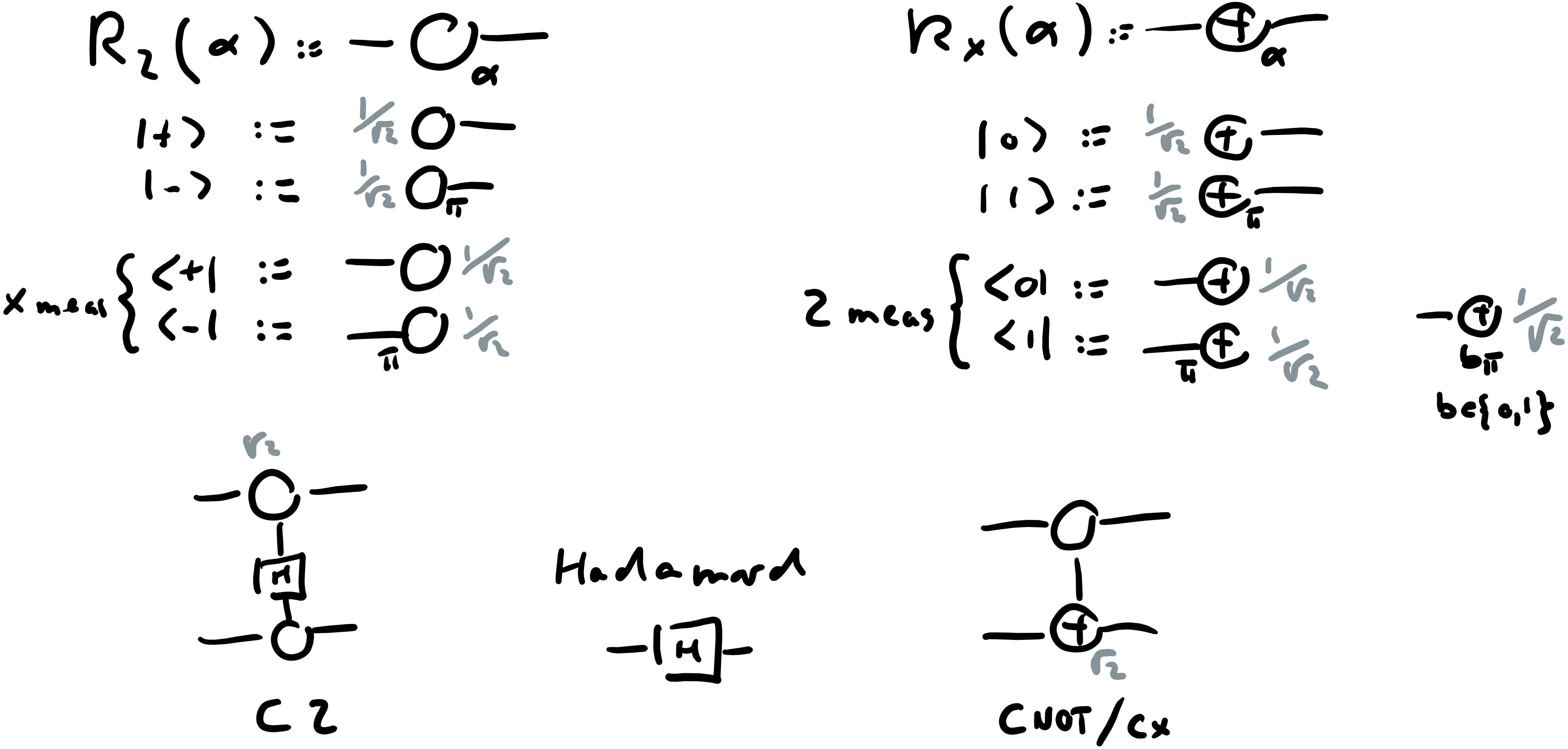
Quantum in Pictures Lecture Series

Lecturer: Stefano Gogioso

Thu 29 June 2023 –Morning Lecture

INDIANA UNIVERSITY BLOOMINGTON



$R_Z(\alpha) :=$
α
$R_X(\alpha) :=$
α
$|+\rangle :=$ $\frac{1}{\sqrt{2}}$
$|-\rangle :=$ $\frac{1}{\sqrt{2}}$ π
$|0\rangle :=$ $\frac{1}{\sqrt{2}}$
$|1\rangle :=$ $\frac{1}{\sqrt{2}}$ π
X meas
$\langle +| :=$ $\frac{1}{\sqrt{2}}$
$\langle -| :=$ π $\frac{1}{\sqrt{2}}$
Z meas
$\langle 0| :=$ $\frac{1}{\sqrt{2}}$
$\langle 1| :=$ π $\frac{1}{\sqrt{2}}$
$\frac{1}{\sqrt{2}}$
$b\pi$
$b\in\{0,1\}$
$\sqrt{2}$
H
CZ
Hadamard
H
$\sqrt{2}$
CNOT/CX

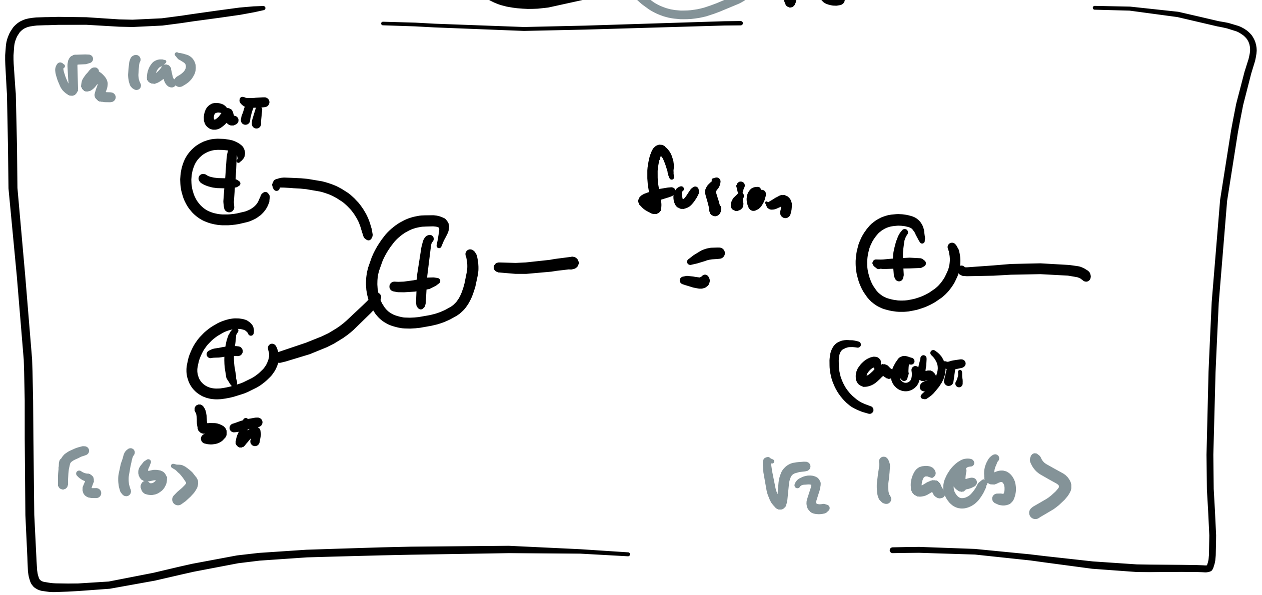
$$ {}_{\alpha}\bigcirc\!\!-\!\!- \;:=\; |0\rangle + e^{i\alpha}|1\rangle \qquad \left({}_{\alpha}\bigcirc\!\!-\!\!- \right)^{\dagger} = \left(|0\rangle + e^{i\alpha}|1\rangle\right)^{\dagger} $$

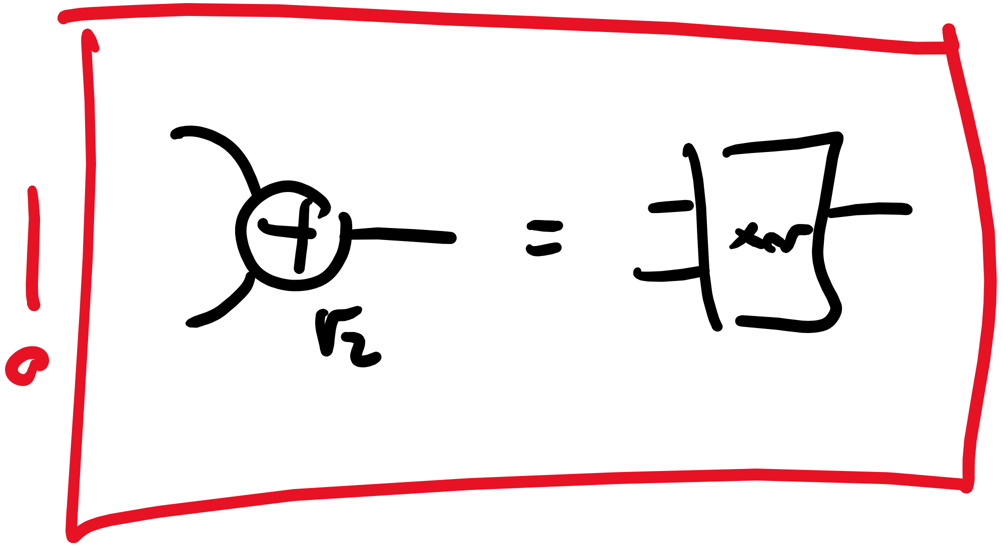
$$ -\!\!-\!\!\bigcirc_{\alpha} \;:=\; \langle 0| + e^{i\alpha}|1\rangle \qquad = \langle 0| + e^{-i\alpha}\langle 1| = -\!\!-\!\!\bigcirc_{-\alpha} $$

