Quantum in Pictures Lecture Series

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Wed 28 June 2023 – Afternoon Lecture

INDIANA UNIVERSITY BLOOMINGTON

$\mathbb{C}[\mathbb{Z}_d]$ $\quad (|j\rangle)_{j=0,\dots,d-1}$ Comp basis

$(|\chi_k\rangle)_{k=0,\dots,d-1}$ Fourier basis

$$|\chi_k\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \chi_k(j) |j\rangle \qquad \chi_k(j) = e^{i 2\pi \frac{k \cdot j}{d}} \in S^1$$

Mult. character

$$\chi_k : \mathbb{Z}_d \to S^1$$

$$\chi_k(i+j) = \chi_k(i) \cdot \chi_k(j)$$

$$\chi_k(0) = 1$$

$$\chi_k(-j) = \chi_k(j)^*$$

$$2\pi \frac{(k+ad)(j+bd)}{d} =$$

$$= 2\pi \frac{kj}{d} + 2\pi \frac{[\text{illegible}]\, \cancel{d}}{\cancel{d}} = 2\pi \frac{kj}{d}$$

$\in \mathbb{Z}_d$

↑ mod 2π

$$\mathbb{Z}_n \longleftrightarrow \hat{\mathbb{Z}_n}$$

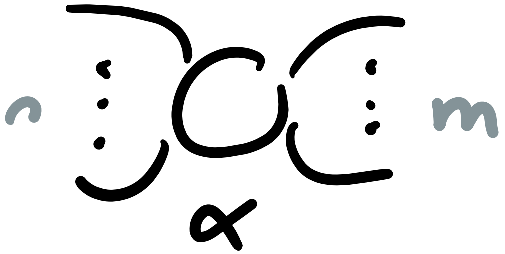
$$j \longmapsto x_j$$

$$j \longmapsto x_{pj} \qquad p \in \mathbb{Z}_n^\times$$

$$\mathbb{R} \longleftrightarrow \mathbb{R}^{\wedge}$$

$$x \longmapsto \left(y \mapsto e^{ixy} \right)$$

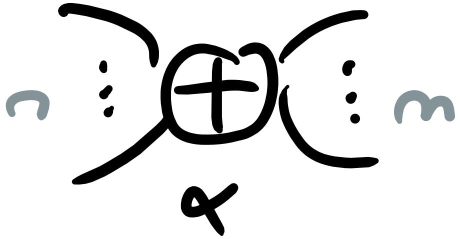
$$x \longmapsto \left(y \mapsto e^{i\frac{xy}{\hbar}} \right) \qquad \hbar \neq 0$$



$$:= \sum_{j=0}^{d-1} e^{i\alpha_j} |\underbrace{j\ldots j}_{m}\rangle\langle\underbrace{j\ldots j}_{n}|$$

$$\alpha_0 := 0$$

$$\begin{pmatrix} 1 & & & \\ & e^{i\alpha_1} & & \\ & & e^{i\alpha_2} & \\ & & & \ddots & \\ & & & & e^{i\alpha_{d-1}} \end{pmatrix}$$



$$:= \sum_{j=0}^{d-1} e^{i\alpha_j} |\underbrace{x_j\ldots x_j}_{m}\rangle\langle\underbrace{x_j\ldots x_j}_{n}|$$

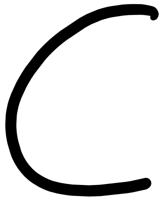
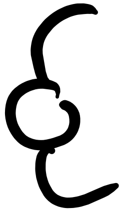
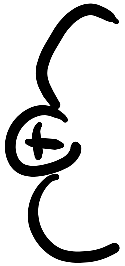
$$:= \sum_{j=0}^{d-1} |x_j\rangle\langle j|$$

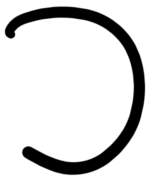
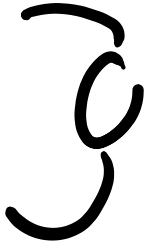
$$= \sum_{j=0}^{d-1} |j\rangle\langle x_j|$$

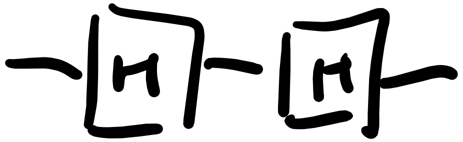
$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix}^T = \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & k \end{pmatrix}$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad |2\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

for $d > 2$

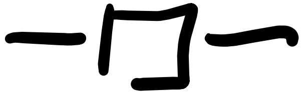
 :=  $= \sum_{j=0}^{d-1} |jj\rangle \neq \sum_{j=0}^{d-1} |x_j x_j\rangle =$ 

 :=  $= \sum_{j=0}^{d-1} \langle jj|$



$$= \sum_{j'=0}^{d-1} \sum_{j=0}^{d-1} |x_{j'}\rangle \underbrace{\langle j' | x_j\rangle}\langle j|$$

