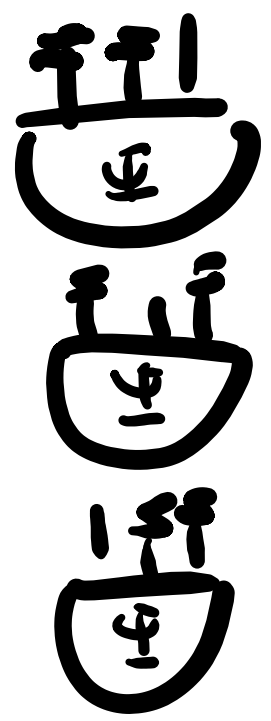
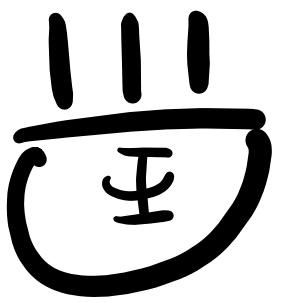
st.  $\frac{\text{cup}}{\Psi} = \text{line}$

$\Rightarrow \frac{\text{cup}}{\Psi} = \text{cup}$   
 for some U unitary

Proof:

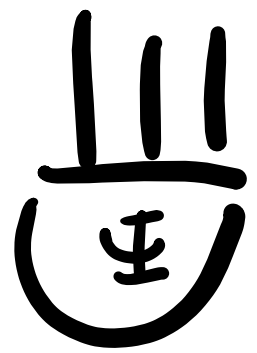
$\frac{\text{cup}}{\Psi} = \text{line} = \text{cup}$





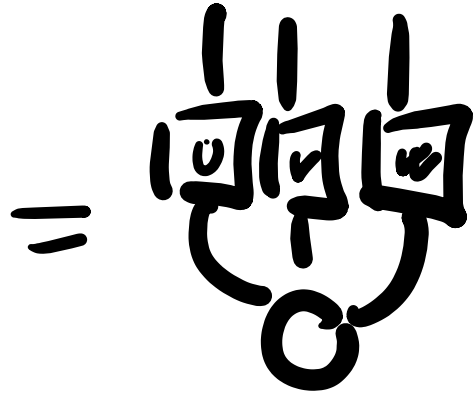
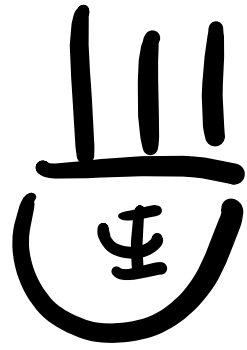
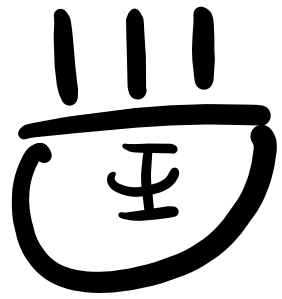
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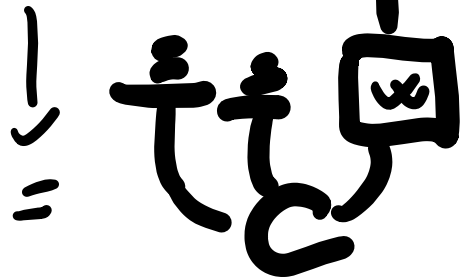
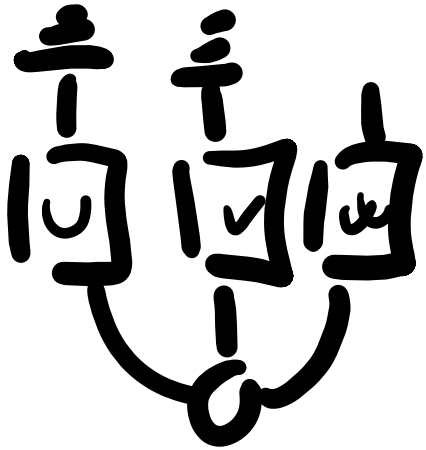


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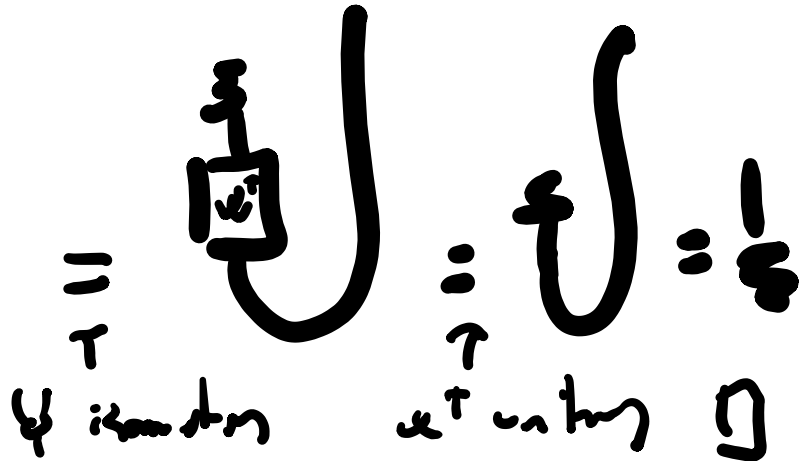




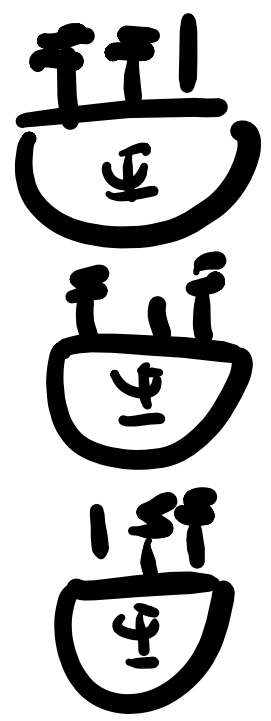
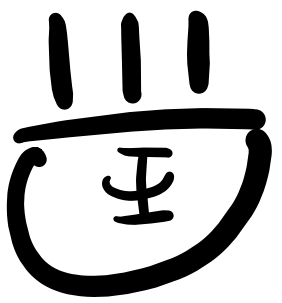
u, v using



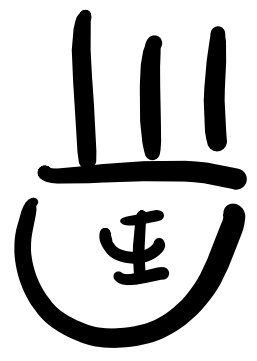
u, v using  
so u^T u = 1



u^T u = 1



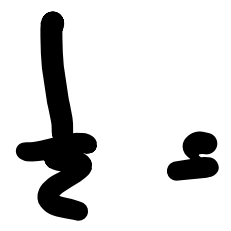
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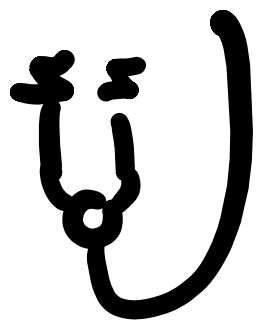
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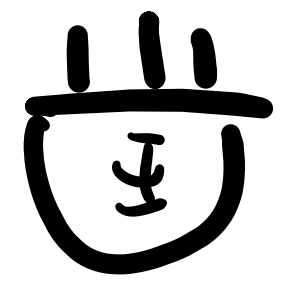
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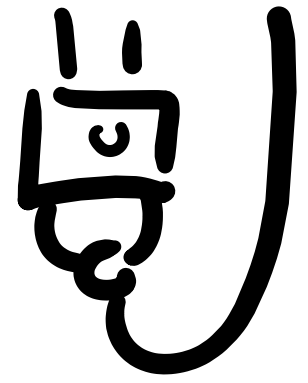
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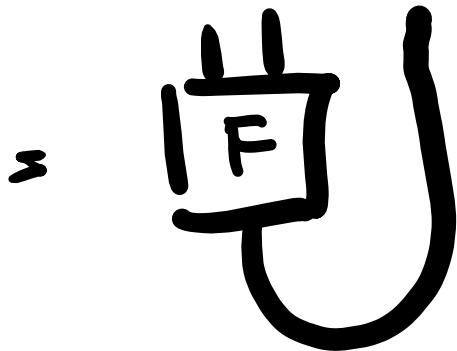
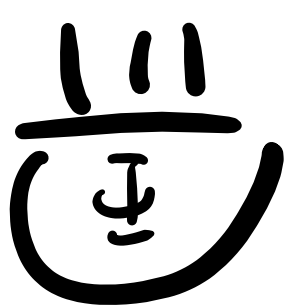
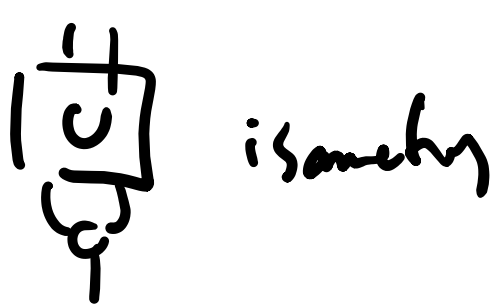
sur  
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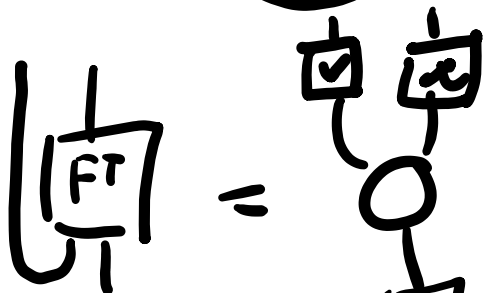
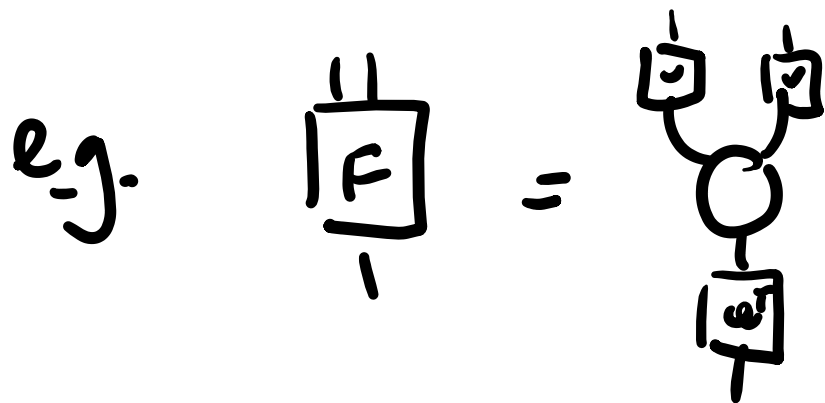
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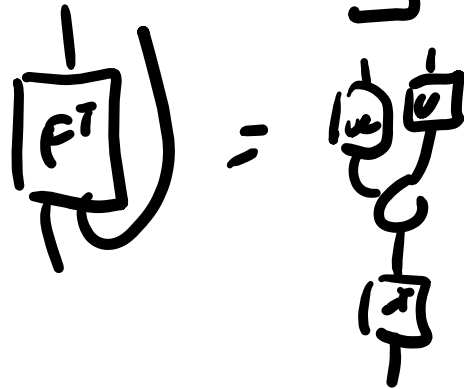
U  
U  
U