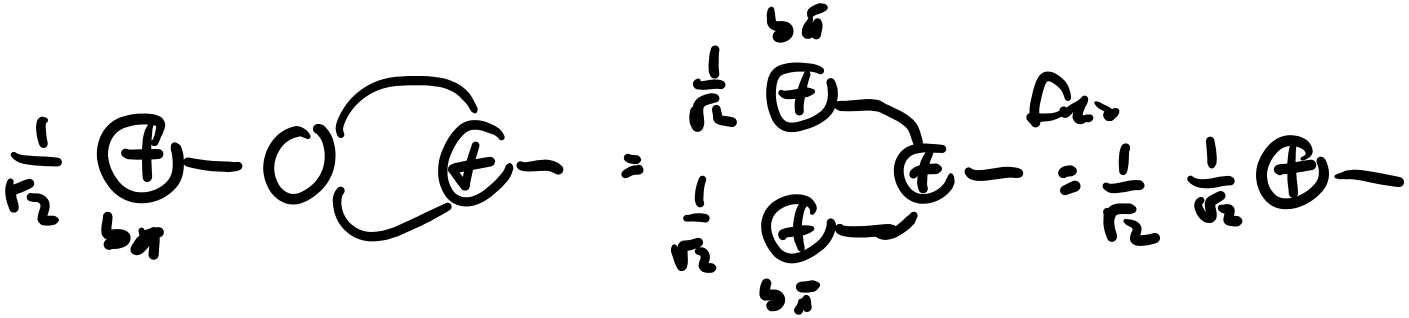
$$ |+\rangle = \frac{1}{\sqrt{2}}\,\bigcirc\!\!-\!\!- = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) $$

$$ \bigcirc\!\!-\!\!- \;:=\; |0\rangle + |1\rangle \quad \Rightarrow \quad \langle 0|\!-\!\!\bigcirc = \left(\langle 0| + \langle 1|\right)|0\rangle = 1 $$


|b⟩
|b⟩
|b⊕b⟩ = |0⟩
|b⟩
√2
|b⟩
|0⟩
1/√2

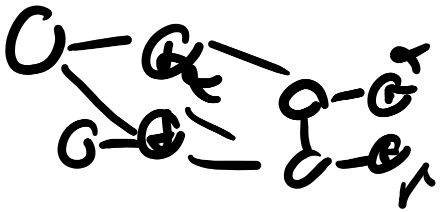

√2 |a⟩
aπ
bπ
fusion
(a⊕b)π
√2 |b⟩
√2 |a⊕b⟩


!
√2
xor


1/√2
bπ
1/√2
bπ
1/√2
bπ
fusion
1/√2
1/√2

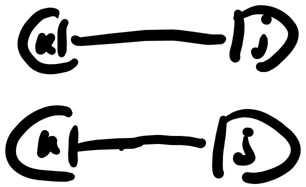
Amplitudes

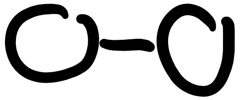
$(Z) \longleftrightarrow \mathbb{C}$

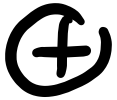
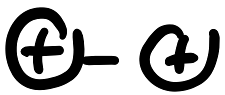
 $\in \mathbb{C}$

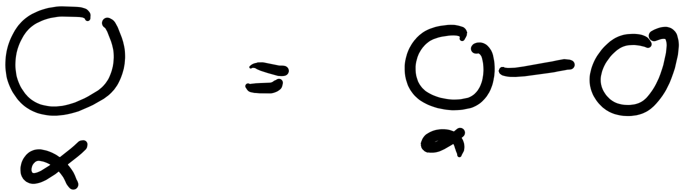
$\mathrm{Tr}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

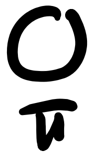
$\|$

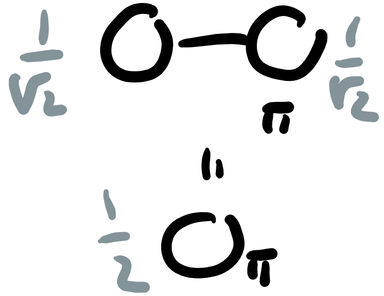
 $= \sum_{k=0,1} \sum_{j=0,1}$  $= \sum_{k=0,1} \sum_{j} \delta_{jk}^2 = 2$

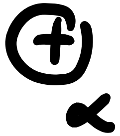
 $=$  $= \big((\langle 0| + \langle 1|)(|0\rangle + |1\rangle) \big) = 2$

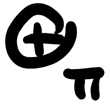
 $=$  $= \big((\langle + | + \langle - |)(|+\rangle + |-\rangle) \big) = 2$

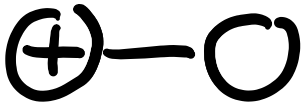
 $= (\langle 0| + \langle 1|)(|0\rangle + e^{i\alpha}|1\rangle) = 1 + e^{i\alpha}$

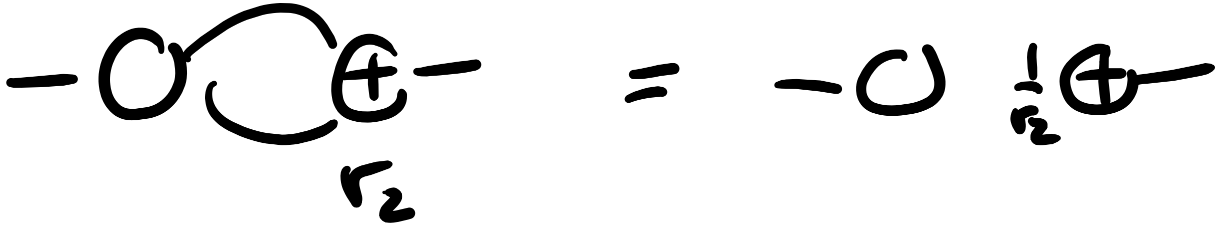
 $= 1 + e^{i\pi} = 1 - 1 = 0$

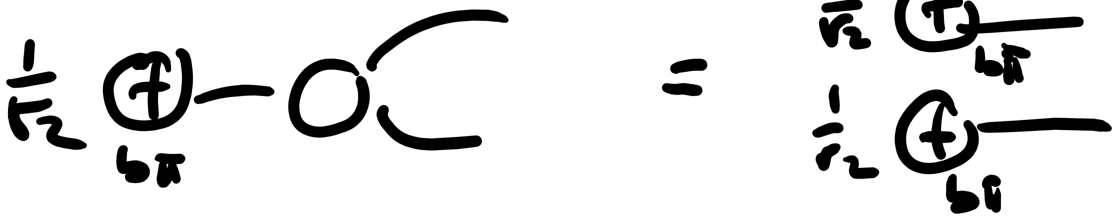
e.g.  $= \langle - | + \rangle = 0$

 $= 1 + e^{i\alpha}$

 $= 0$

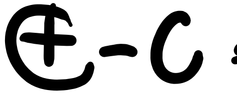
 $= (\langle 0| + \langle 1|)(|+\rangle + |-\rangle) = \underset{1/\sqrt{2}}{\langle 0|+\rangle} + \underset{1/\sqrt{2}}{\langle 1|+\rangle} + \underset{1/\sqrt{2}}{\langle 0|-\rangle} + \underset{-1/\sqrt{2}}{\langle 1|-\rangle} = \sqrt{2}$