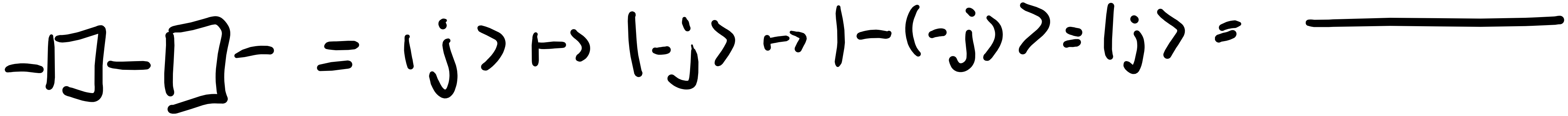
$$(\langle x_j | j'\rangle)^\dagger = x_j(j')^\dagger = x_j(-j')$$

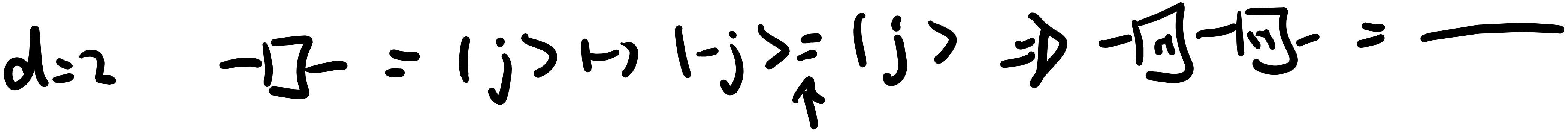
$$= \sum_{j=0}^{d-1} |-j\rangle\langle j| =:$$ 

$$\sum_{j'=0}^{d-1} x_j(-j') = \begin{cases} 0 & \text{if } j \neq 0 \\ 1 & \text{if } j = 0 \end{cases}$$

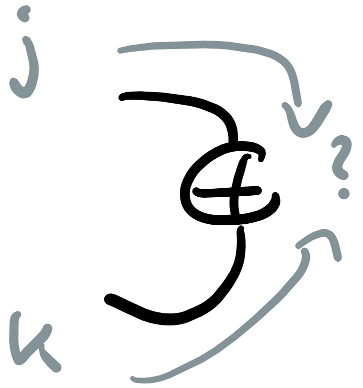
Antipode

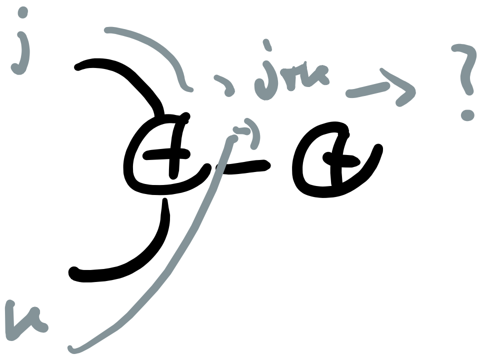
= group inverse



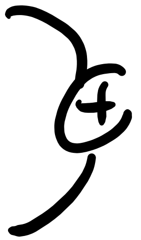


$-1 = 1 \mod 2$

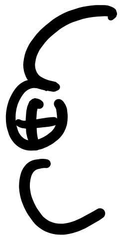




$$-\oplus = (\oplus -)^\dagger = (|0\rangle)^\dagger = \langle 0|$$



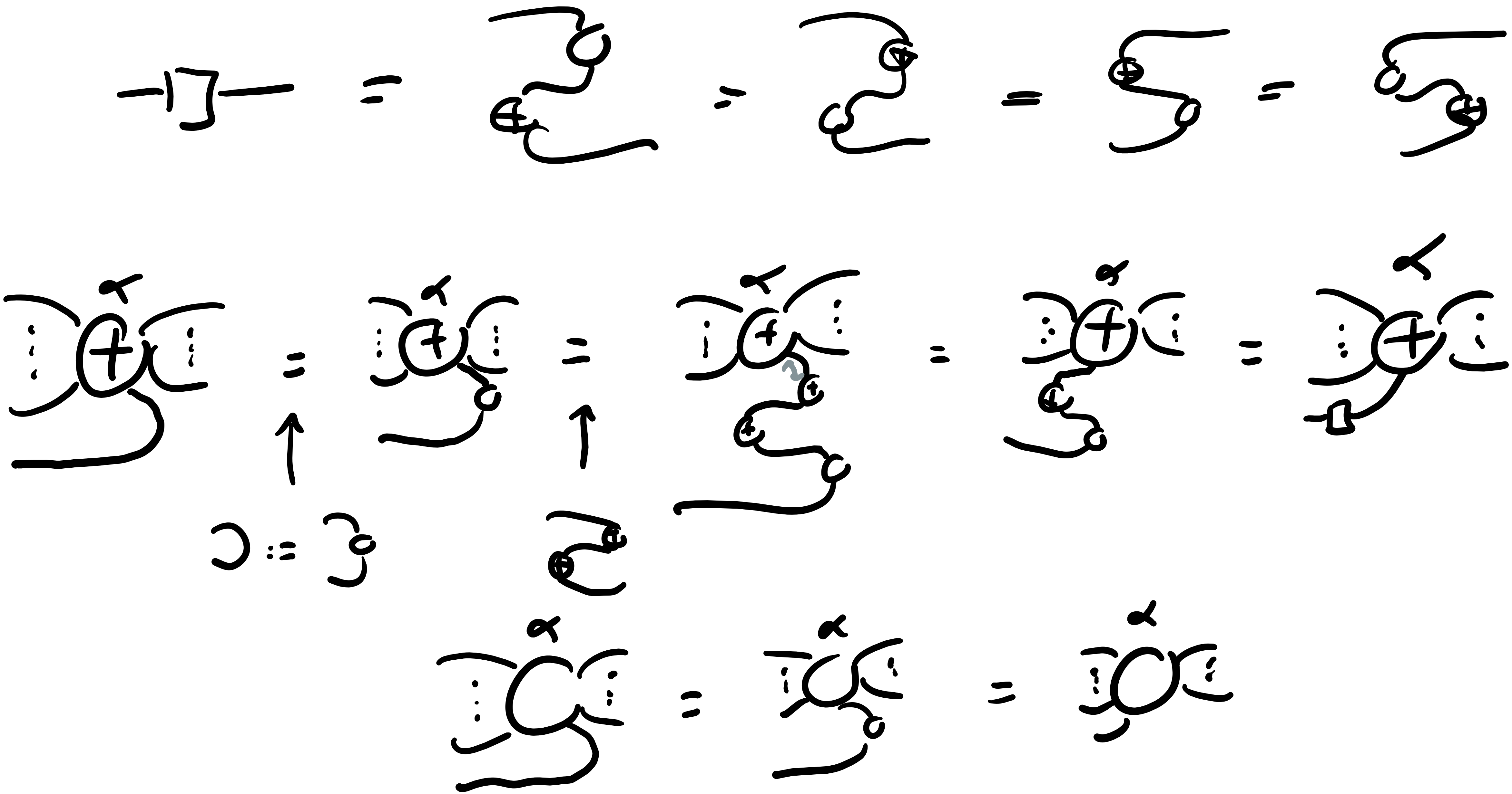
$$= |jk\rangle \mapsto \begin{cases} 1 & j=-k \\ 0 & \text{otherwise} \end{cases} \in \mathbb{C}$$