F isometry

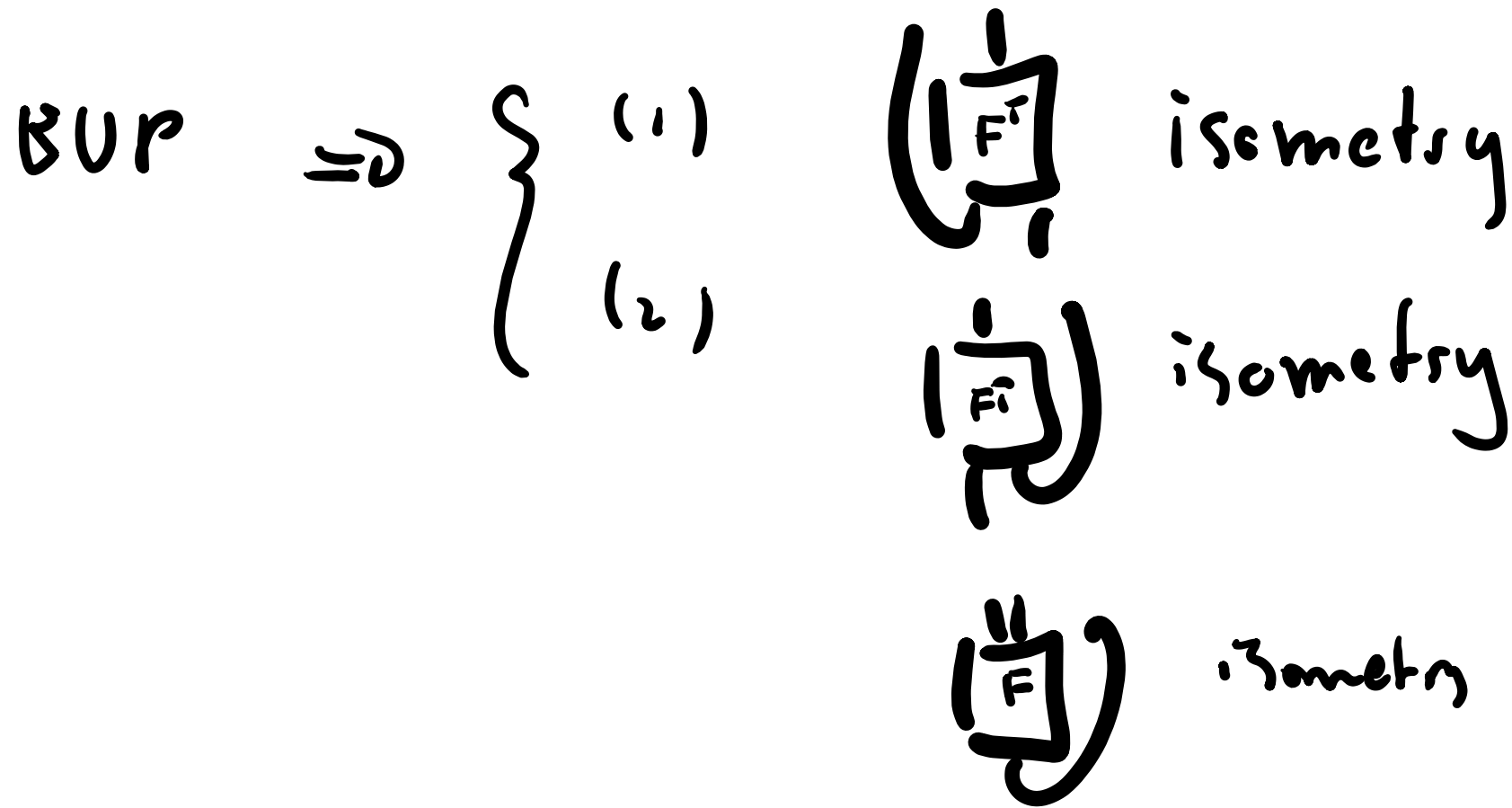
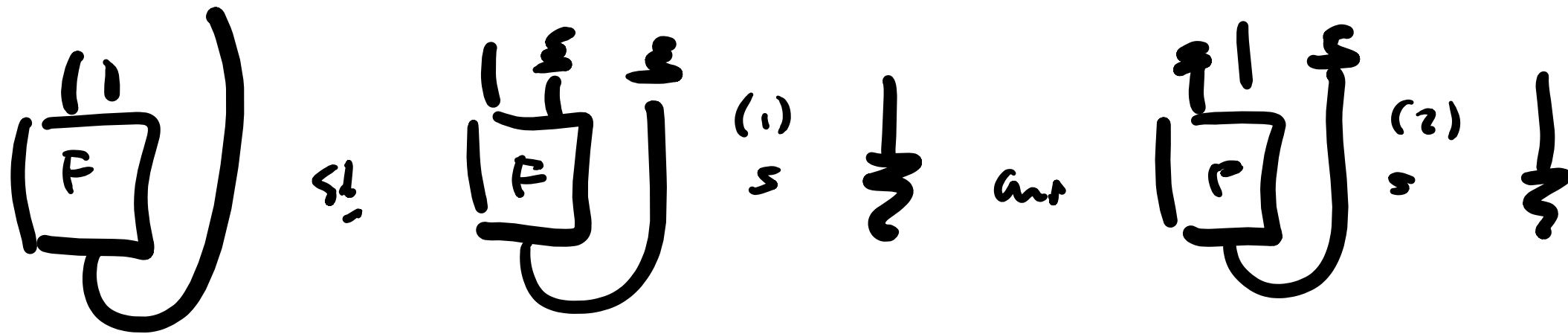


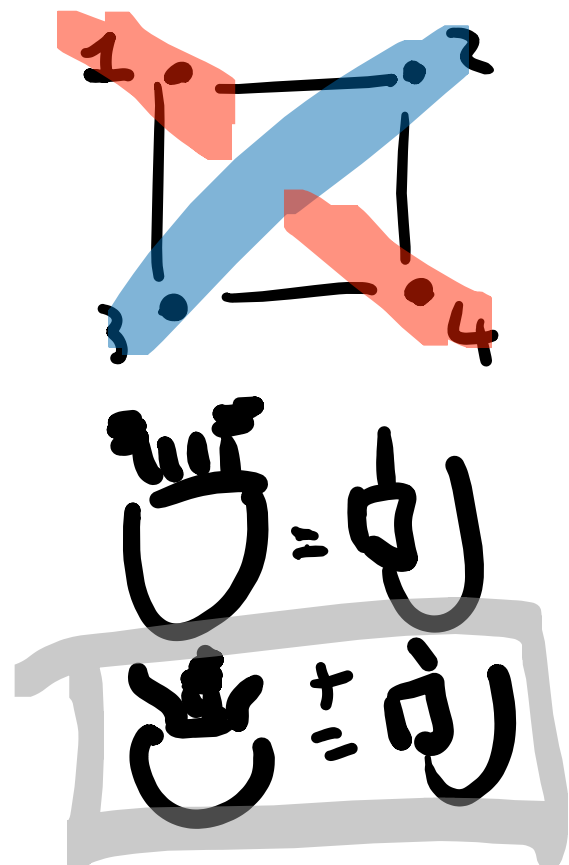
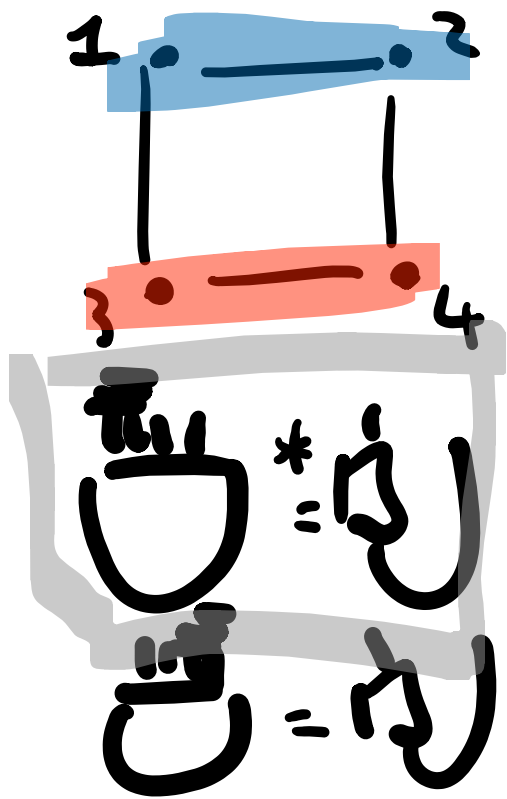
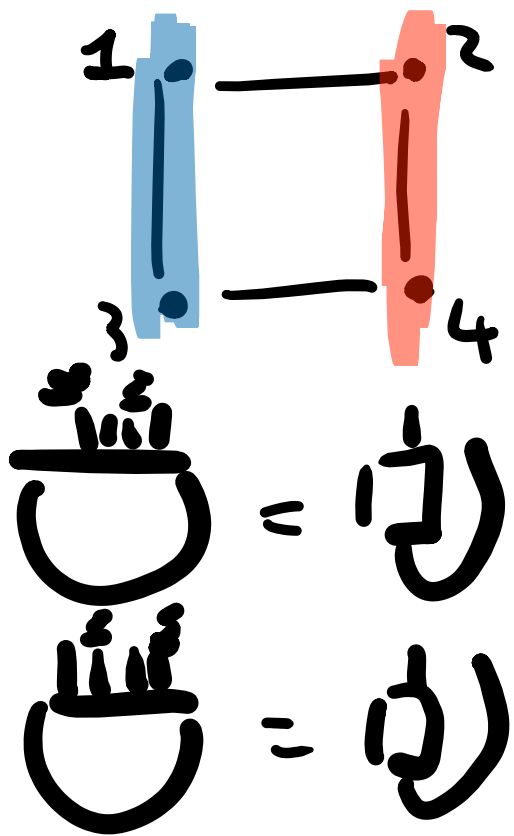
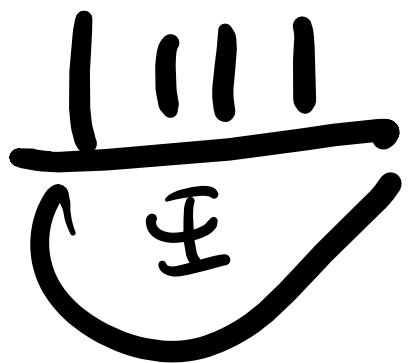
isometry