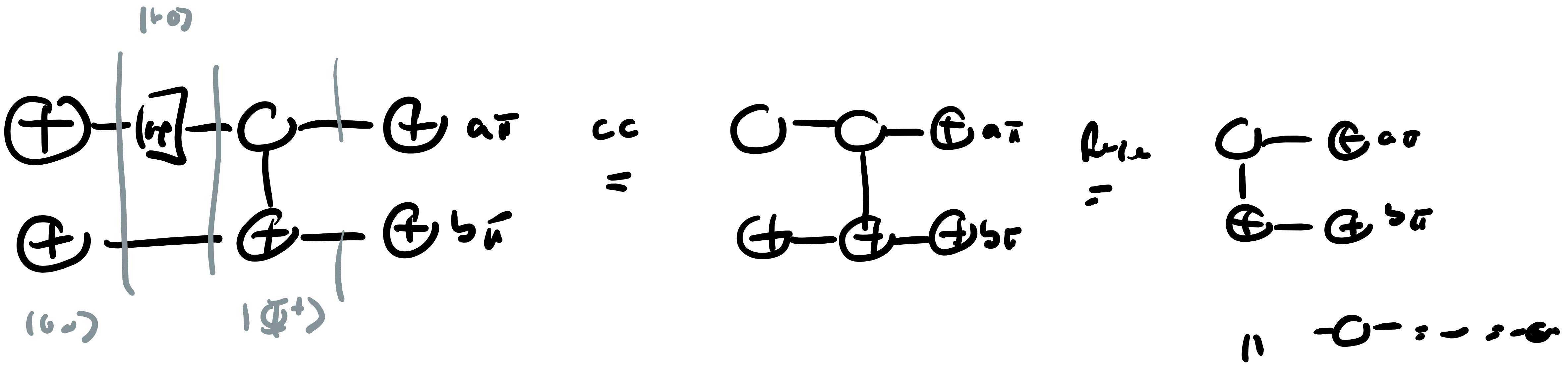


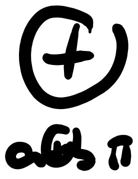
$$|b\rangle \mapsto |bb\rangle$$

$$= \left(\langle 0| + \langle 1|\right)|b\rangle = 1 \quad \Rightarrow$$



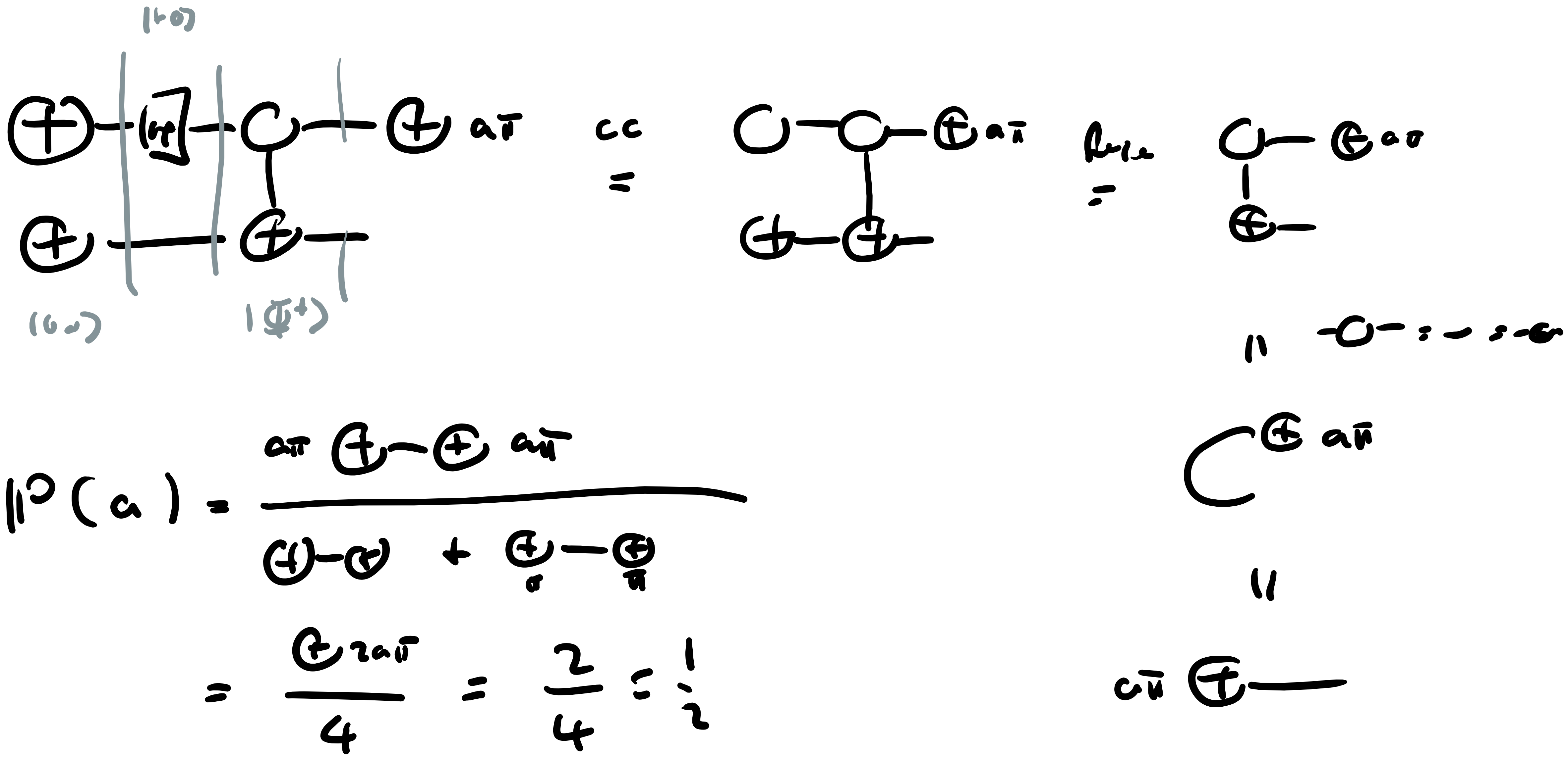
$$= \sqrt{2}$$



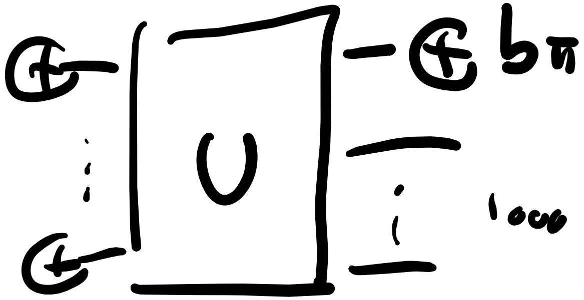


$$= \begin{cases} 2 & \text{if } a \oplus b = 0 \\ 0 & \text{if } a \oplus b = 1 \end{cases}$$

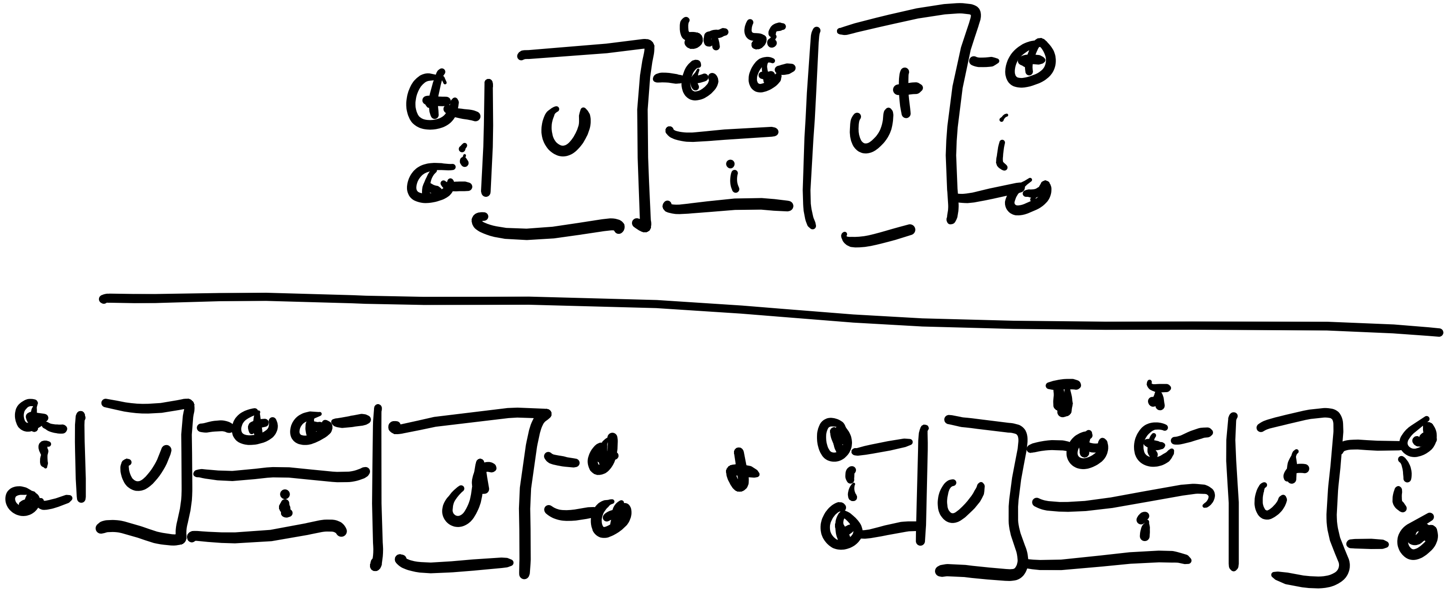
$$\begin{cases} \mathbb{P}(00) = \dfrac{2}{2+0+2+0} = \dfrac{1}{2} \\ \mathbb{P}(01) = \dfrac{0}{2+0+2+0} = 0 \end{cases}$$



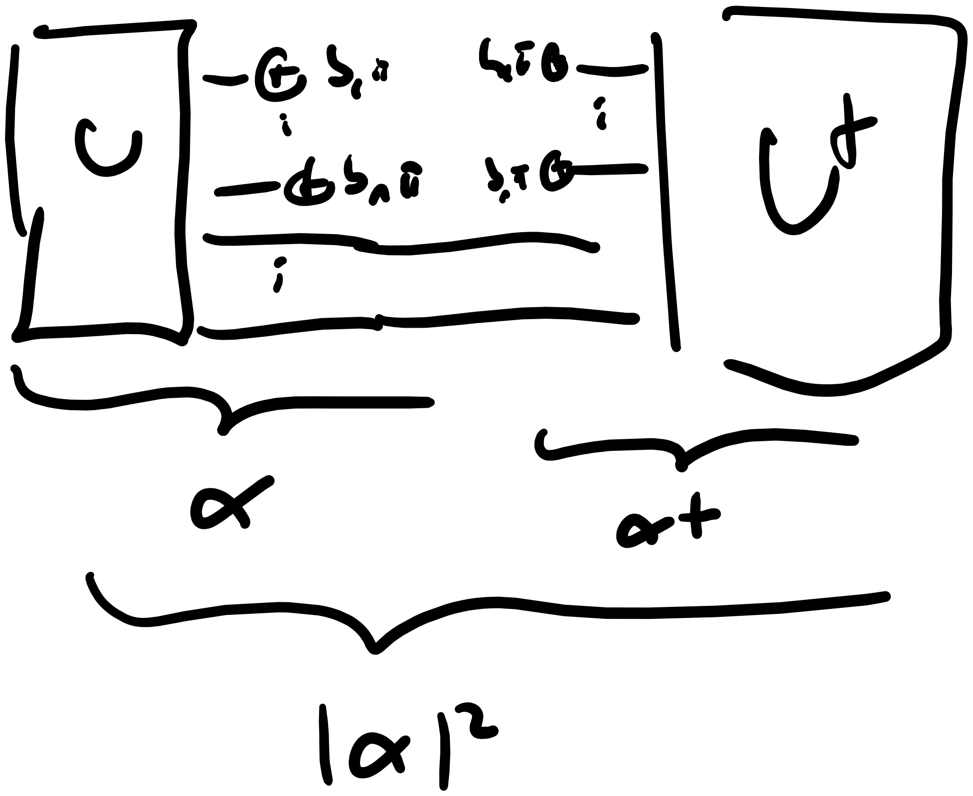
H
aπ
cc
=
P(a) =
4
= 2/4 = 1/2



$\mathbb{P}(b) =$



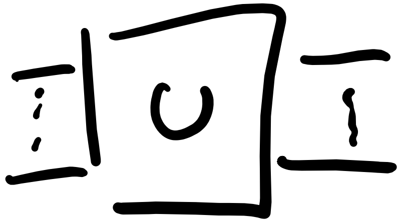
$$\mathbb{P}(\underline{y}) =$$

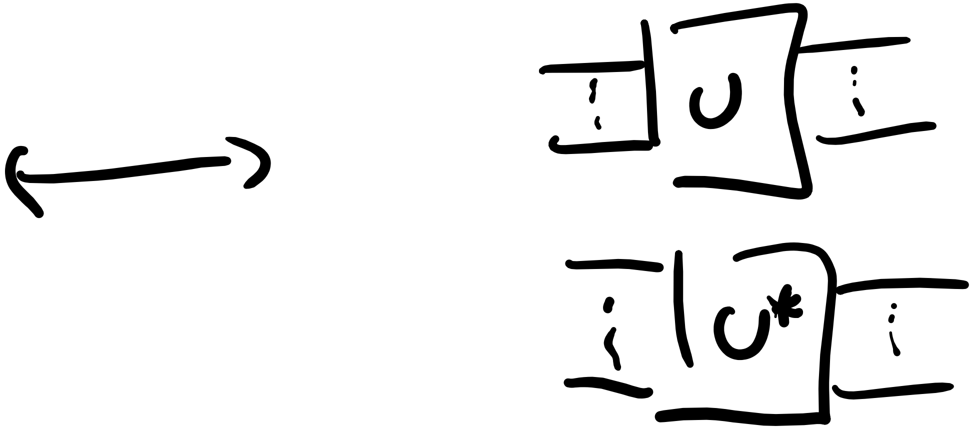


(up to normalization)

<u>Born Rule</u>

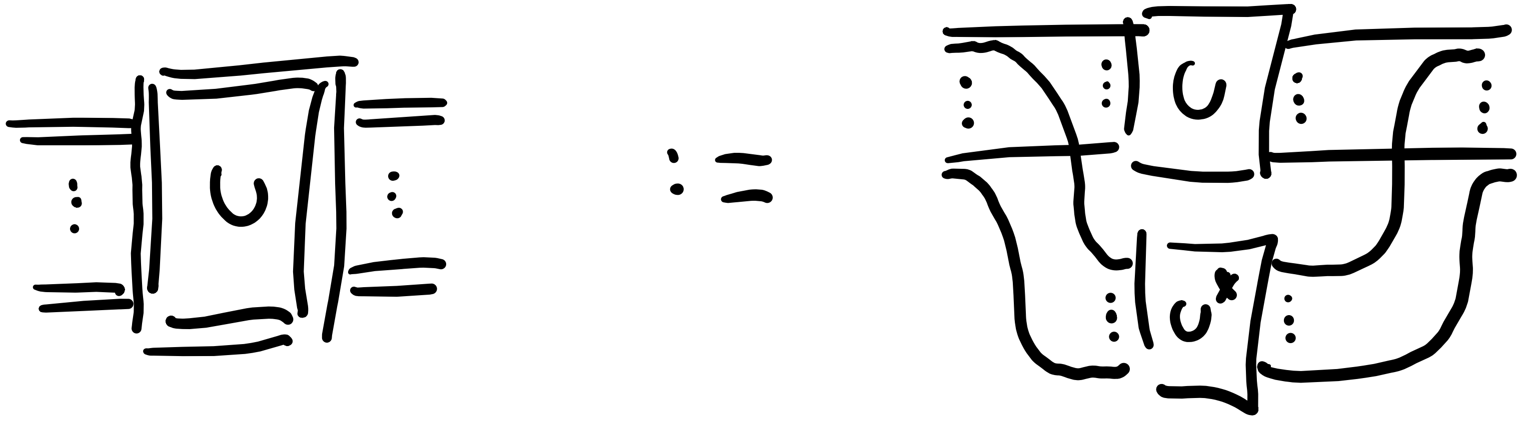
Doubled ZX





ℂ-linear map

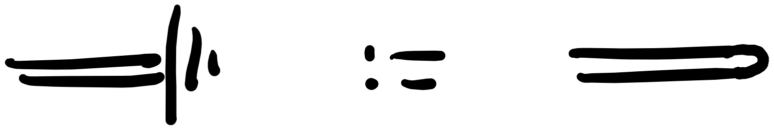
state	$\vert 0\rangle$	$\longmapsto$	$\vert 0\rangle\langle 0\vert$	density matrix
bra	$\langle 0\vert$	$\longmapsto$	$\langle 0\vert \;\square\; \vert 0\rangle$	Kraus op
linear map	U	$\longmapsto$	$U \;\square\; U^\dagger$	Kraus op