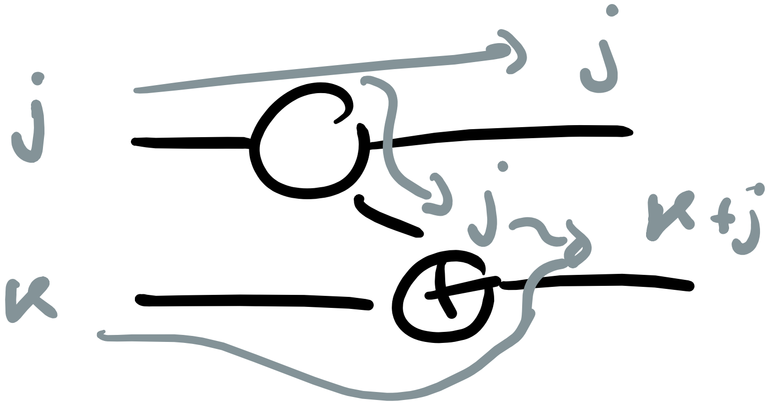


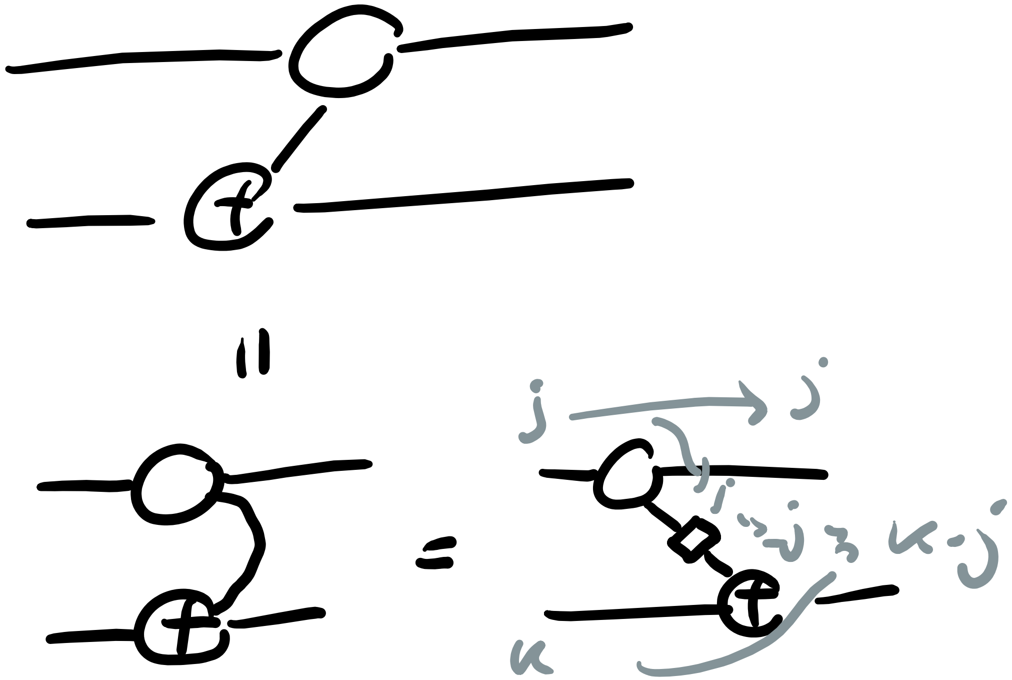
$$= \sum_j |j,-j\rangle$$

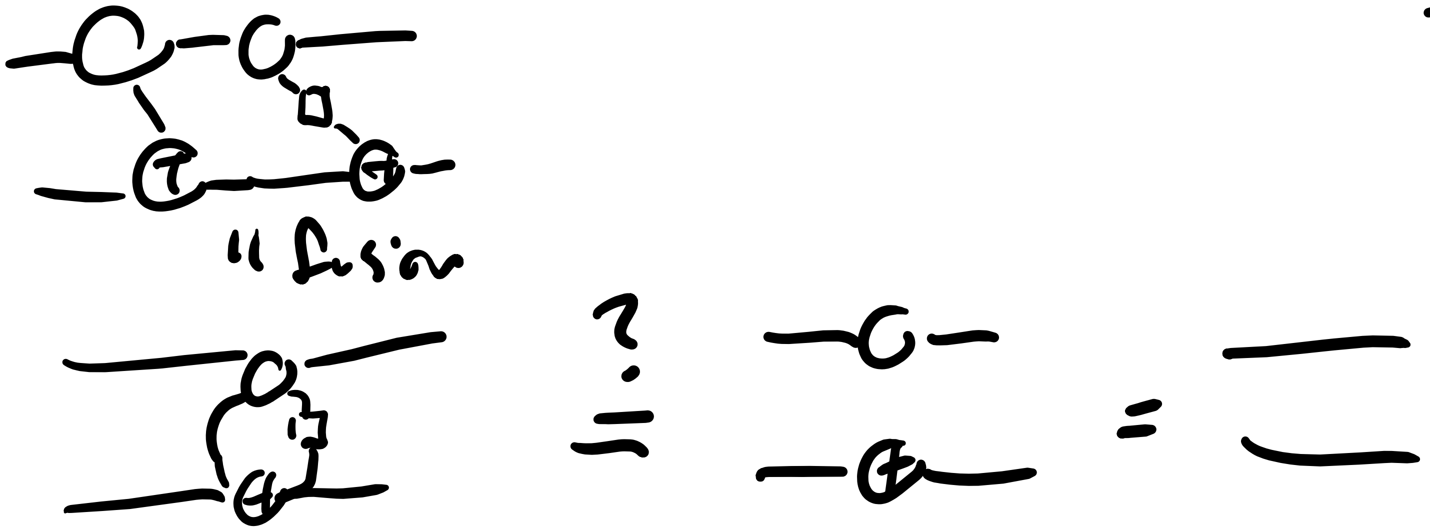
$$|j\rangle \longrightarrow |-j\rangle$$