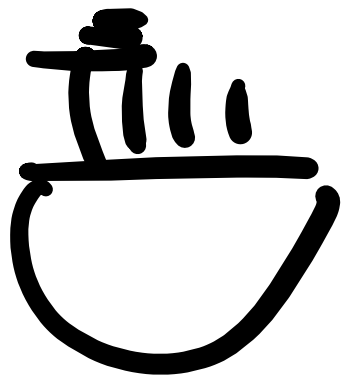


isometry

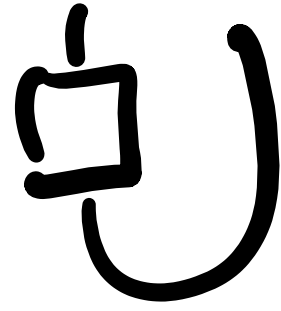




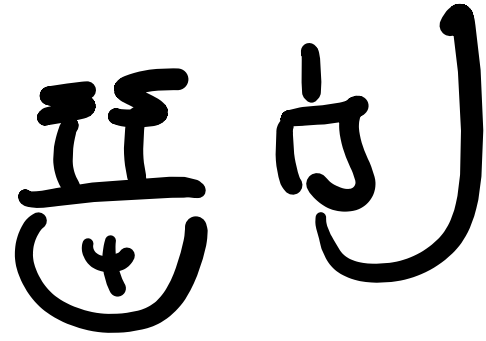




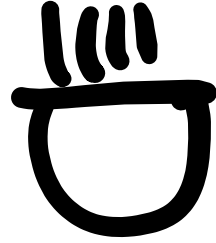
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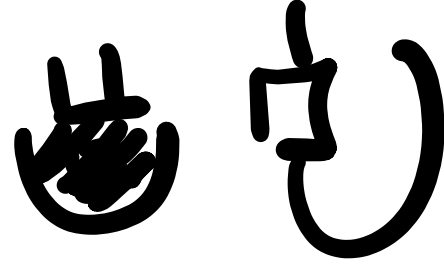
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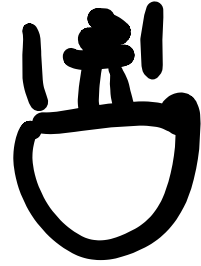
Purich  
=>



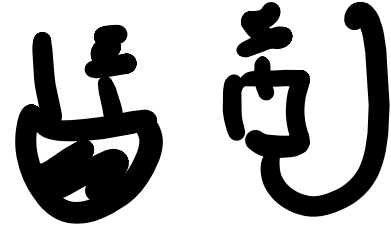
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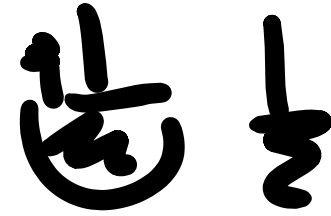
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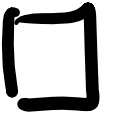
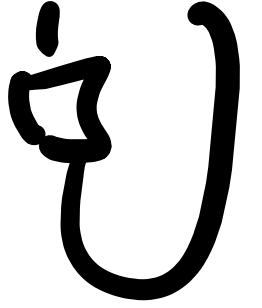
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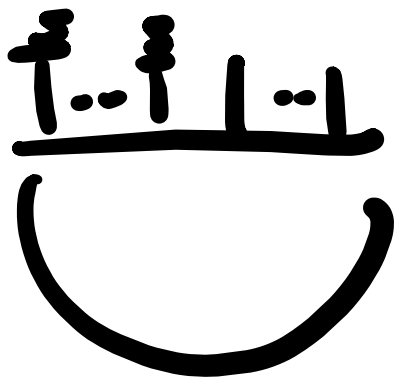
(+)



$$AME(n, m)$$

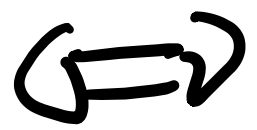
↑  
number  
of parties

↑  
dimension of space

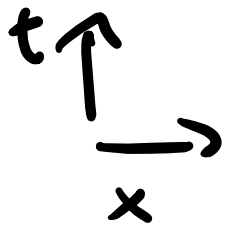


maximally entangled

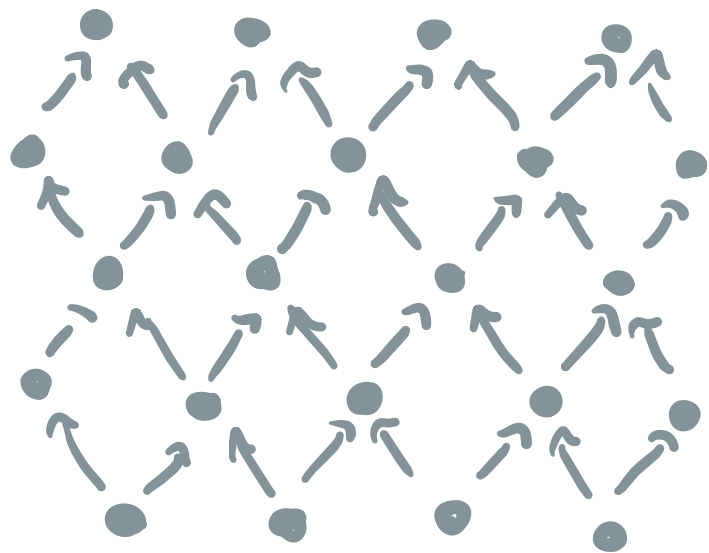
$$\sum_{\mu} \sum_{j} \text{Tr} [\mu_j P]^2 = 0$$



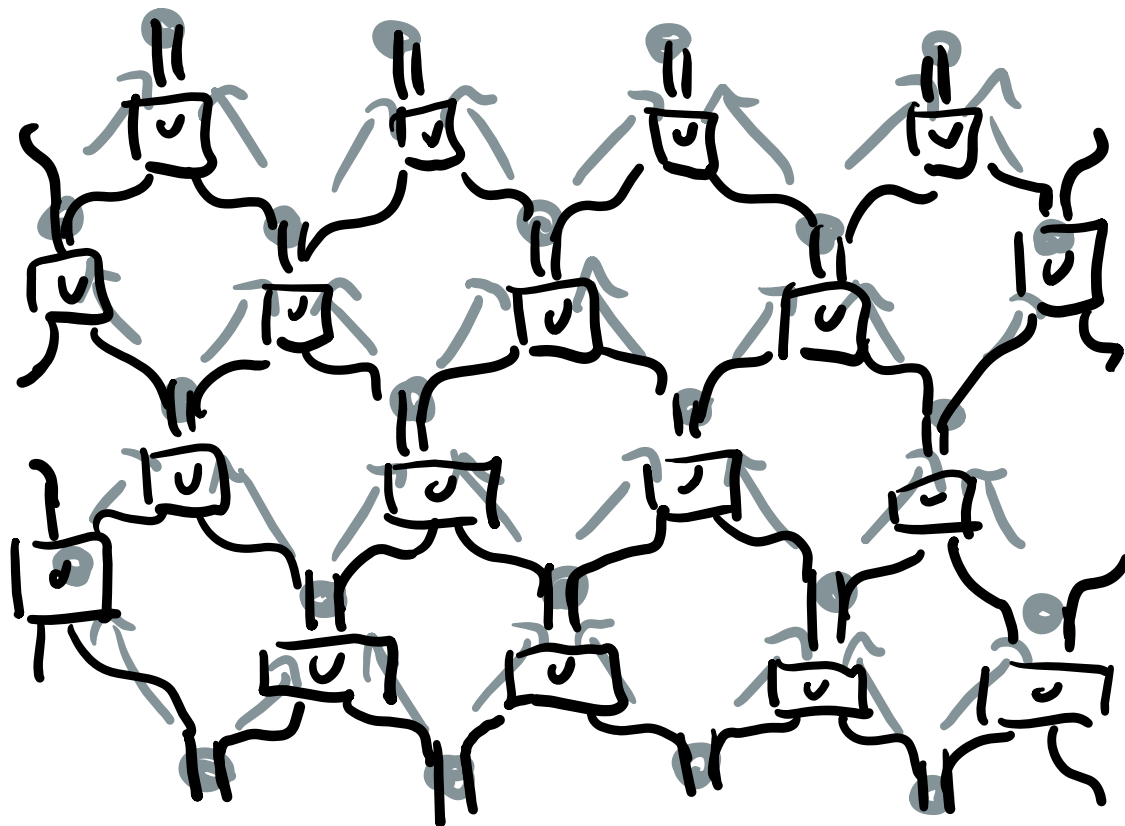
$$\begin{matrix} \frac{\mu_{11}}{P} = \frac{1}{2} \\ \vdots \\ \frac{\mu_{21}}{P} = \frac{1}{2} \end{matrix}$$