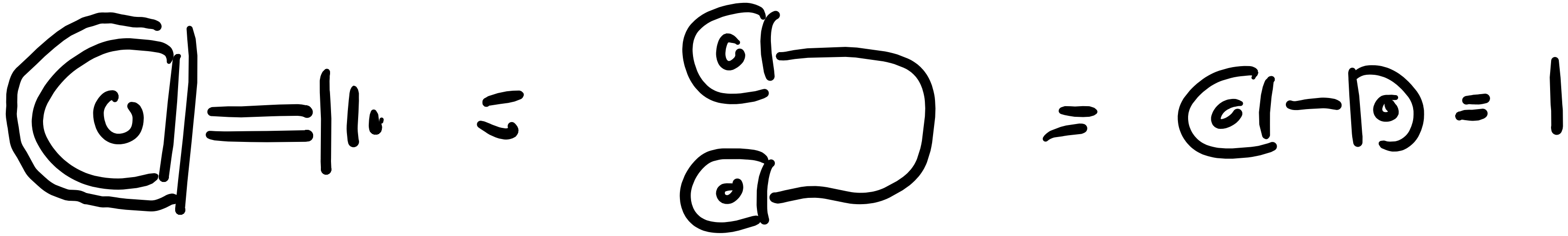
States = density matrices (up to normalisation)

boxes = completely positive maps

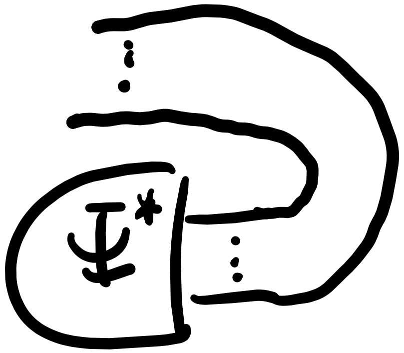
Scalars = $\mathbb{R}^+$



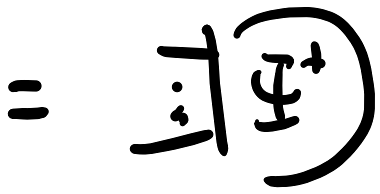
discarding map (aka partial trace)



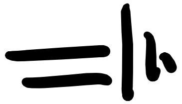
$$\Big[\underbrace{\tfrac{1}{\sqrt{2}}\big(|0\rangle + i|1\rangle\big)}_{|R\rangle}\Big]^* = \underbrace{\tfrac{1}{\sqrt{2}}\big(|0\rangle - i|1\rangle\big)}_{|L\rangle}$$

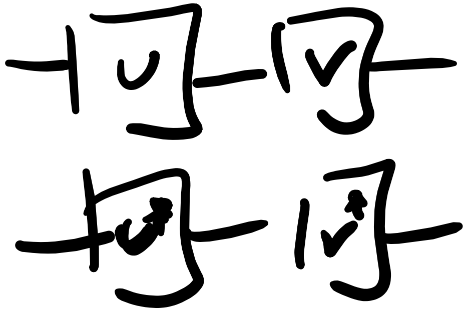


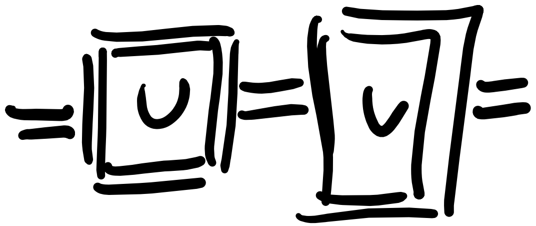
conjugate + transpose

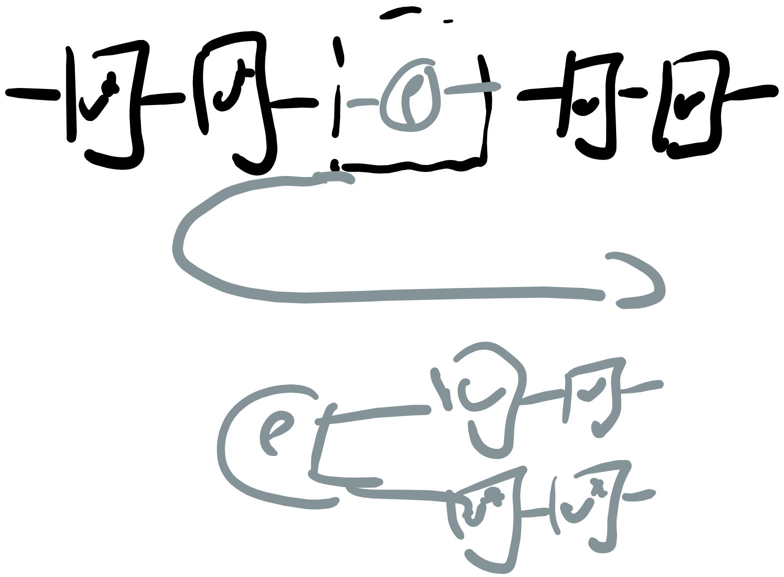


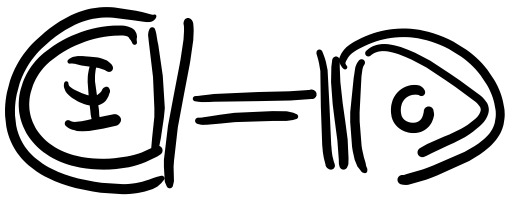
adjoint

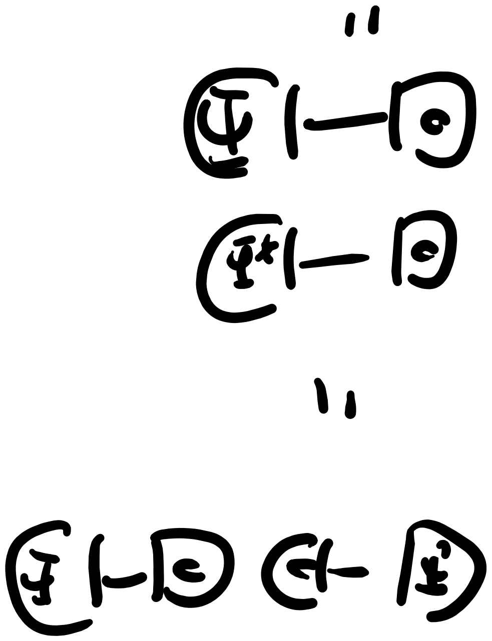
 $= \mathrm{Tr} = \rho \mapsto \sum_{b=0,1} \langle b | \rho | b \rangle$