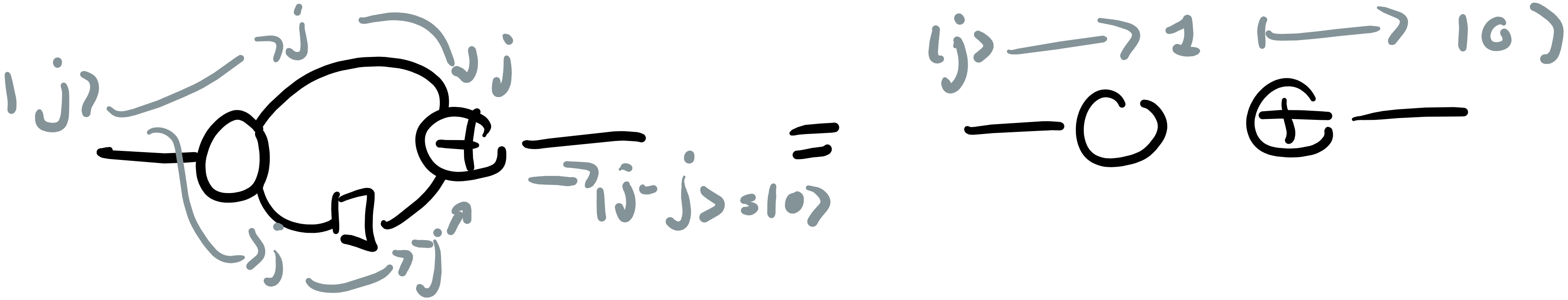
$$-[\,]- =$$



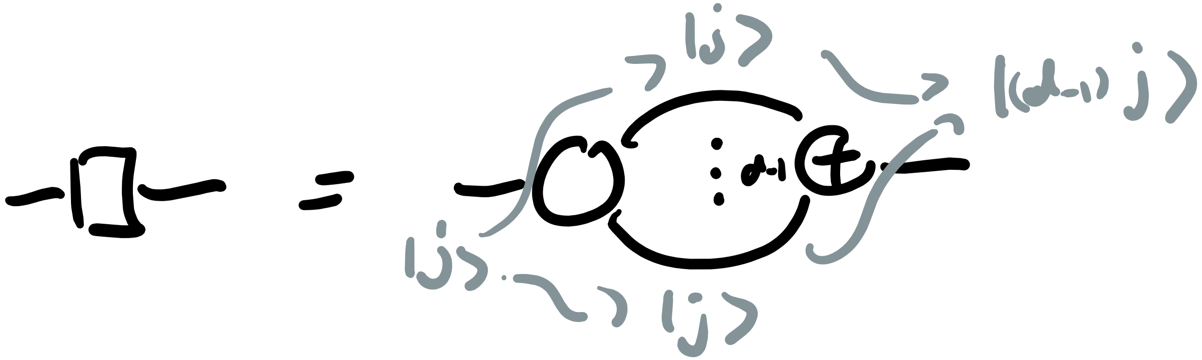

j
j
j
k
k+j