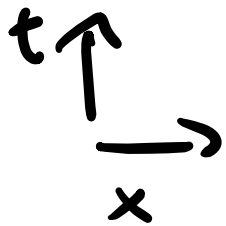
(1+1)-Minkowski



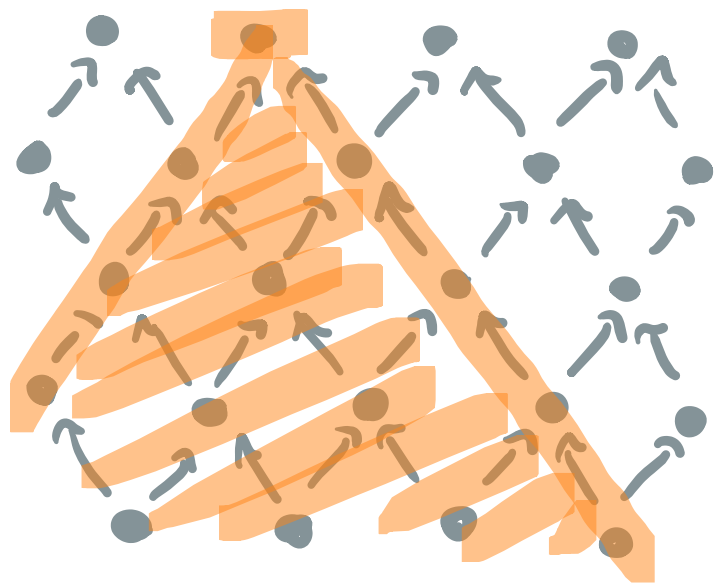
Hasse diagram for causal order



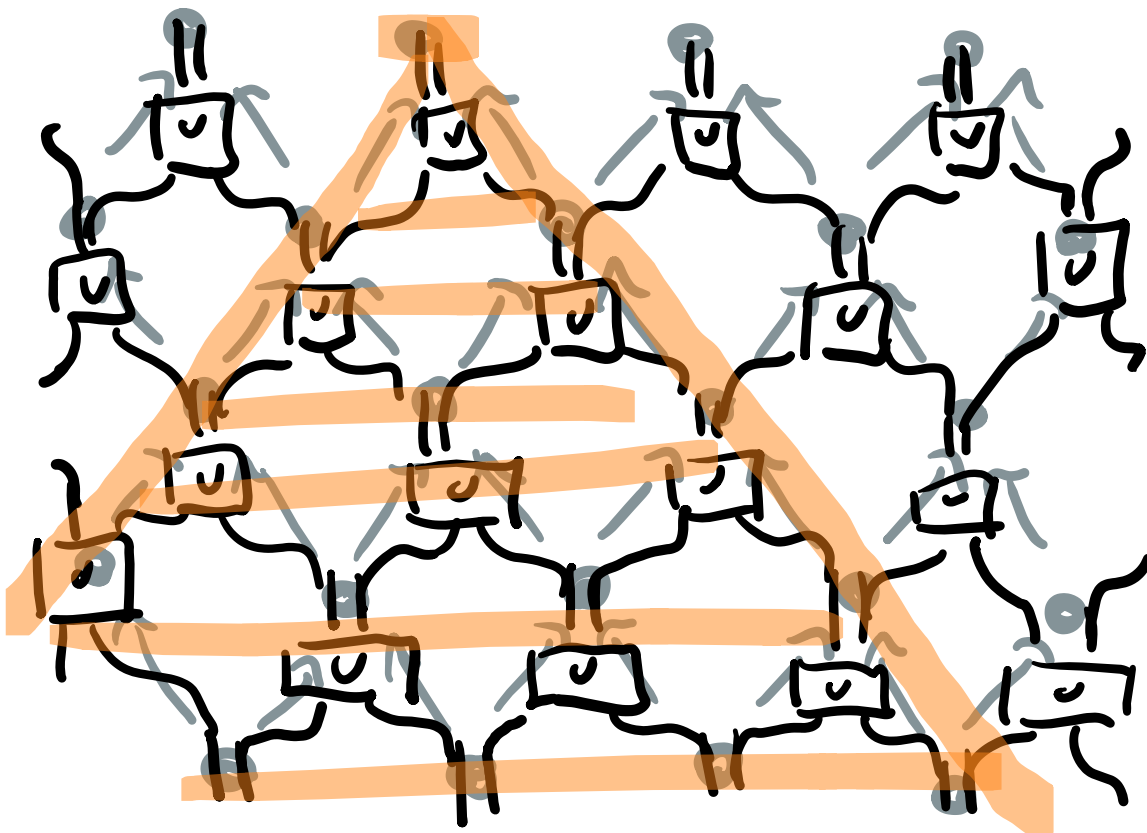
$H \otimes H$



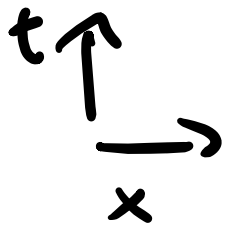
(1+1)-Minkowski



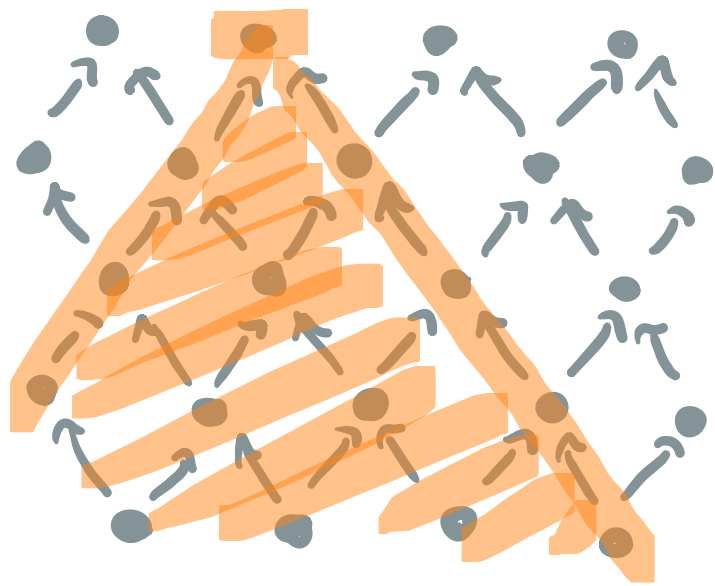
Hasse diagram for causal order



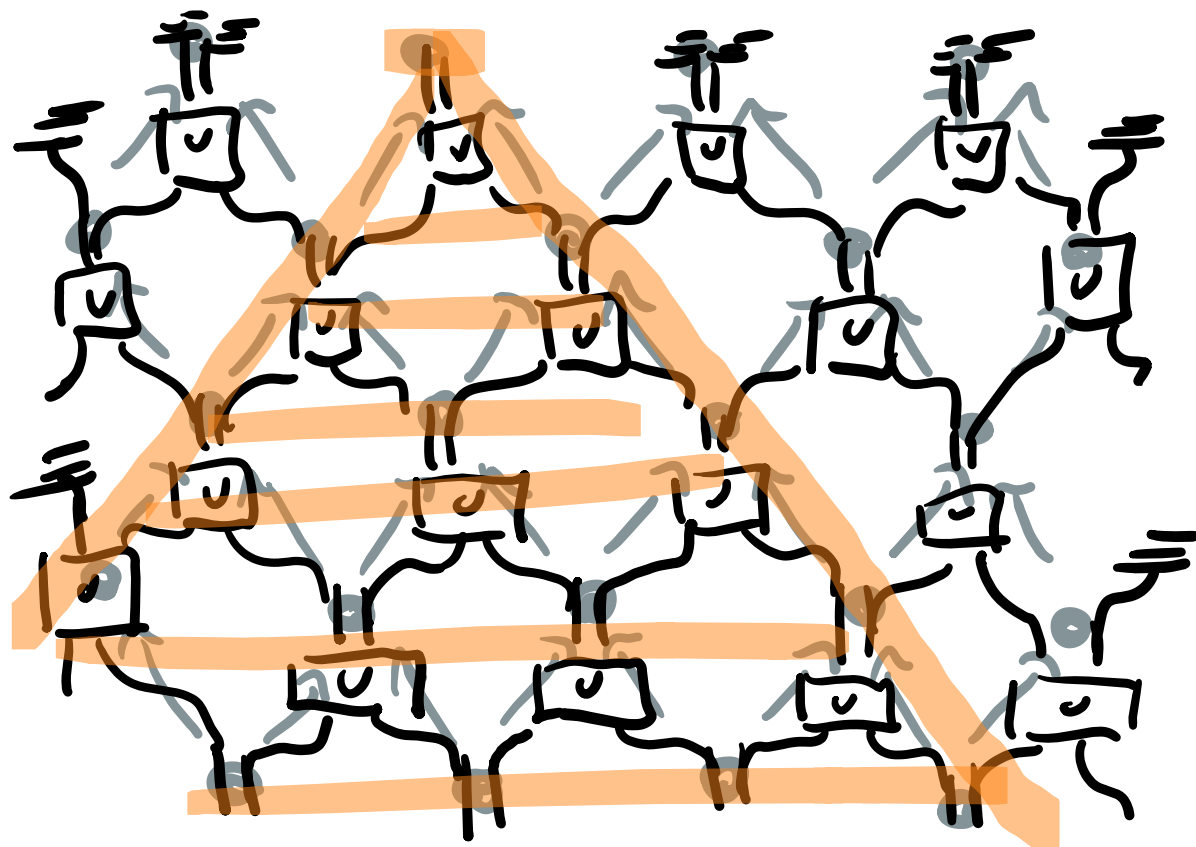
$H \otimes H$



(1+1)-Minkowski

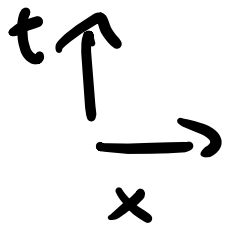