 $= \rho \mapsto V U \rho U^\dagger V^\dagger$

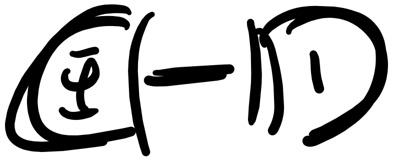


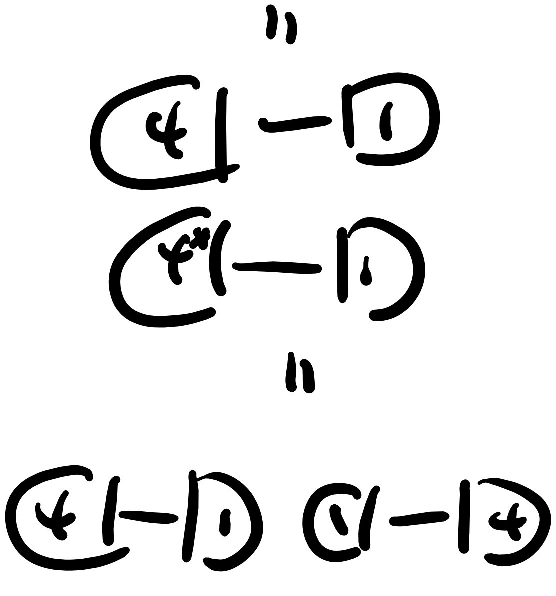


$P(0) =$ 

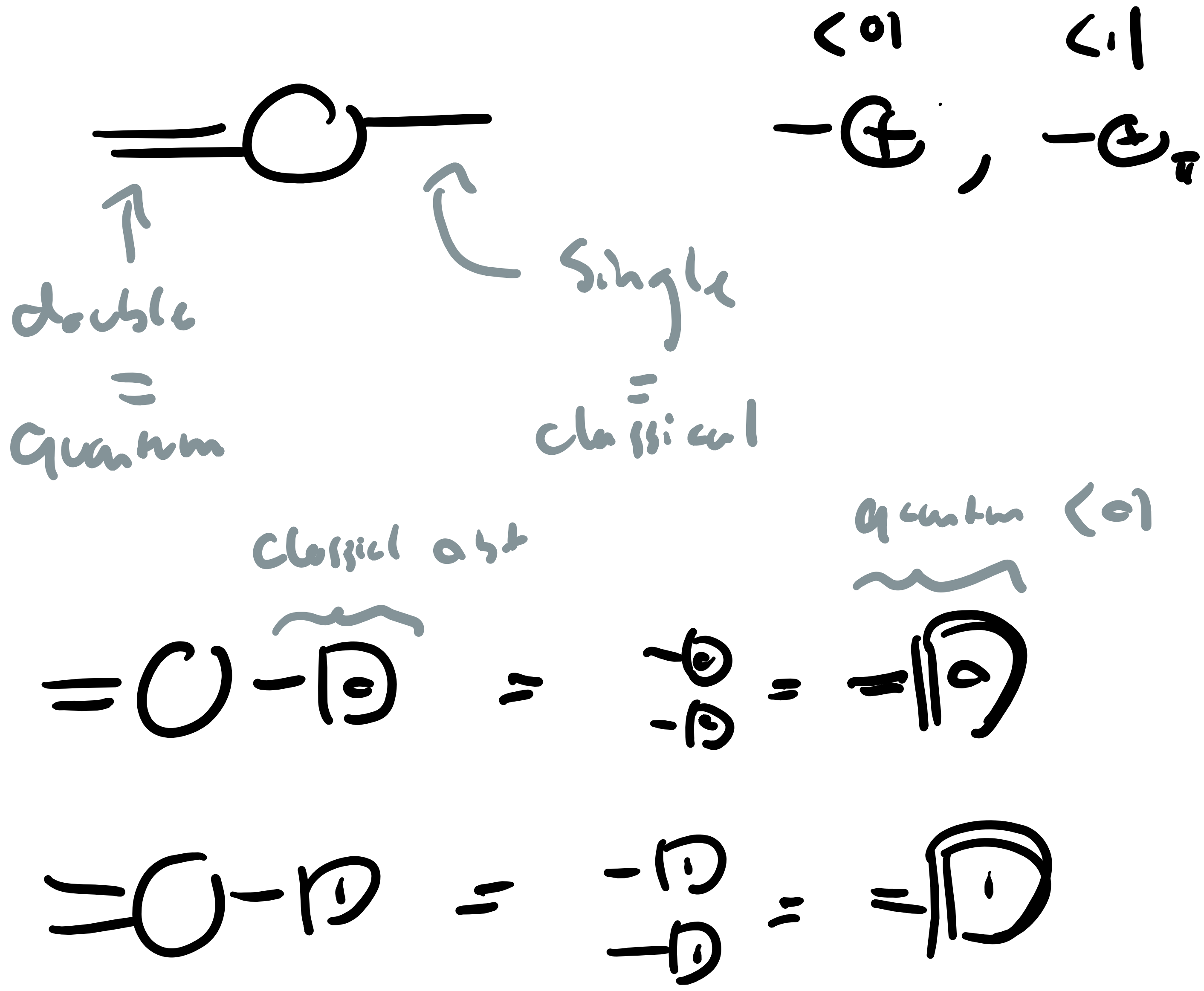


$= |\langle 0|\psi\rangle|^2$

$P(1) =$ 

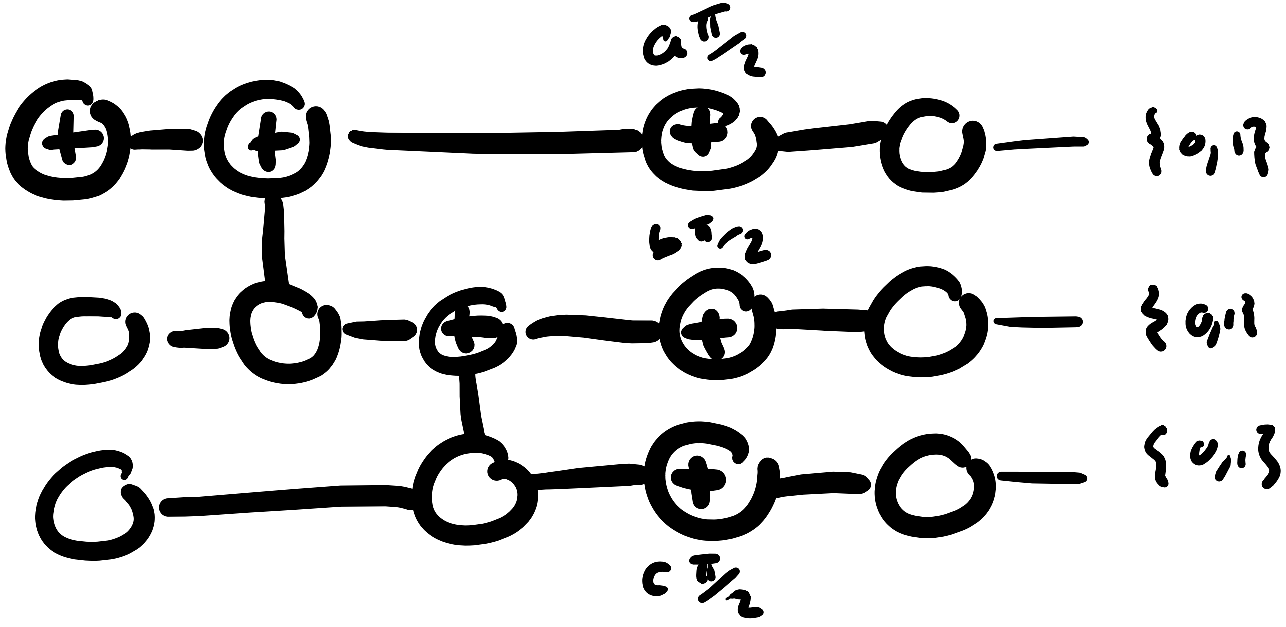


$= |\langle 1|\psi\rangle|$

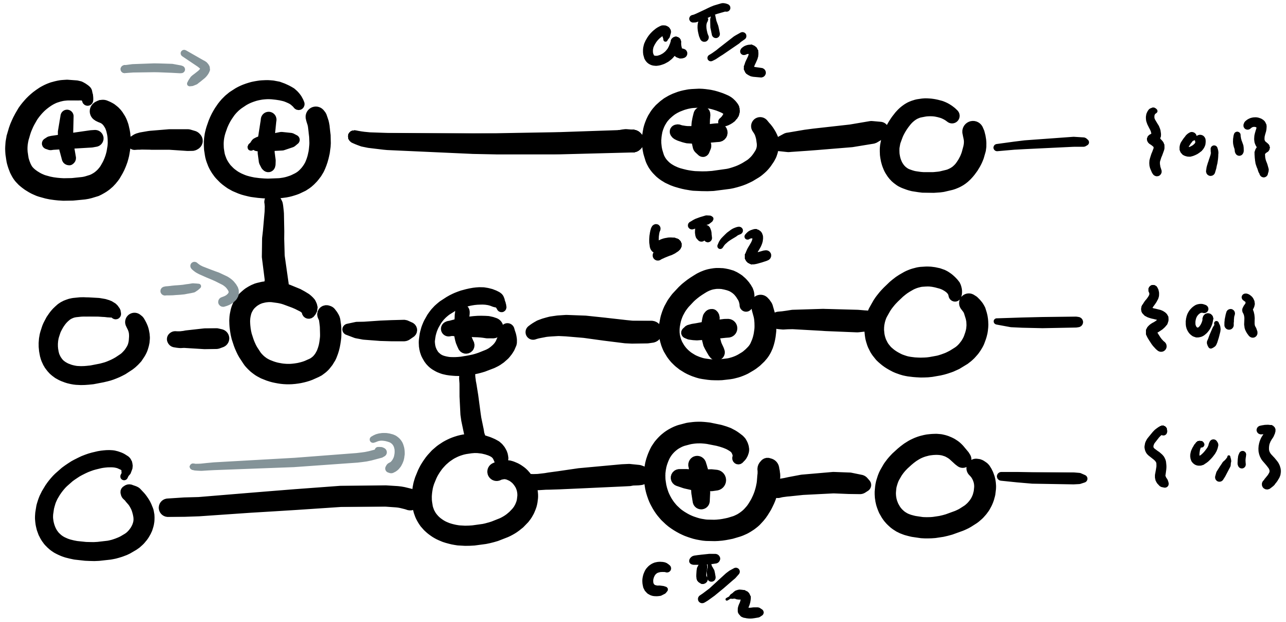


<0|
<1|
double = quantum
single = classical
classical out
quantum <0|

Mermin's non-locality argument

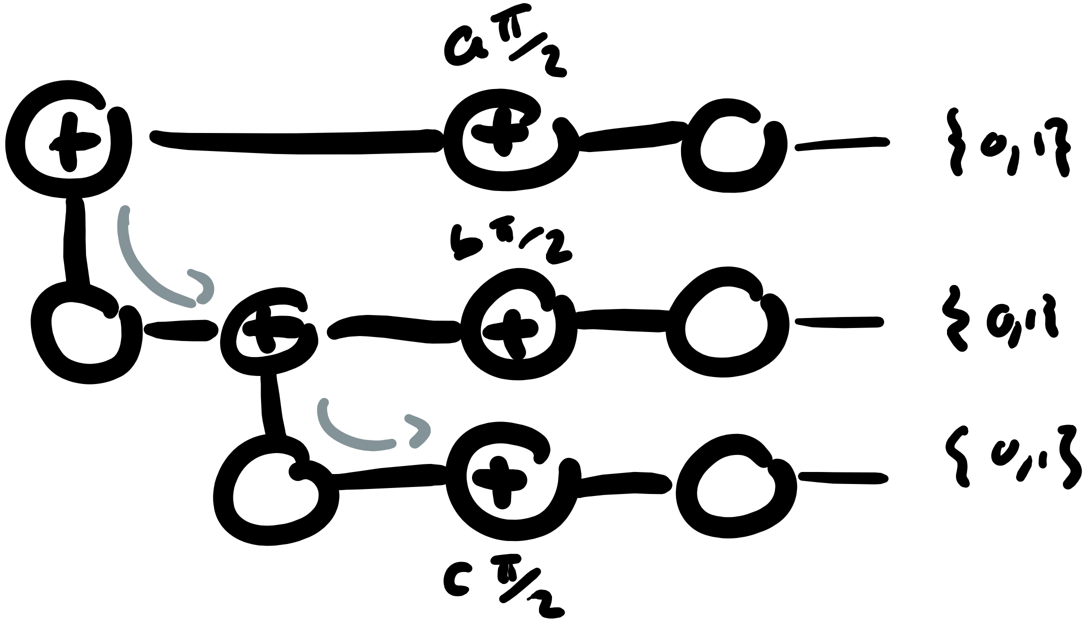