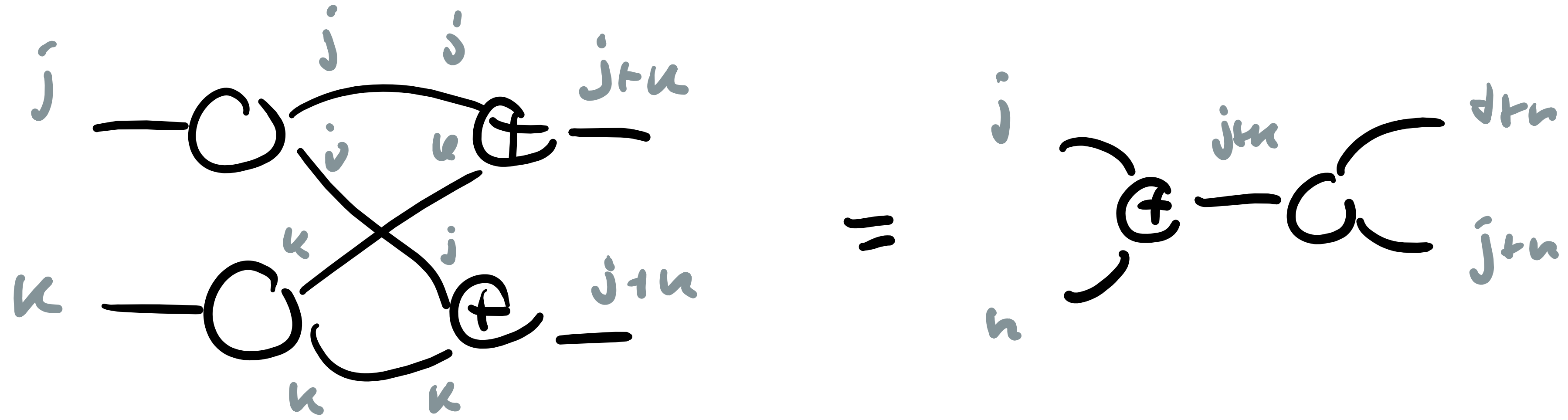

j
j
j
j
k
k+j

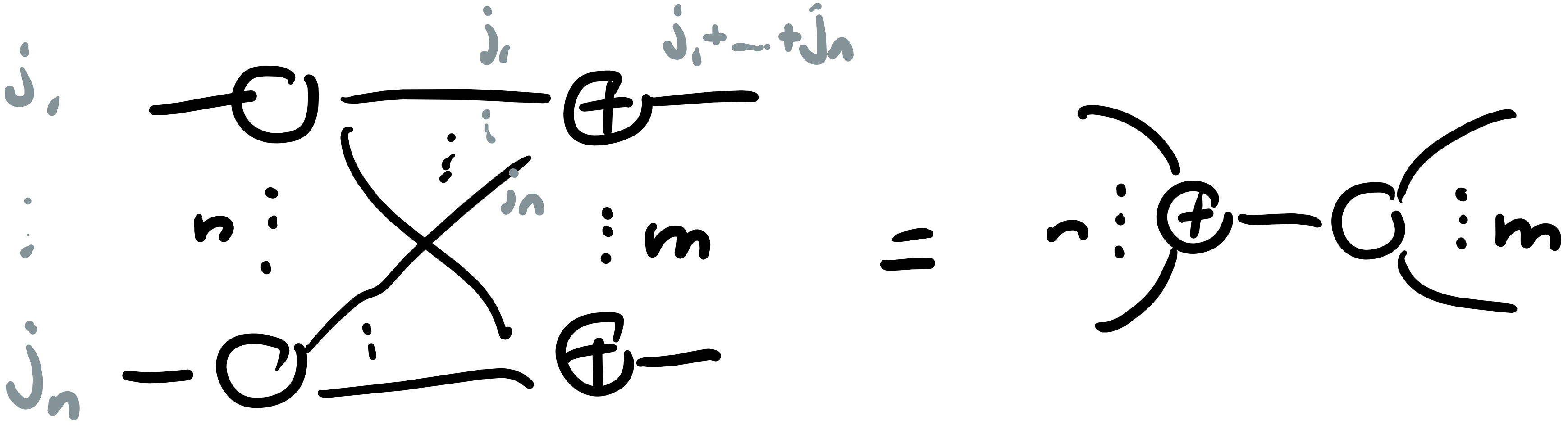

= fusion
?