Hasse diagram for causal order

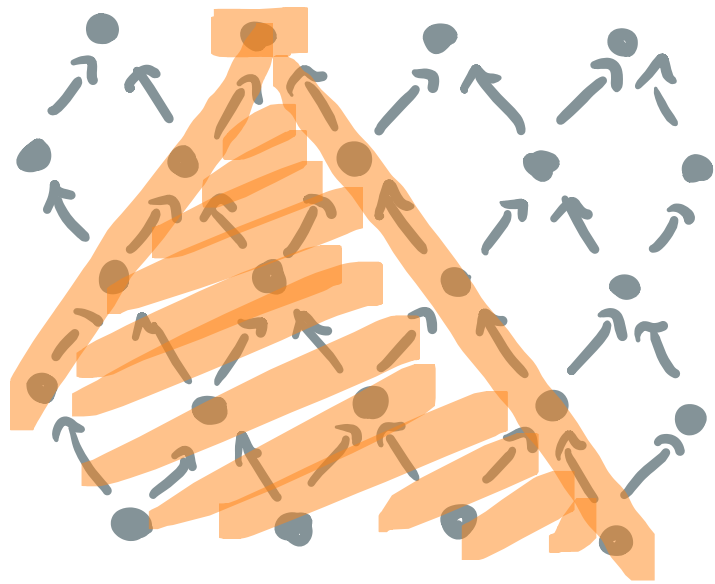


$H \otimes H$

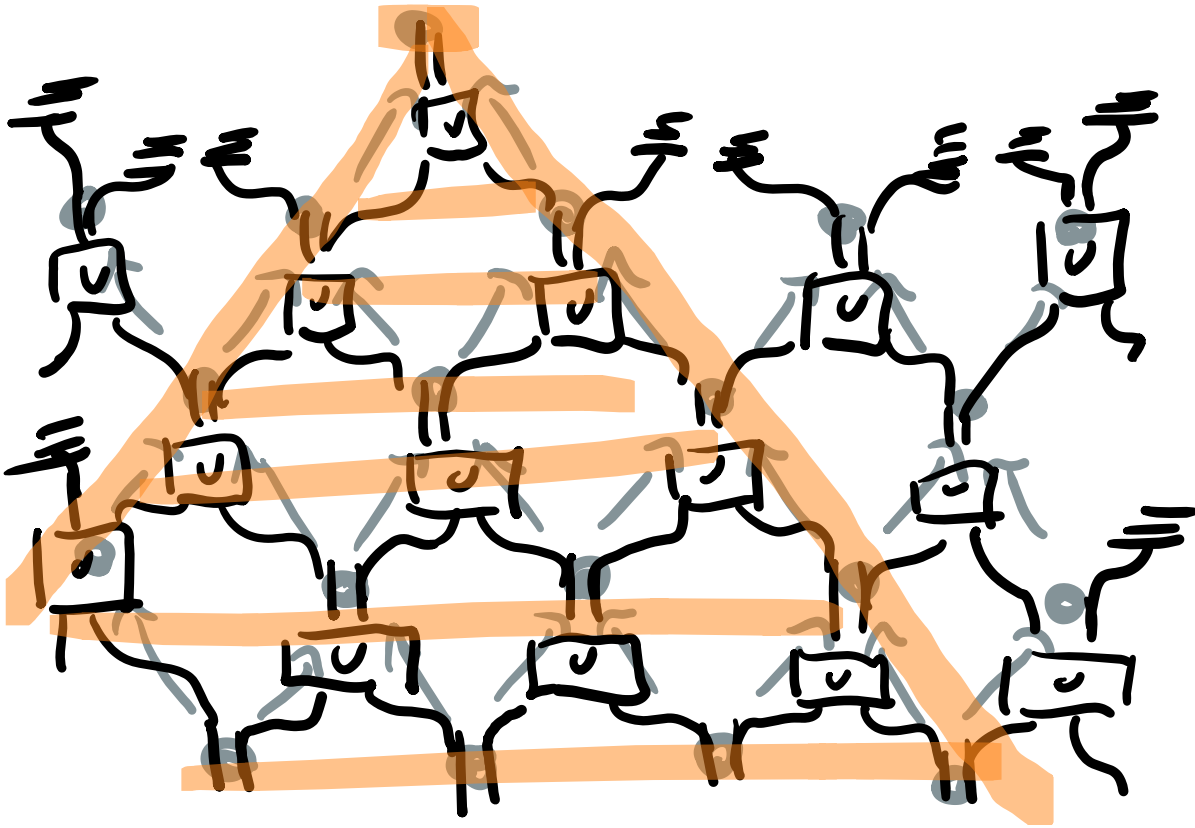




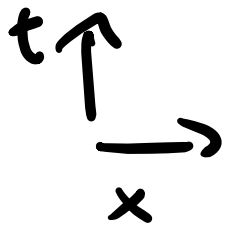
(1+1)-Minkowski



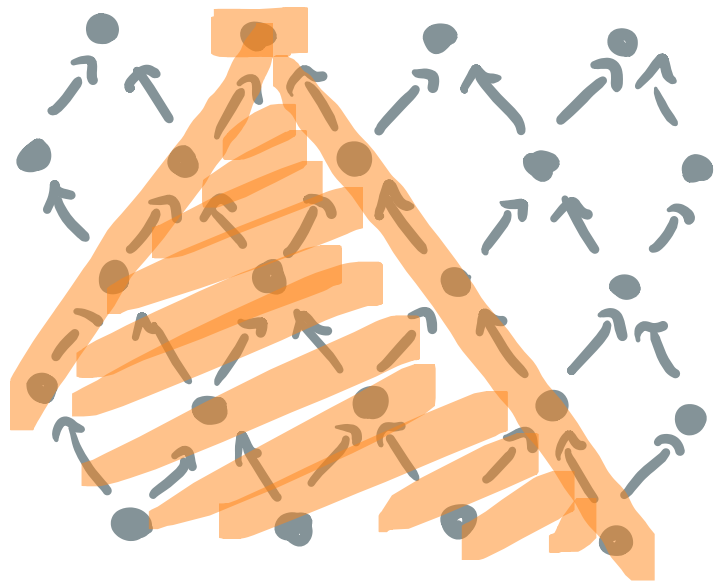
Hasse diagram for causal order



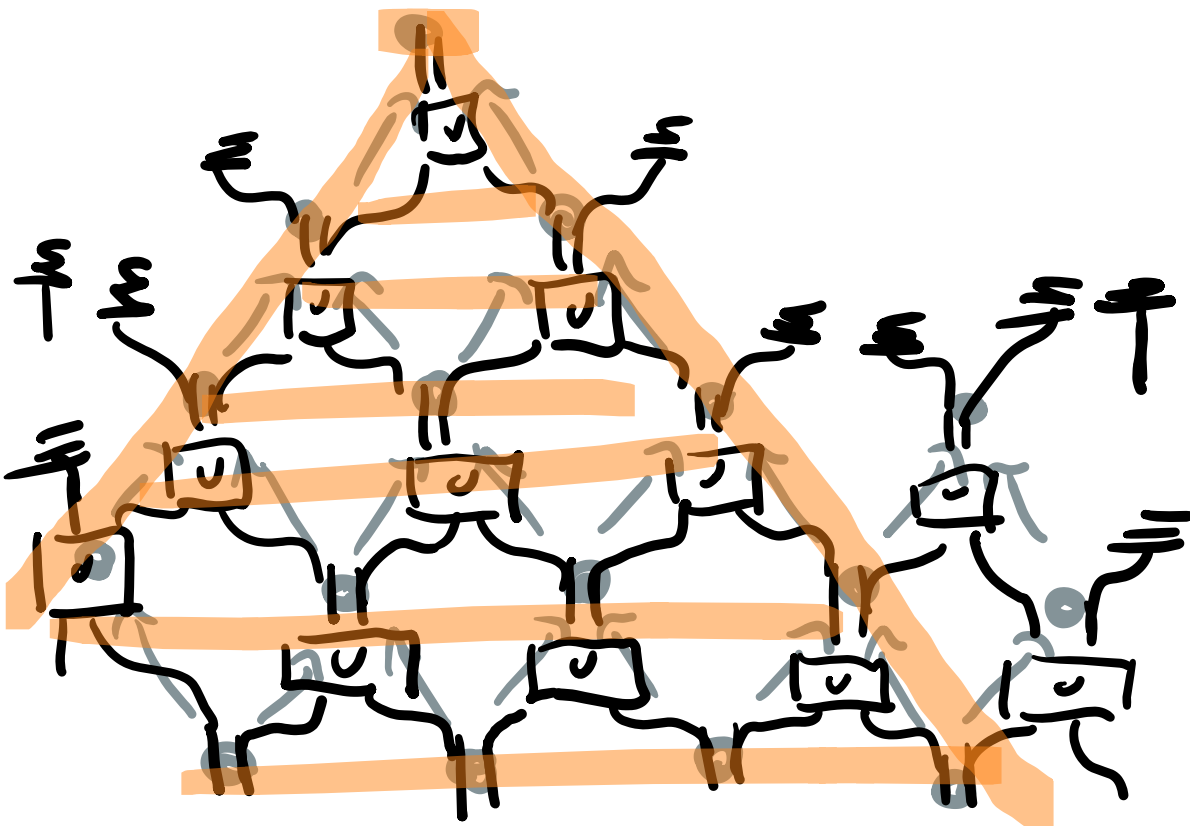
$H \otimes H$



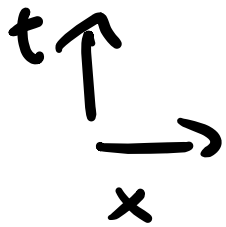
(1+1)-Minkowski



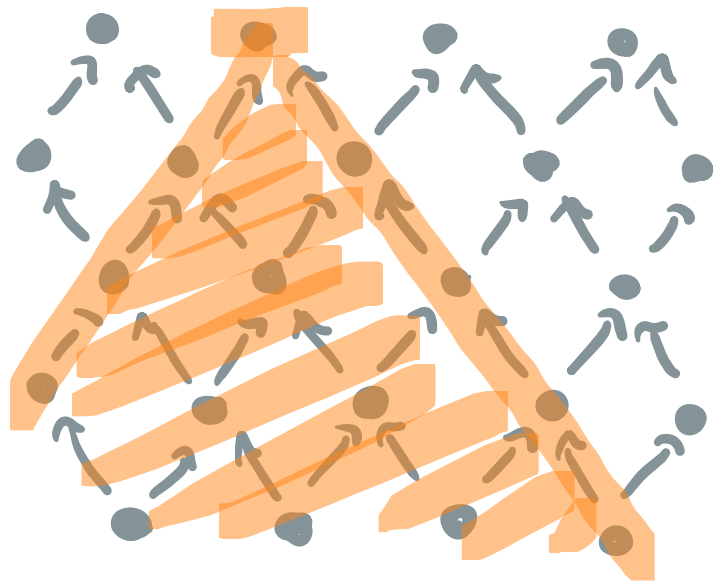
Hasse diagram for causal order