Mermin's non-locality argument

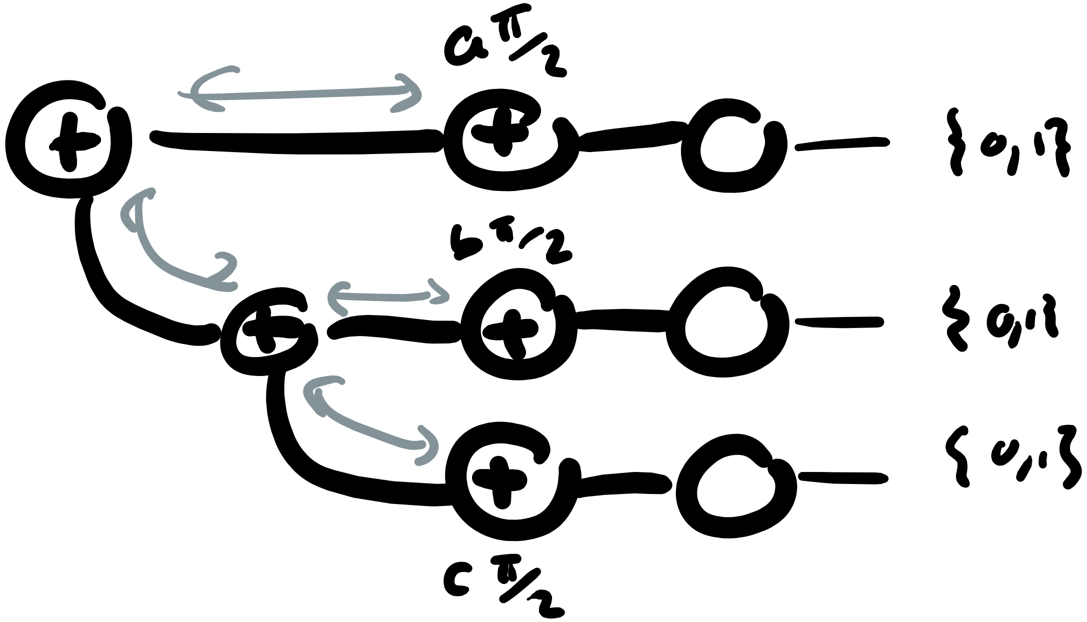


① fusion

Mermin's non-locality argument

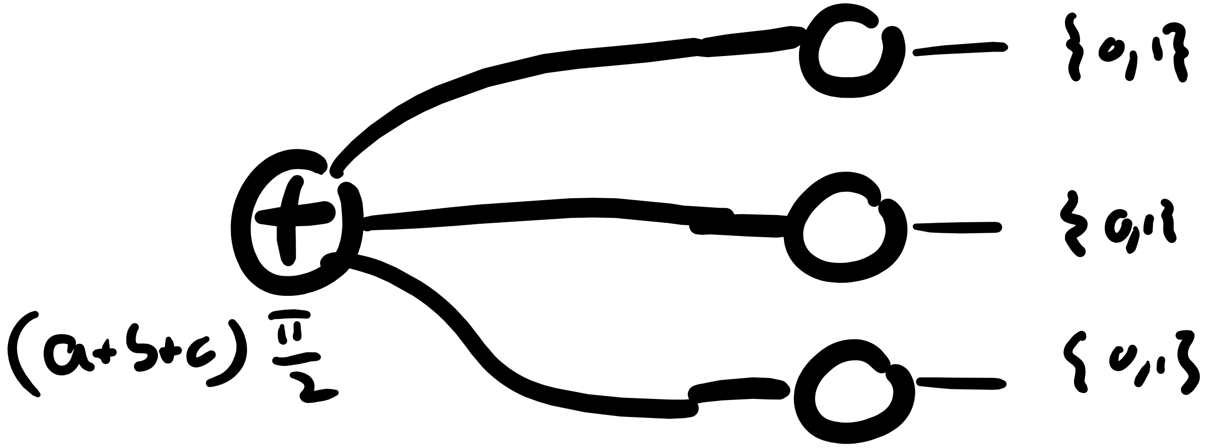


Mermin's non-locality argument

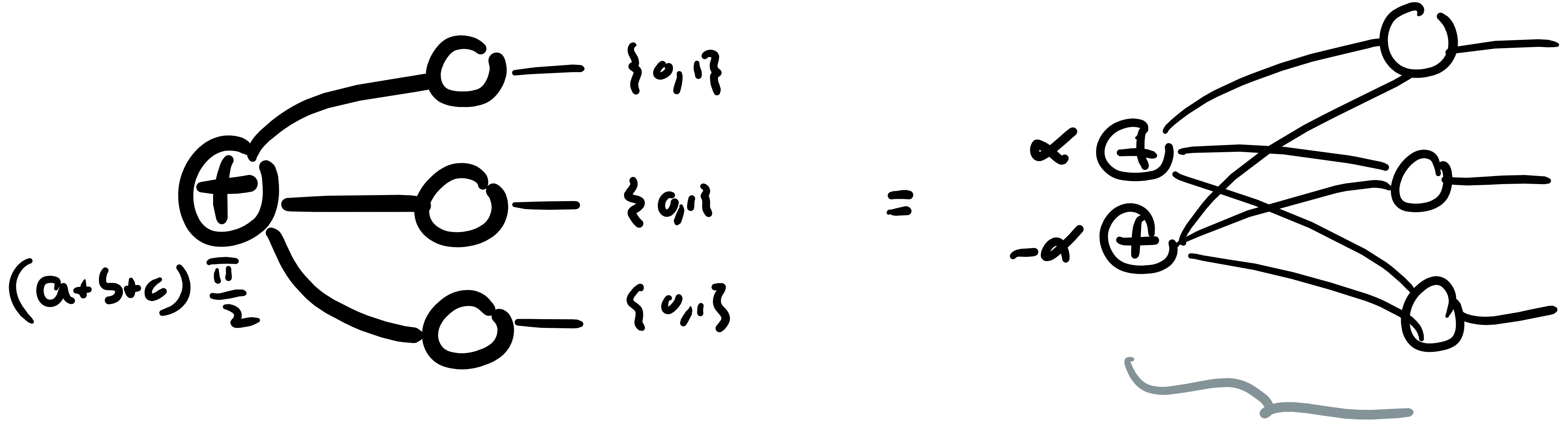


③ Lution

Mermin's non-locality argument



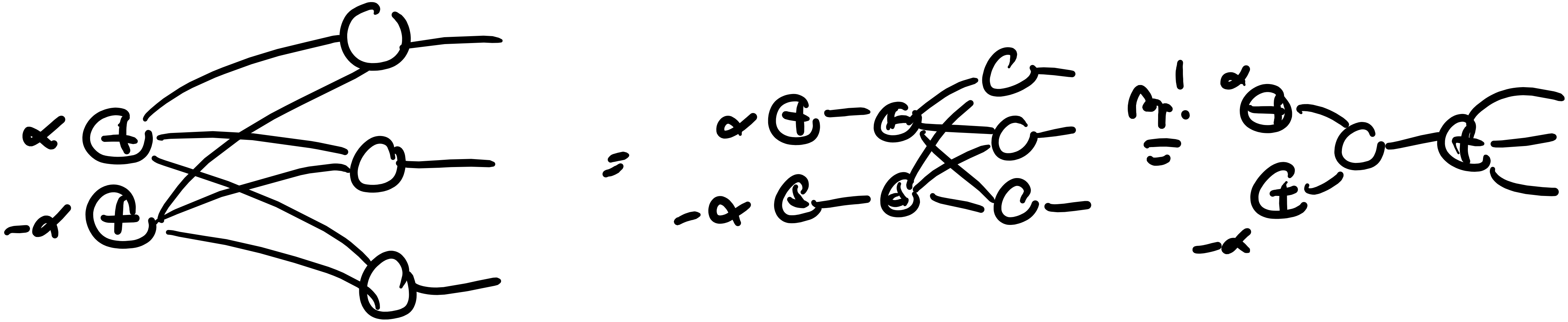
Mermin's non-locality argument



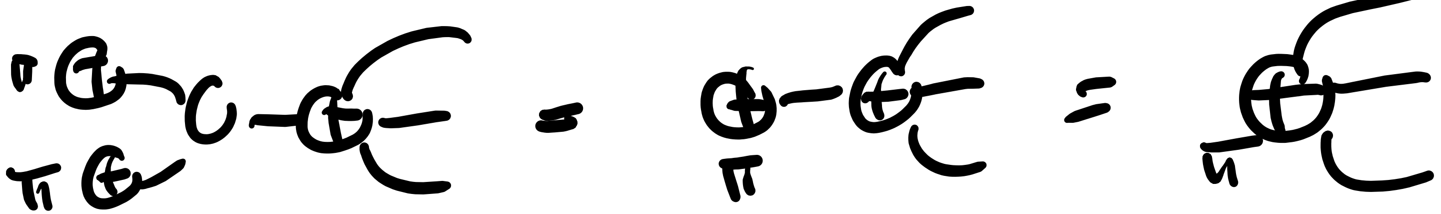
$\alpha := (a+b+c)\frac{\pi}{2}$

$K_{2,3}$ $\Rightarrow$ square pop!