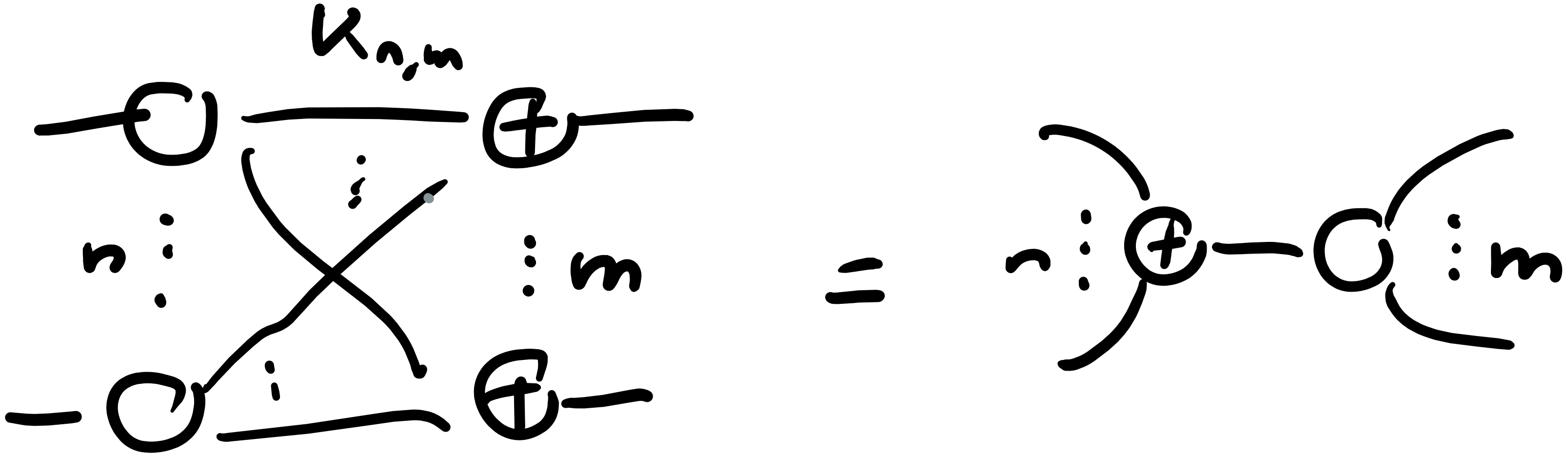
mod d $\quad -j = (d-1) \cdot j$







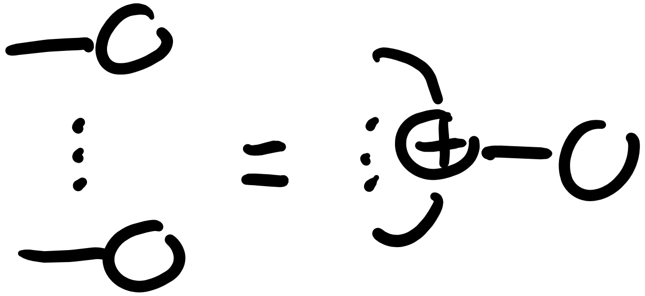
$K_{n,m}$

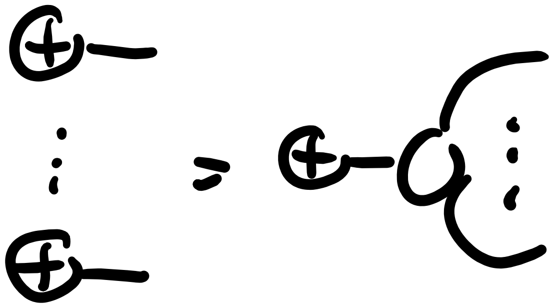


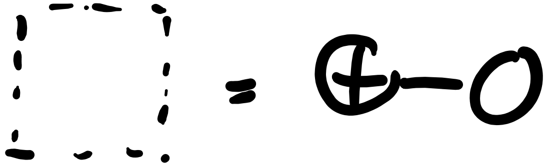
$m = 0, n > 0$

$n = 0, m > 0$

$n = m = 0$







$1/\sqrt{d} \in \mathbb{C}$

$\langle x_0 | 0 \rangle$

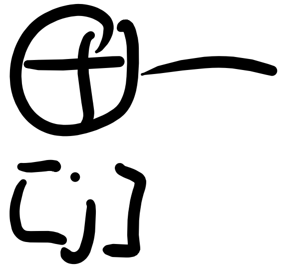
$$\underset{\alpha}{\oplus}\!- \;=\; |x_0\rangle + e^{i\alpha_1}|x_1\rangle + e^{i\alpha_2}|x_2\rangle + \cdots + e^{i\alpha_{d-1}}|x_{d-1}\rangle$$

$$\alpha_j := 2\pi\frac{-jk}{d}$$

$$\underset{\alpha}{\oplus}\!- \;=\; |x_0\rangle + e^{-i2\pi\frac{k}{d}}|x_1\rangle + e^{-i2\pi\frac{2k}{d}}|x_2\rangle \cdots = |k\rangle$$

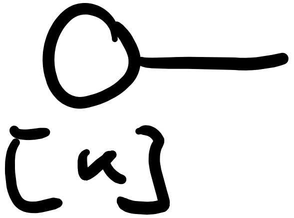
$$\underset{[j]}{\oplus}\!- \;=\; |j\rangle \qquad [j]_k := 2\pi\frac{-jk}{d}$$

$$d=2 \qquad \oplus\!- \;=\; |0\rangle \qquad \underset{\pi}{\oplus}\!- \;=\; |1\rangle$$