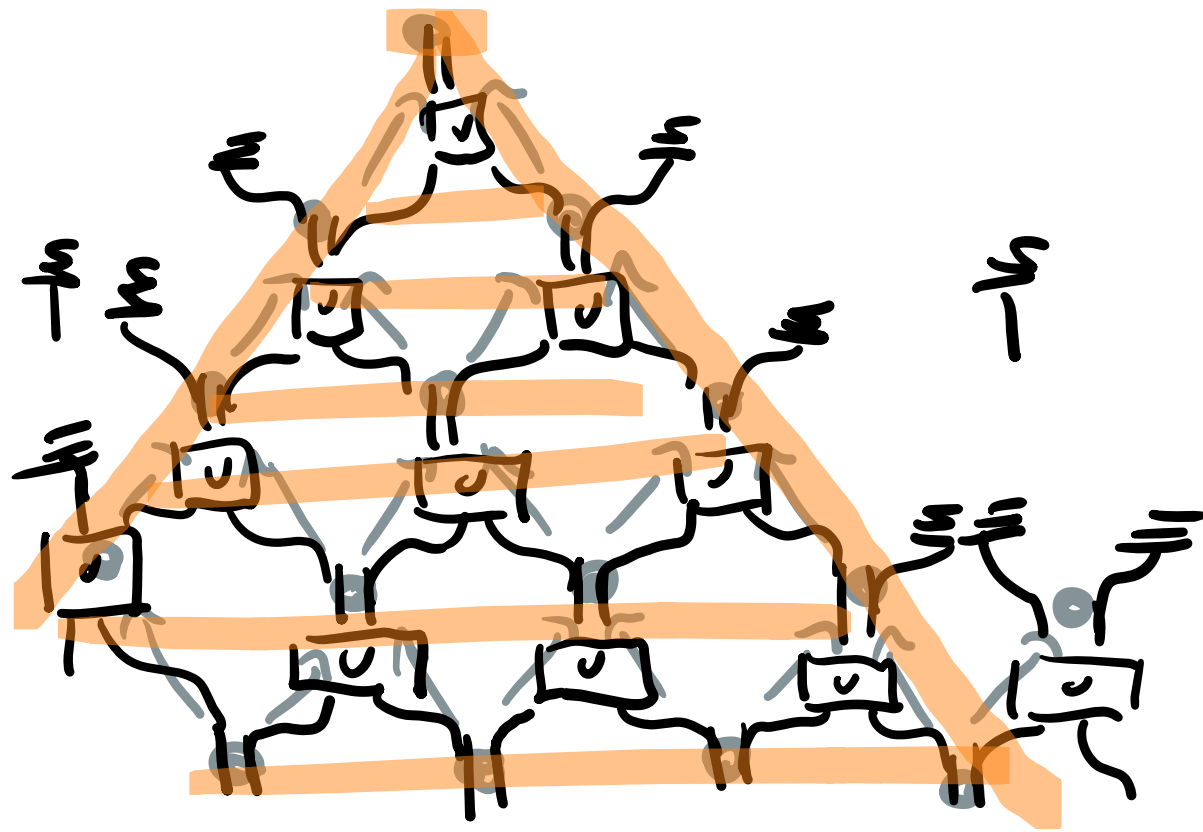
$H \otimes H$



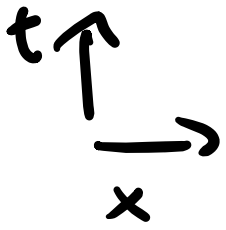
(1+1)-Minkowski



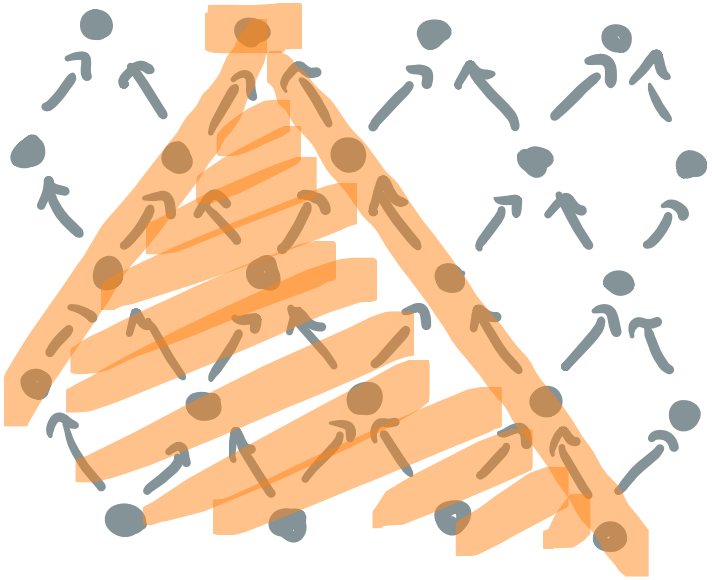
Class diagram for causal order



$H \otimes H$

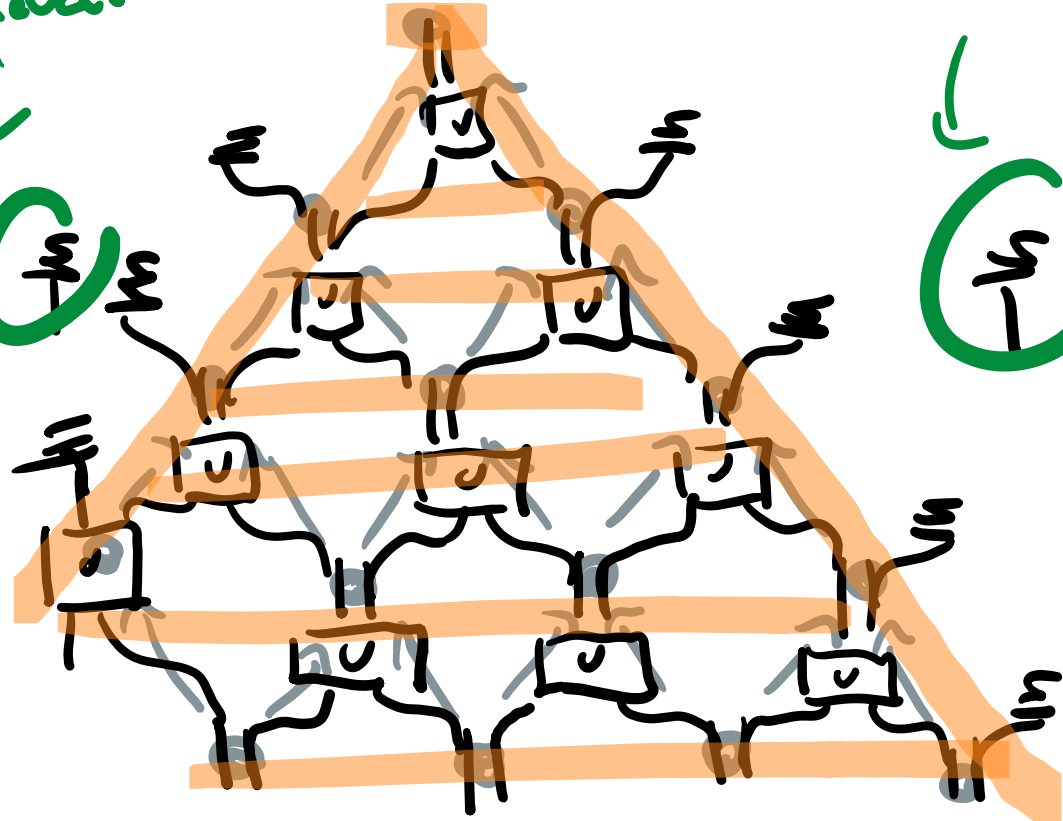


(1+1)-Minkowski

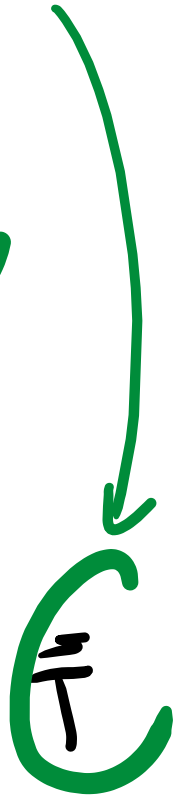


Klasse diagramm für causal order

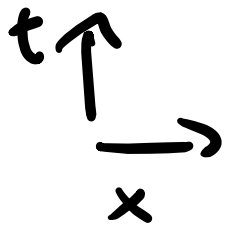
irrelevant



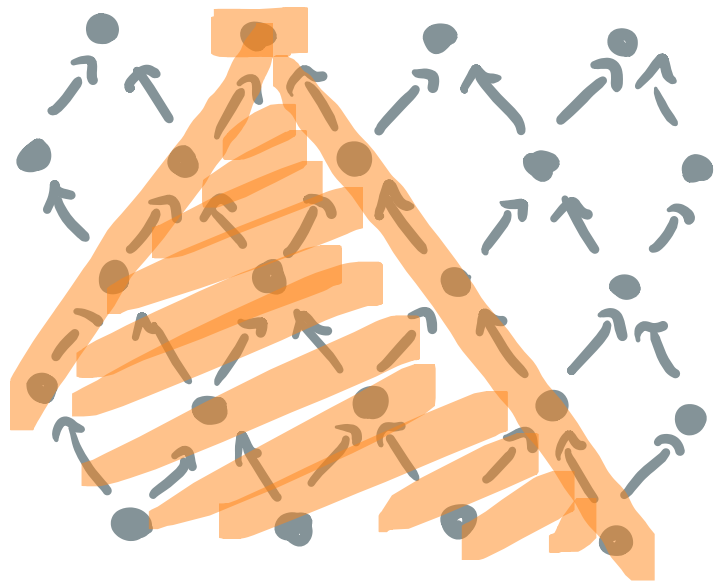
irrelevant