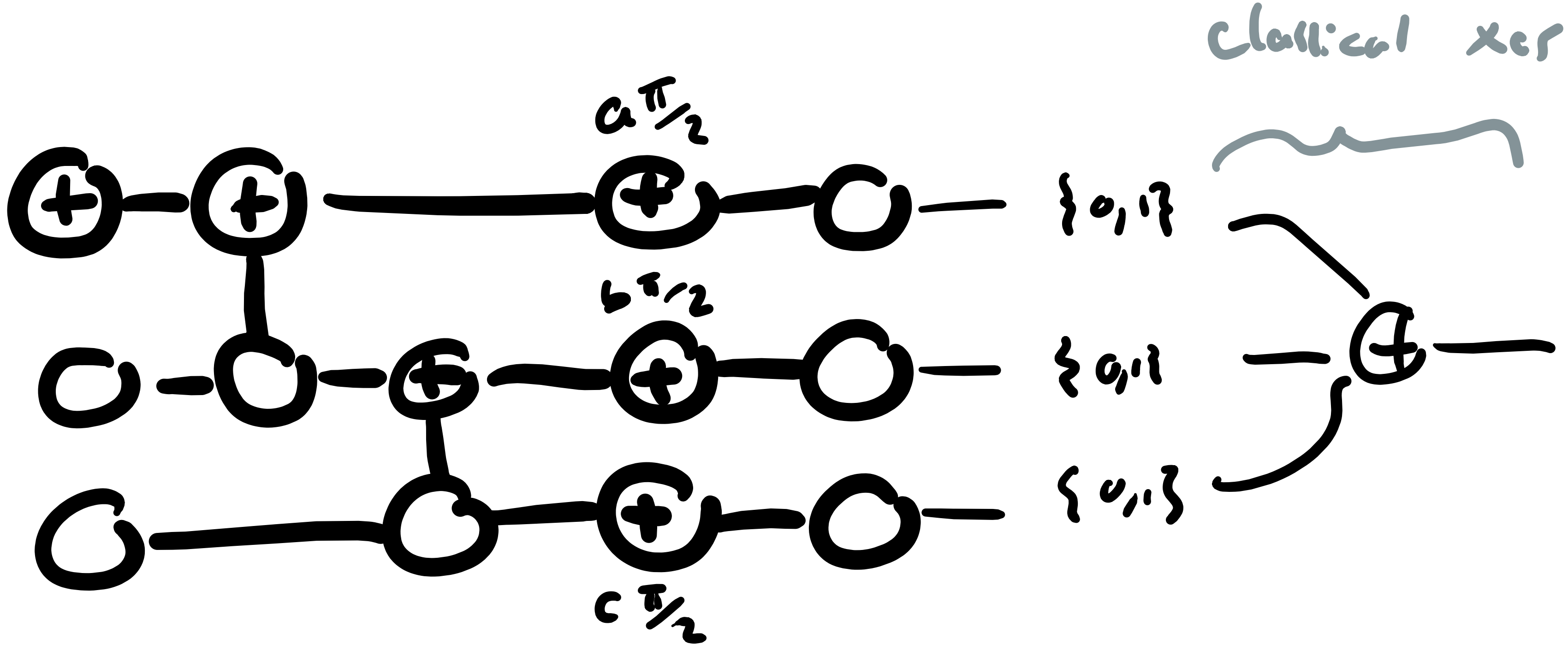
Mermin's non-locality argument



$\alpha = 0, \pi$



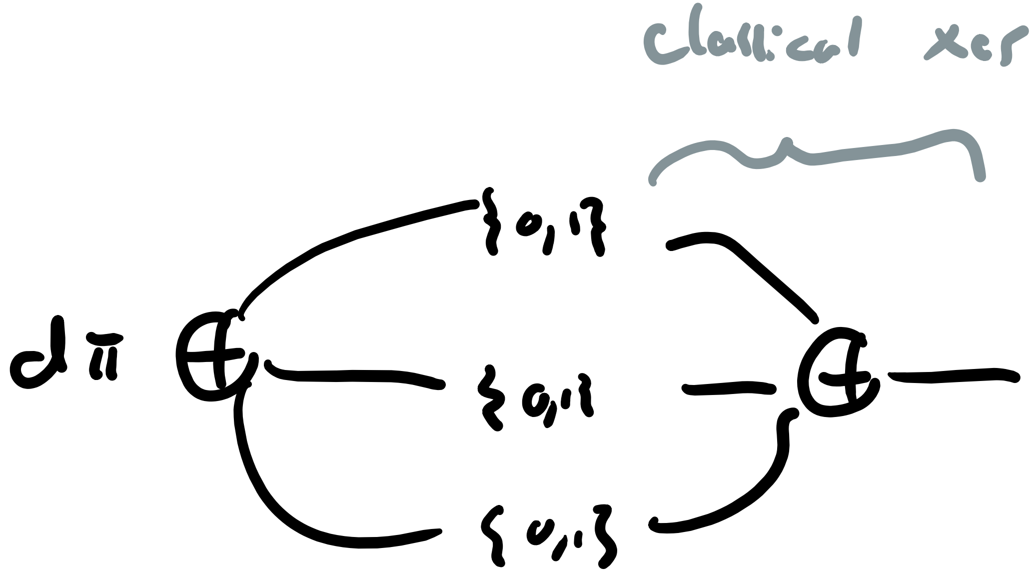
Mermin's non-locality argument



$a, b, c \in \{0,1\}$

restrict to when $a+b+c = 0, 2$

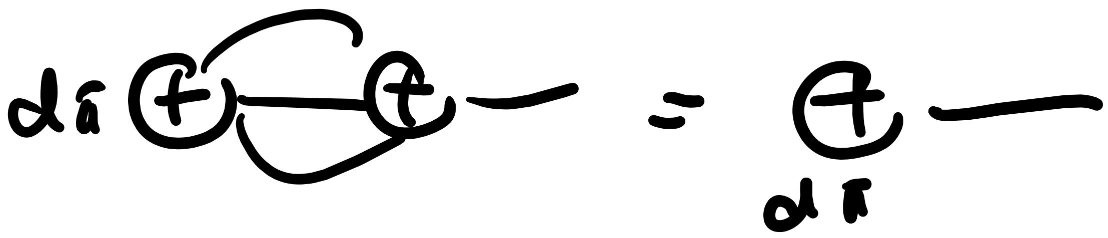
Mermin's non-locality argument



$a, b, c \in \{0,1\}$

restrict to when $a+b+c = \begin{cases} 0 \Rightarrow d=0 \\ 2 \Rightarrow d=1 \end{cases}$