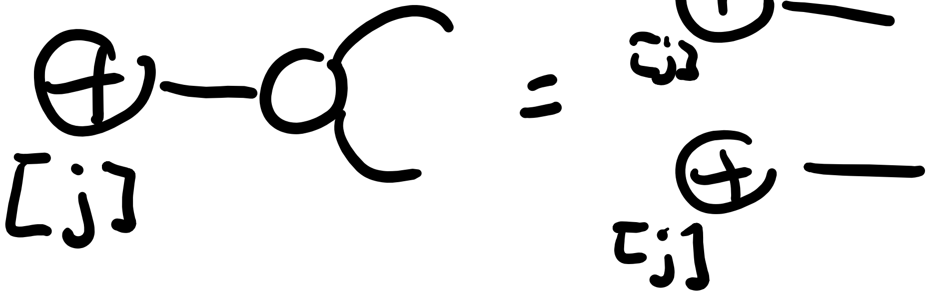
$= |j\rangle$

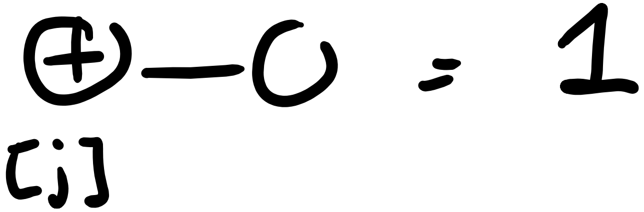
$[j]_k := 2\pi \frac{-jk}{d}$

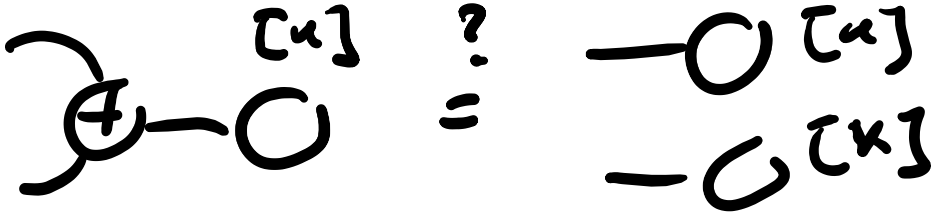


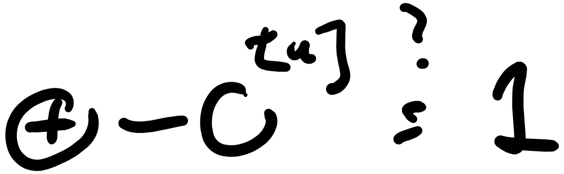
$= |x_k\rangle$