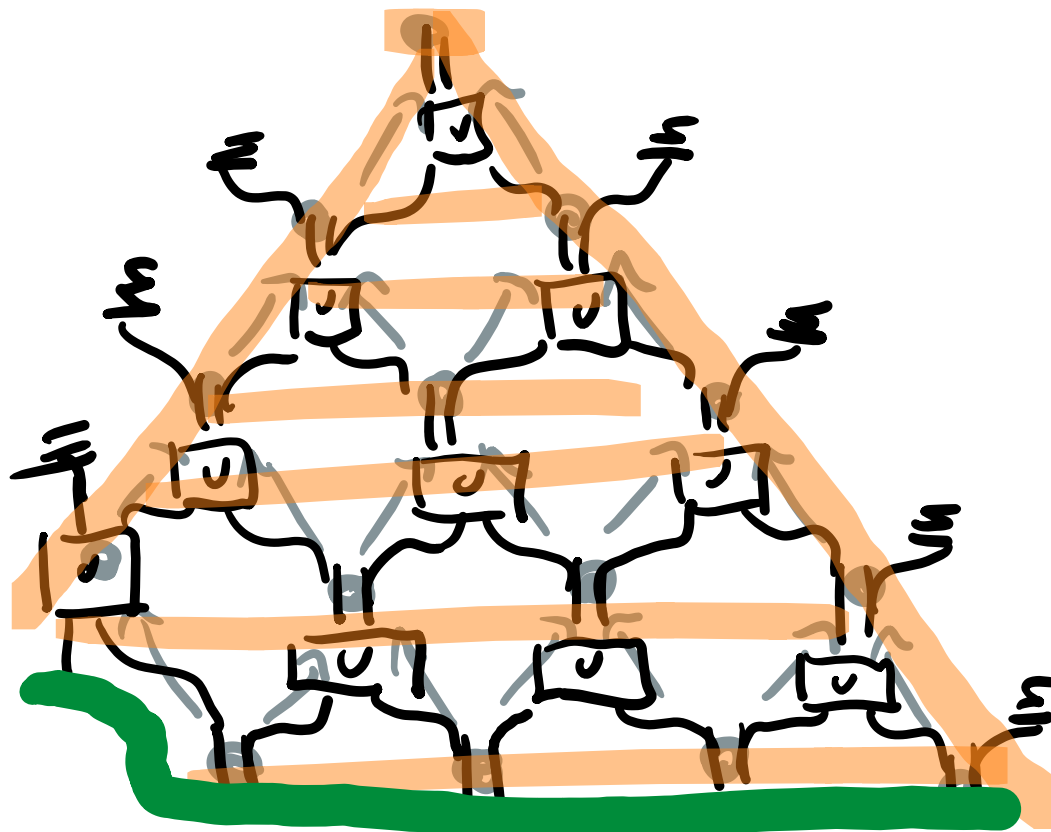
$H \otimes H$



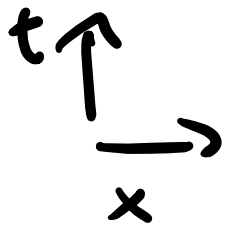
(1+1)-Minkowski



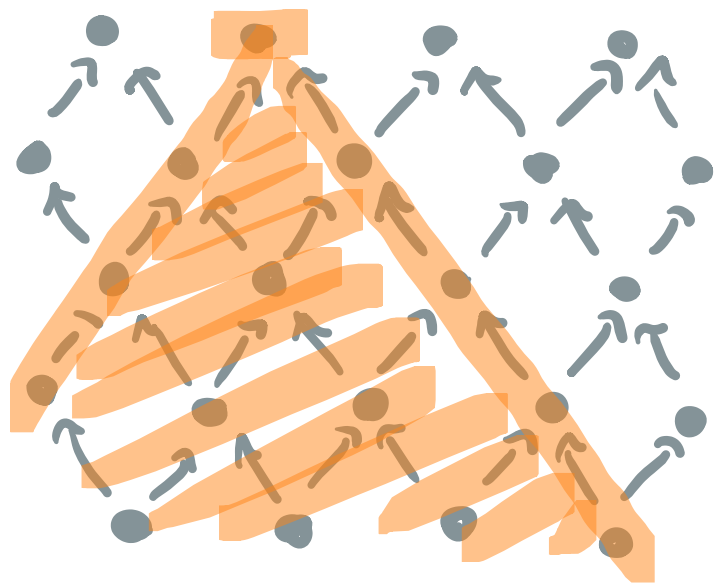
Hasse diagram for causal order



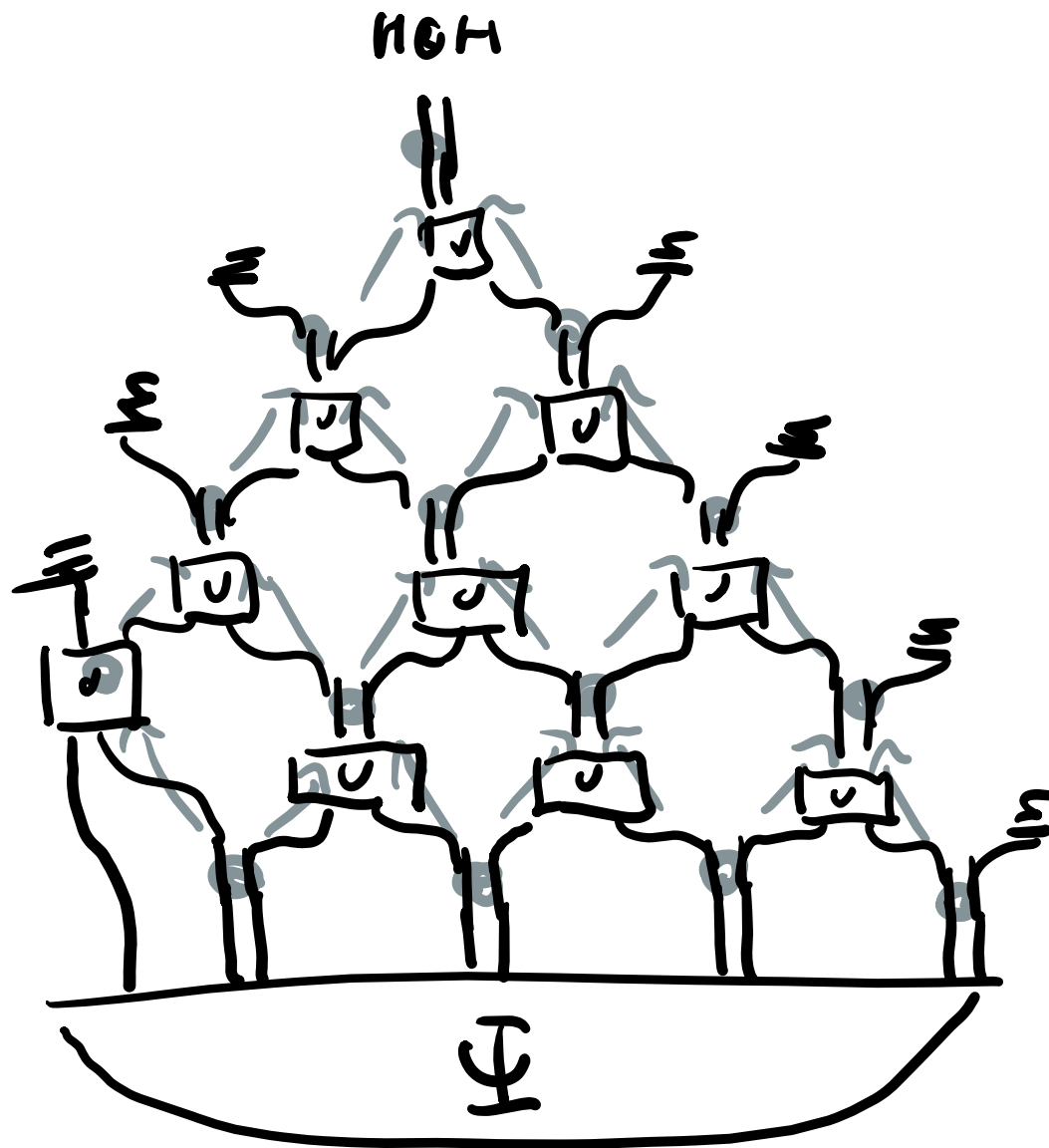
$H \otimes H$

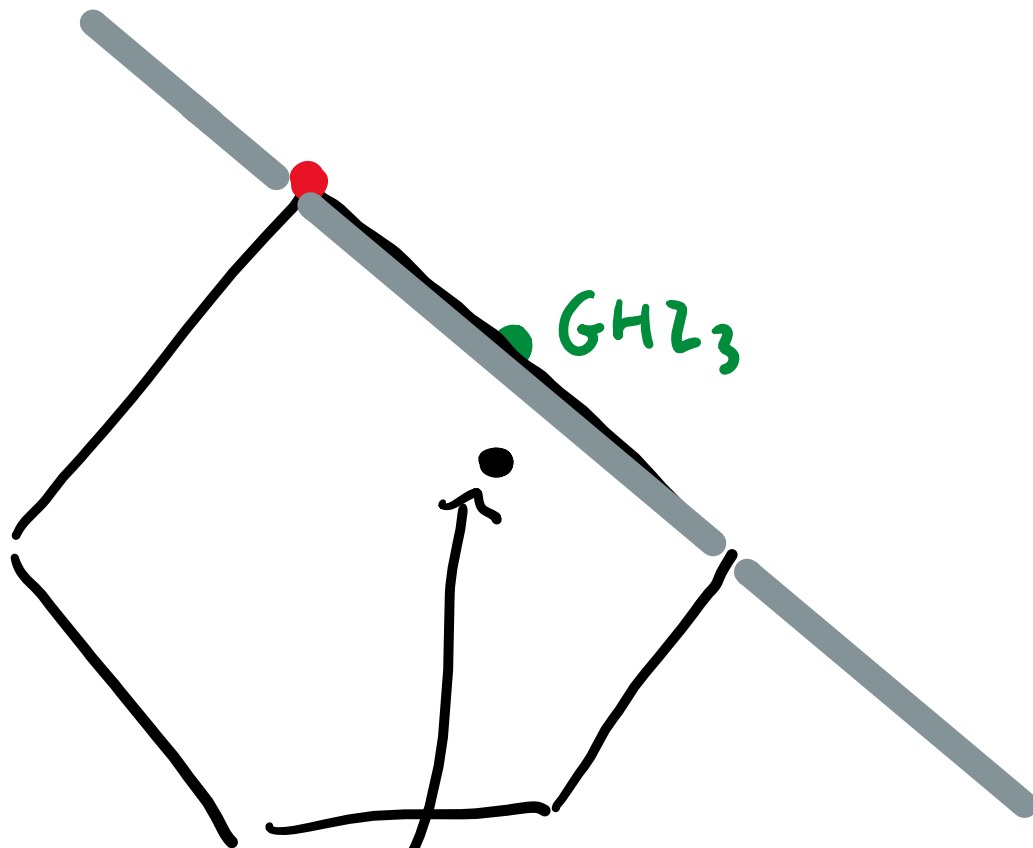
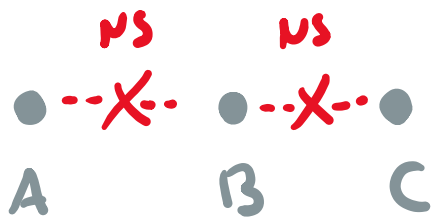
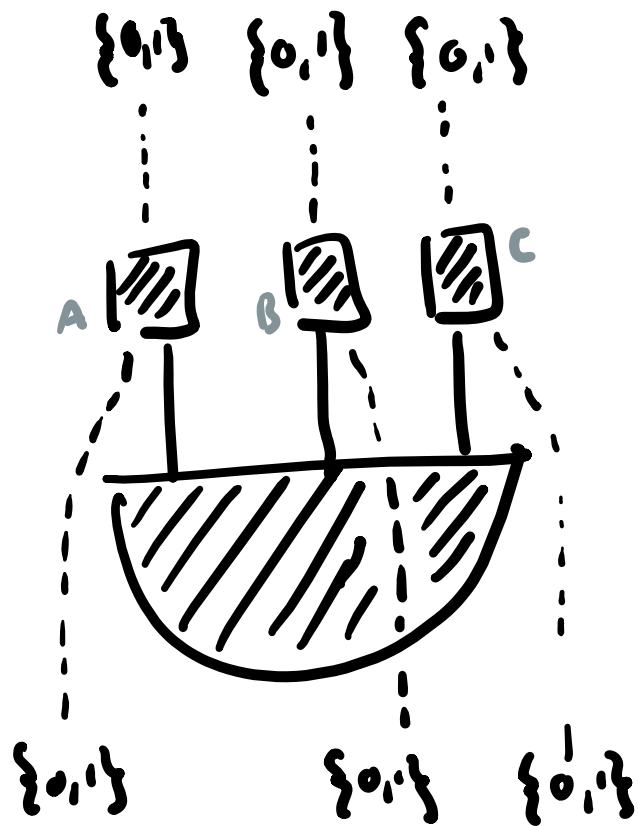


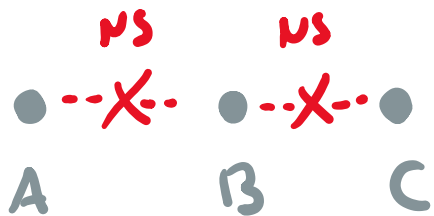
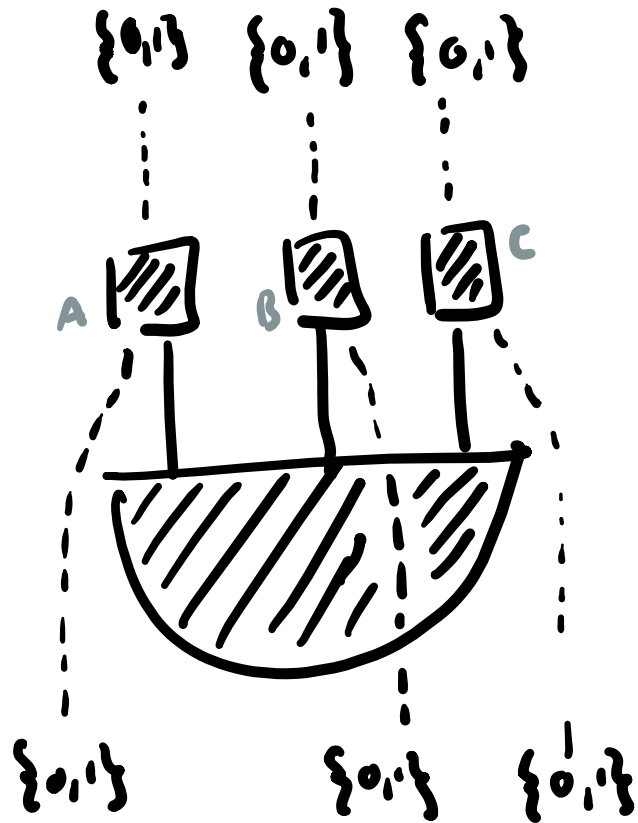
(1+1)-Minkowski