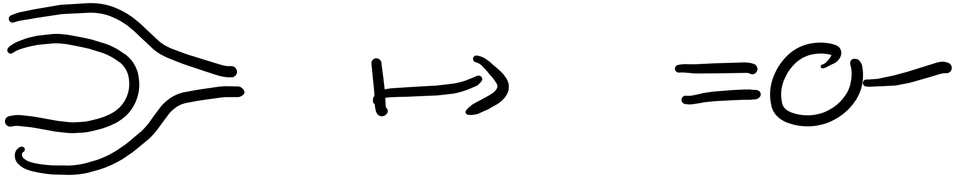
Mermin's non-locality argument



$$= \begin{cases} 0 & \text{if } a=b=c=0 \\ 1 & \text{if } a+b+c=2 \end{cases}$$

abc	out
000	0
011	1
101	1
110	1

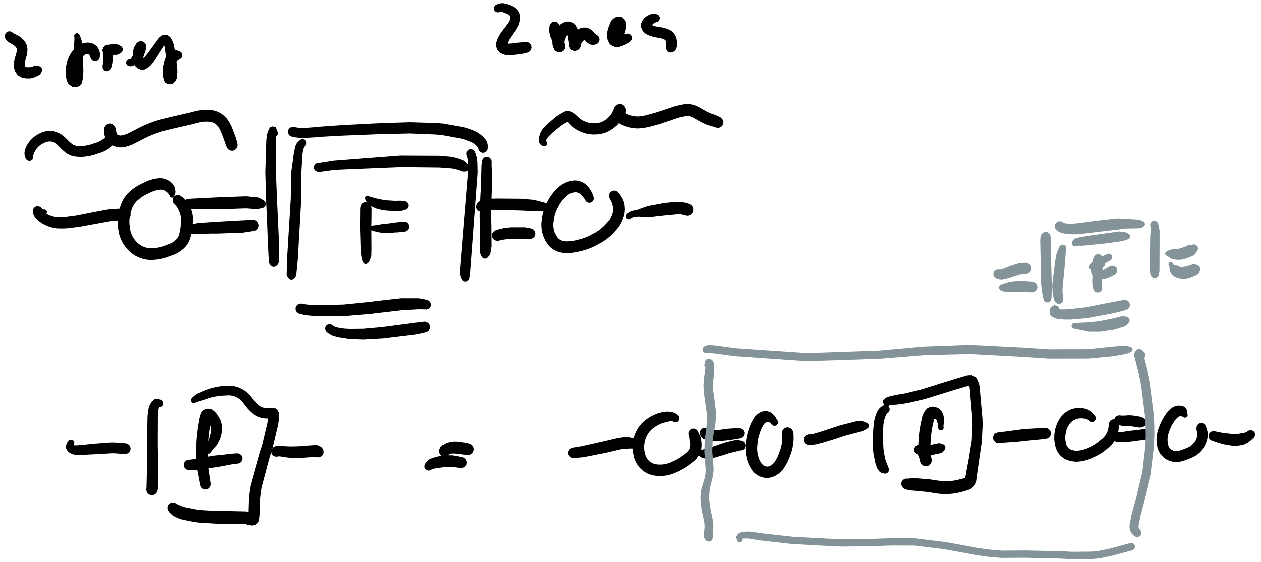
PS

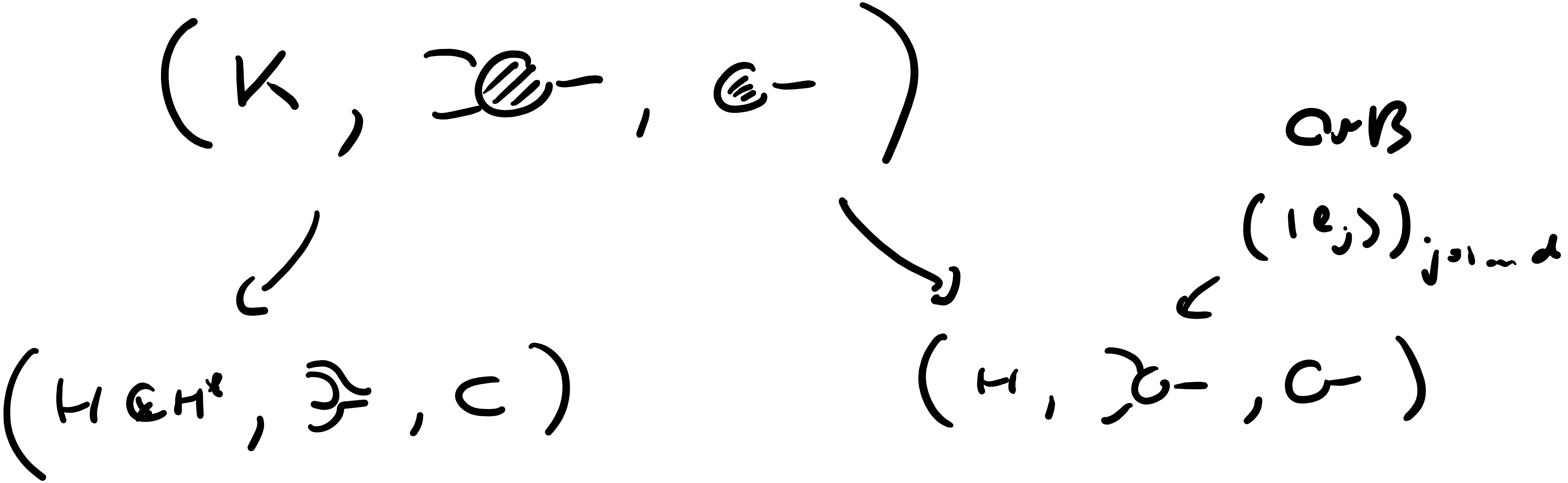


matrix algebra

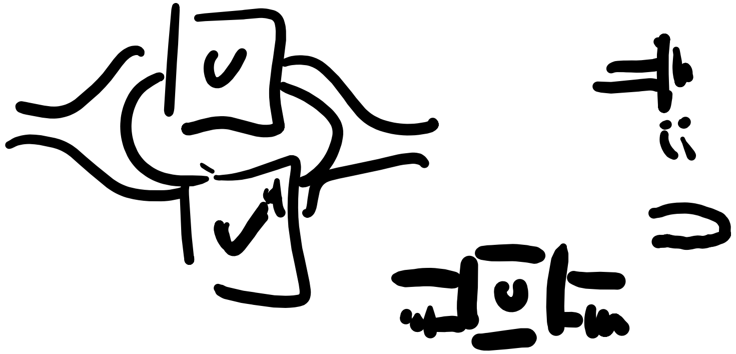
classical basis algebra

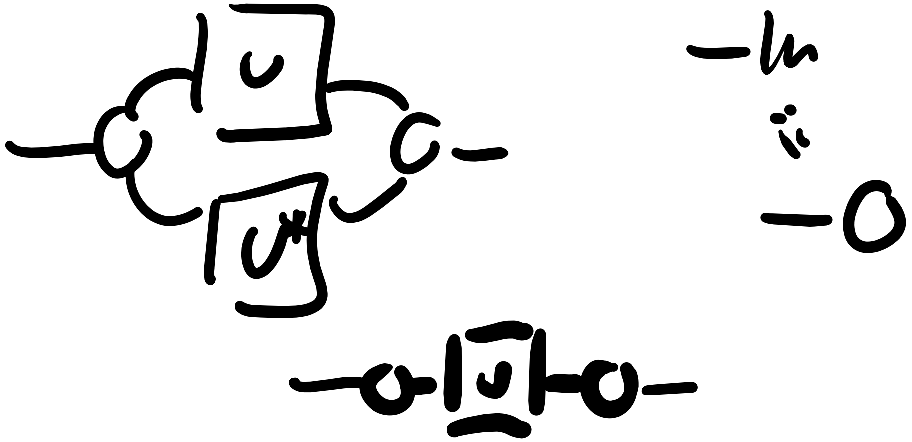
Classical processes



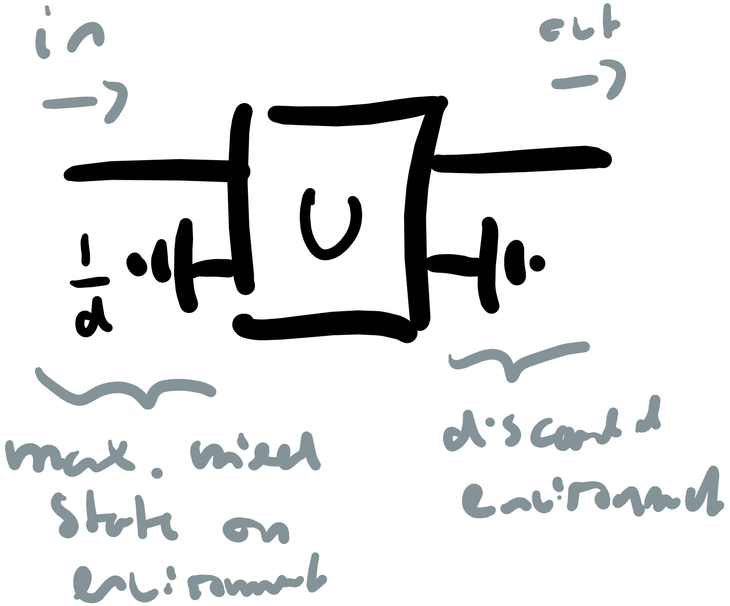


quantum system

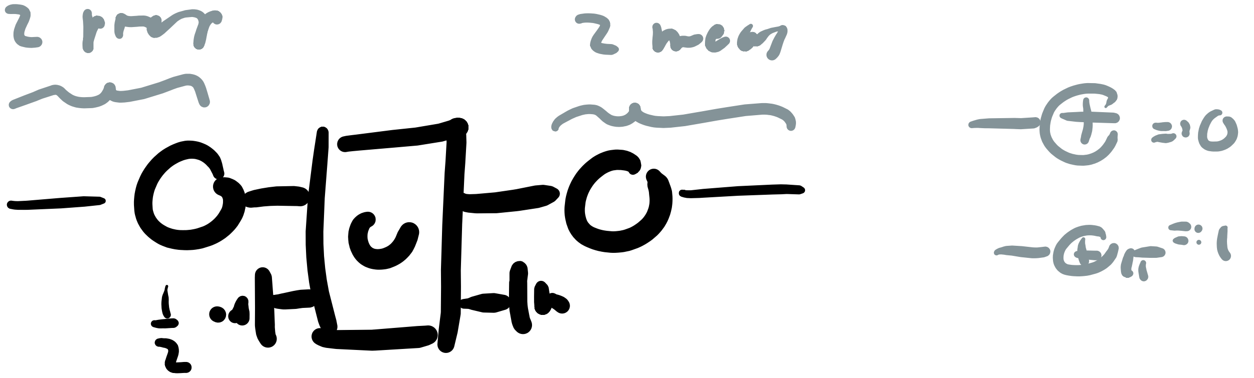




CP map

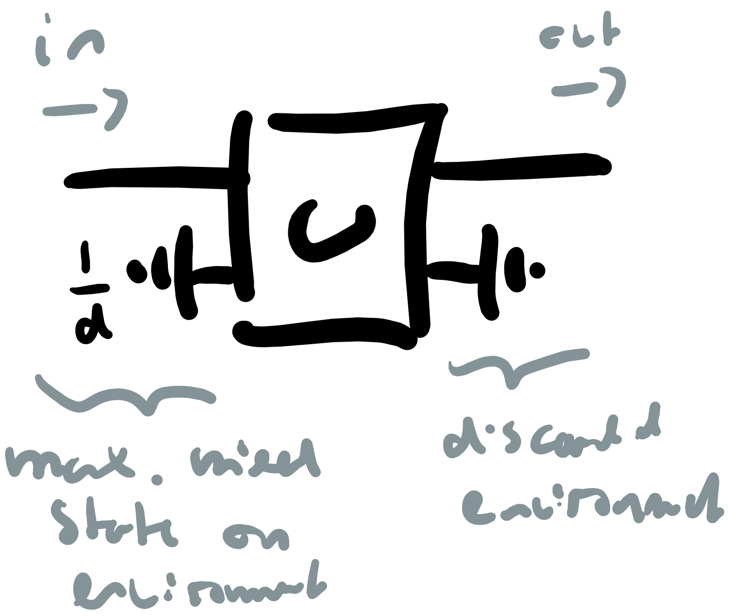


Classical process

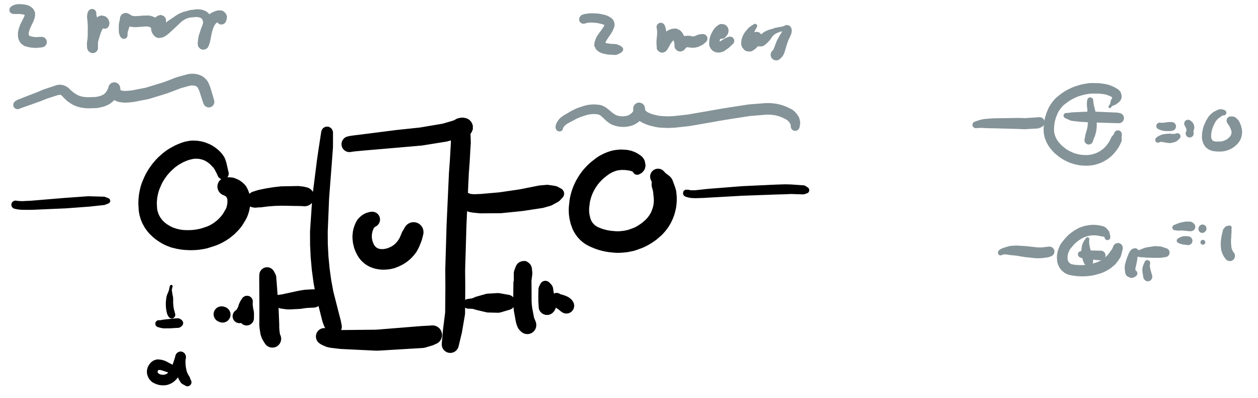


Stinespring Dilation

CP map

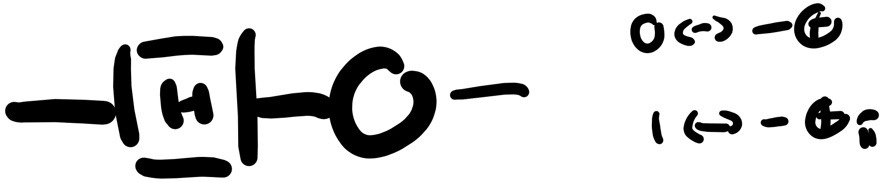


Classical process



Stinespring Dilation

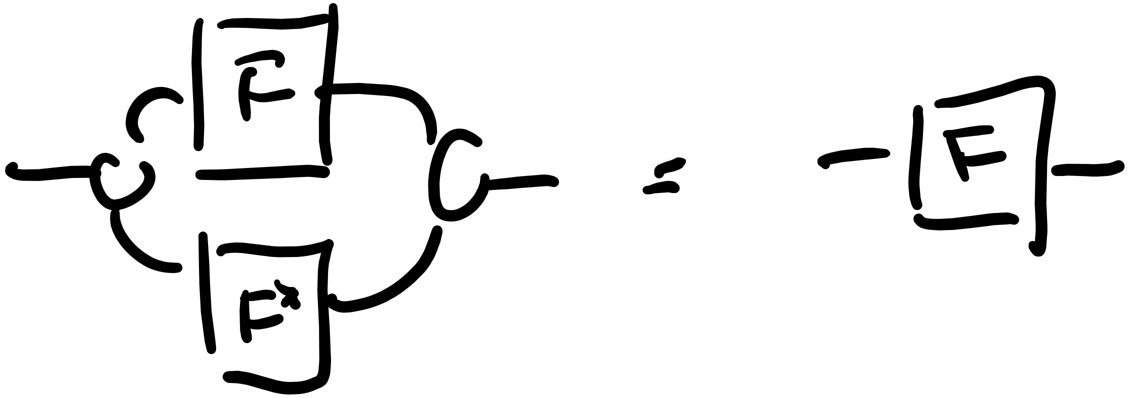
X meas:



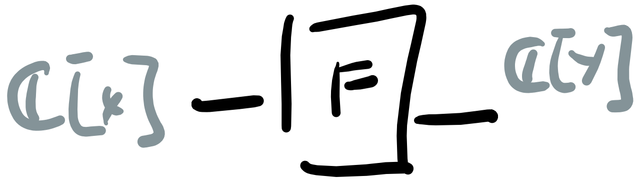
$$\mathbb{R} \xrightarrow{f} \mathbb{R}$$

$$F = \sum_{x \in \mathbb{R}} |f(x)\rangle\langle x|$$

Unbounded operator

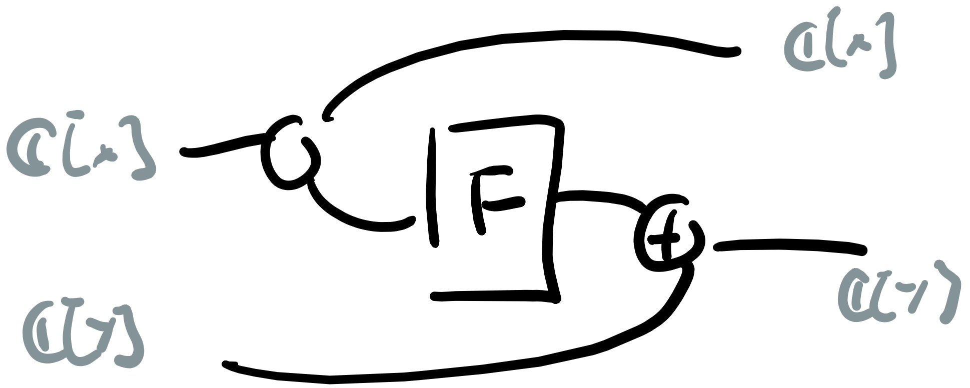


$\in$ Stoch



$= \sum_{x \in X} |f(x)\rangle\langle x|$

$f : X \to Y$



unitary