Hasse diagram for causal order



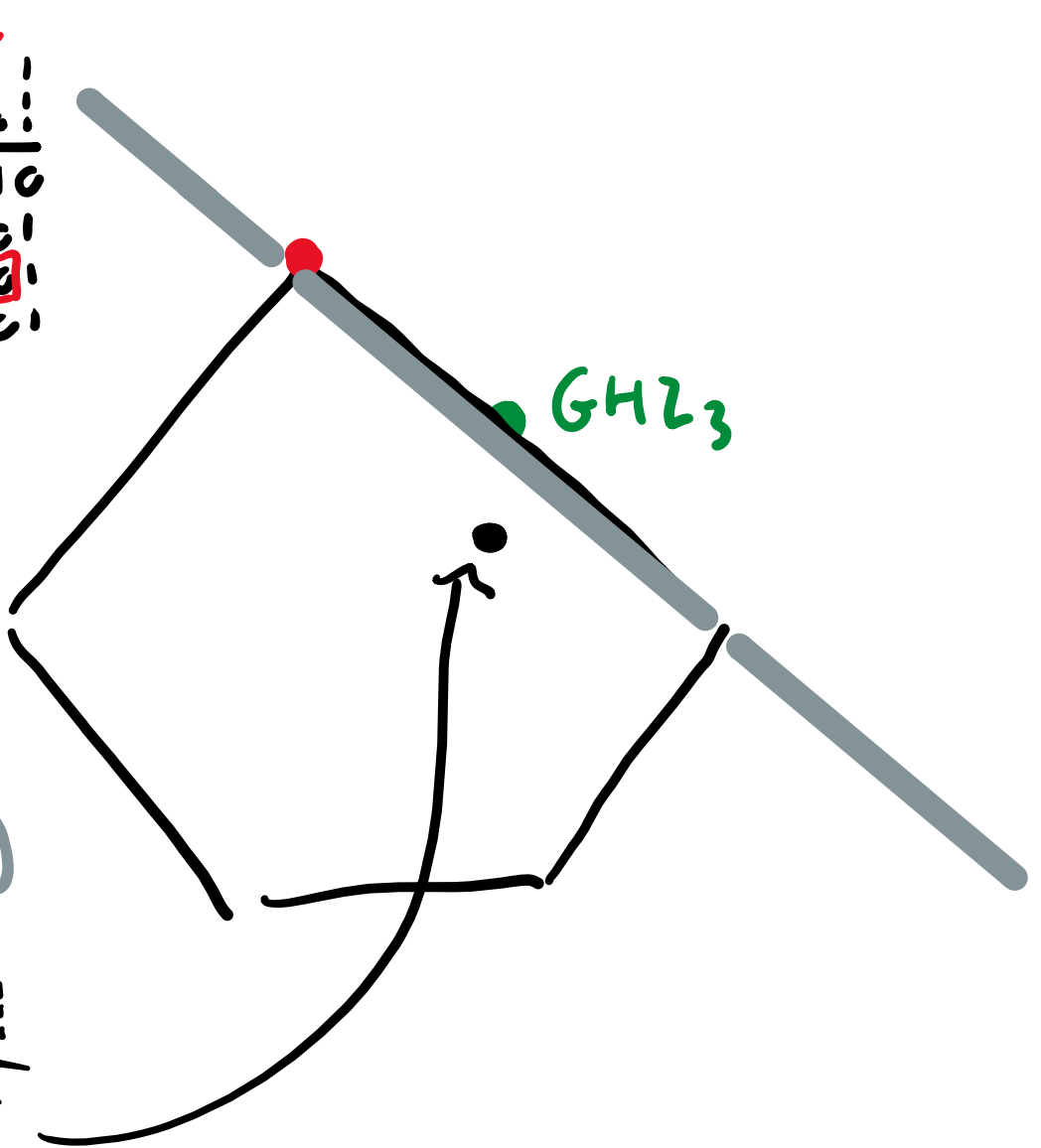




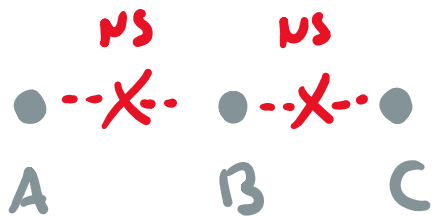
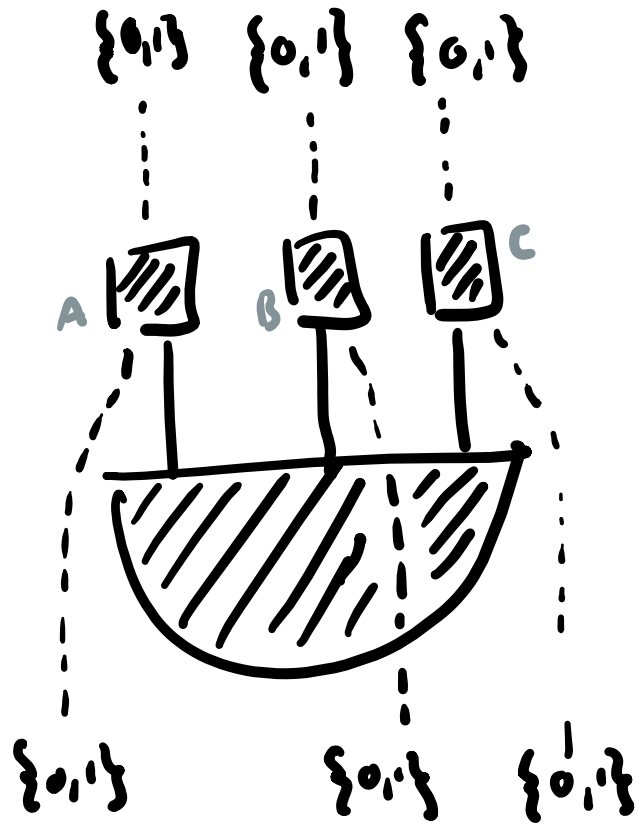
	000	001	010	011	100	101	110	111
→ 000	1	0	0	1	0	1	1	0
011	0	1	1	0	1	0	0	1
→ 101	0	1	0	1	0	1	0	1
110	0	1	1	0	1	0	0	1

000 → 011 ✓  
 101 → 110 ✗  
 Count errors  
 ⇒ fidelity

	000	001	010	011	100	101	110	111
→ 000	1	0	0	1	0	1	1	0
011	0	1	1	0	1	0	0	1
101	0	1	0	1	0	1	0	1
110	0	1	1	0	1	0	0	1



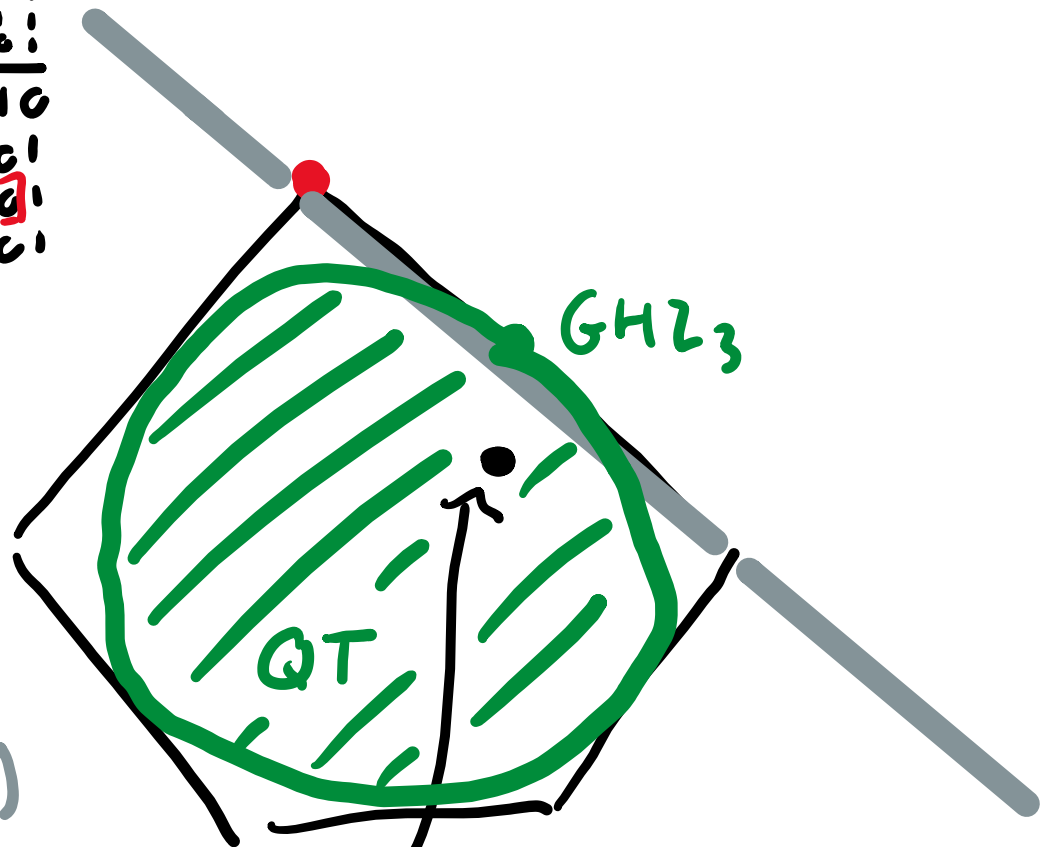


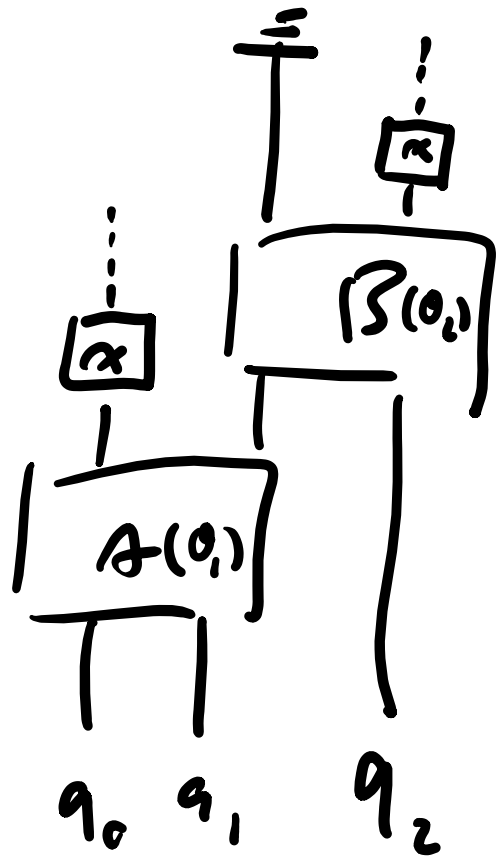


	000	001	010	011	100	101	110	111
→ 000	1	0	0	1	0	1	1	0
011	0	1	1	0	1	0	0	1
→ 101	0	1	0	1	0	0	1	1
110	0	1	1	0	1	0	0	1

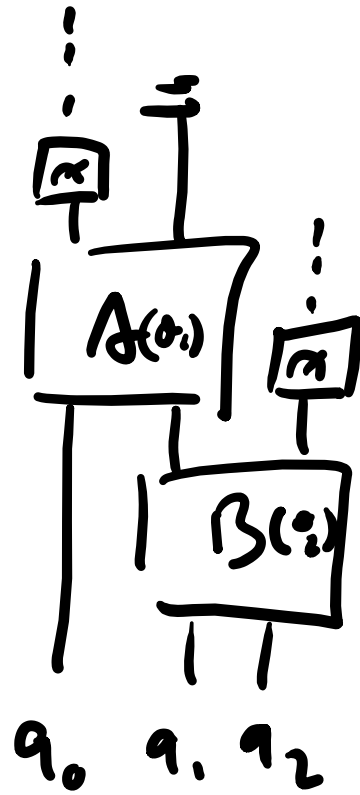
000 → 011 ✓  
 101 → 110 ✗  
 Count errors  
 ⇒ fidelity

	000	001	010	011	100	101	110	111
→	000	001	010	011	100	101	110	111





=



$A(0,1), B(0,2)$   
 simultaneously  
 performed



Contextuality