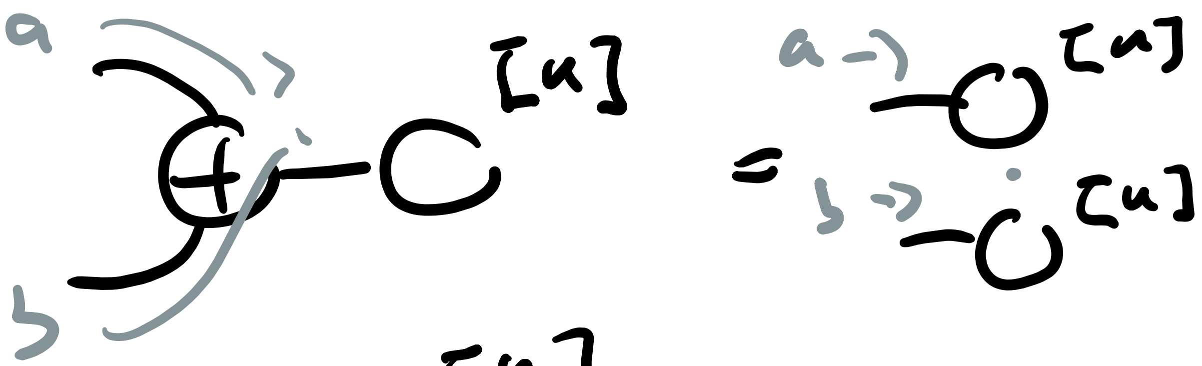
$[k]_j := 2\pi \frac{kj}{d}$



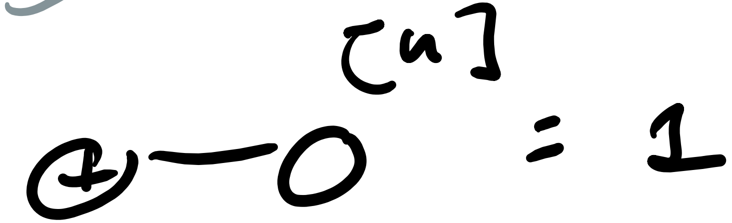




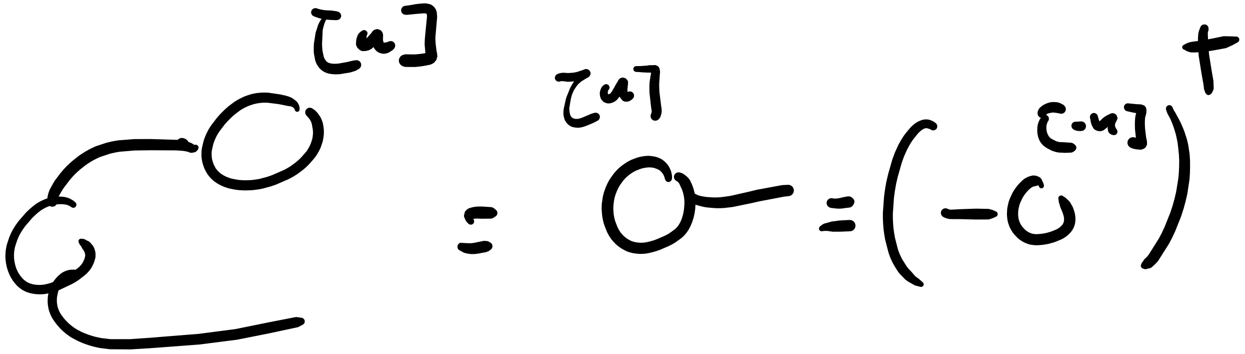




$$\chi_u(a+b) = \chi_u(a) \cdot \chi_u(b)$$

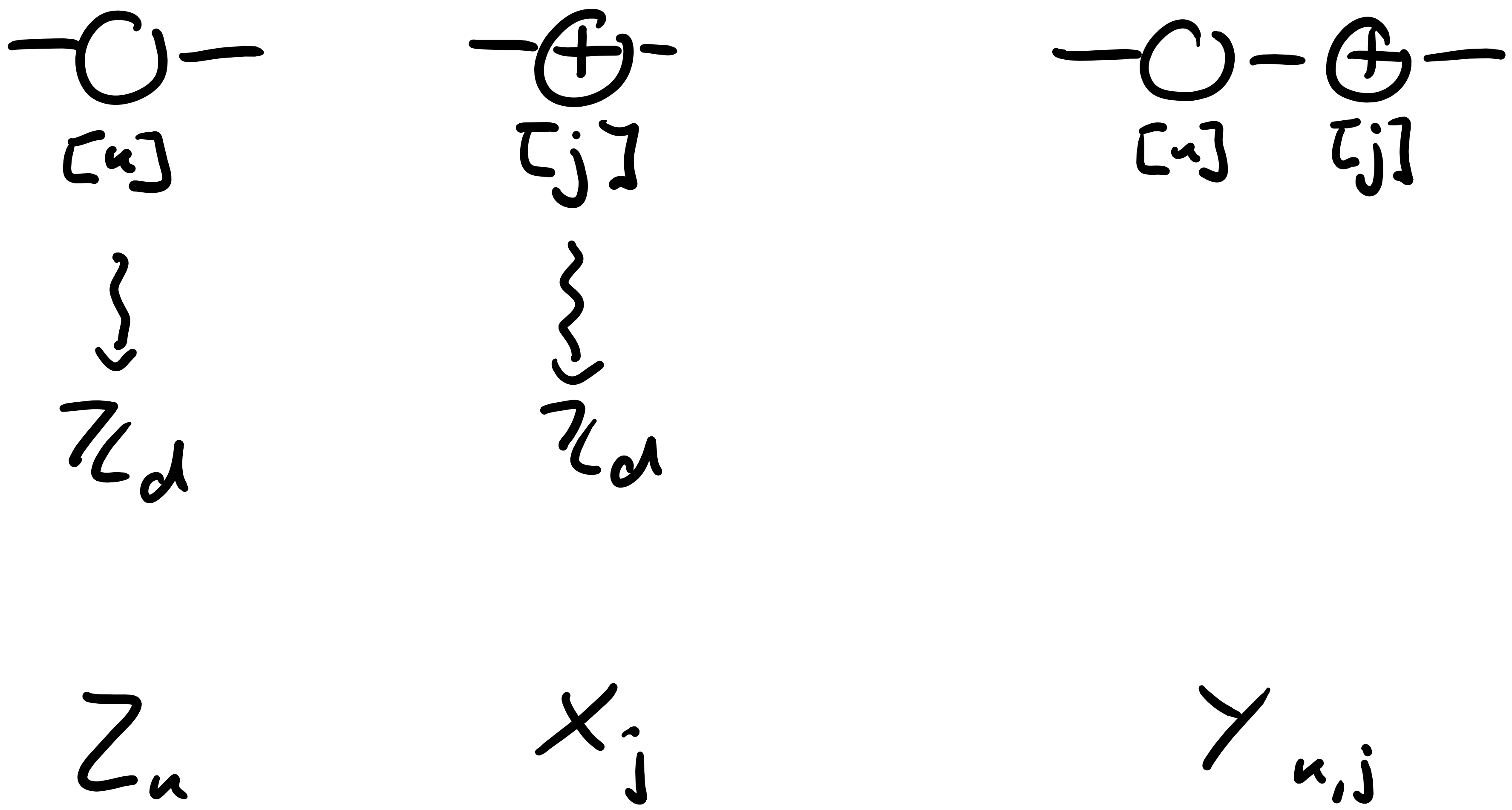


$$\chi_u(0) = 1$$

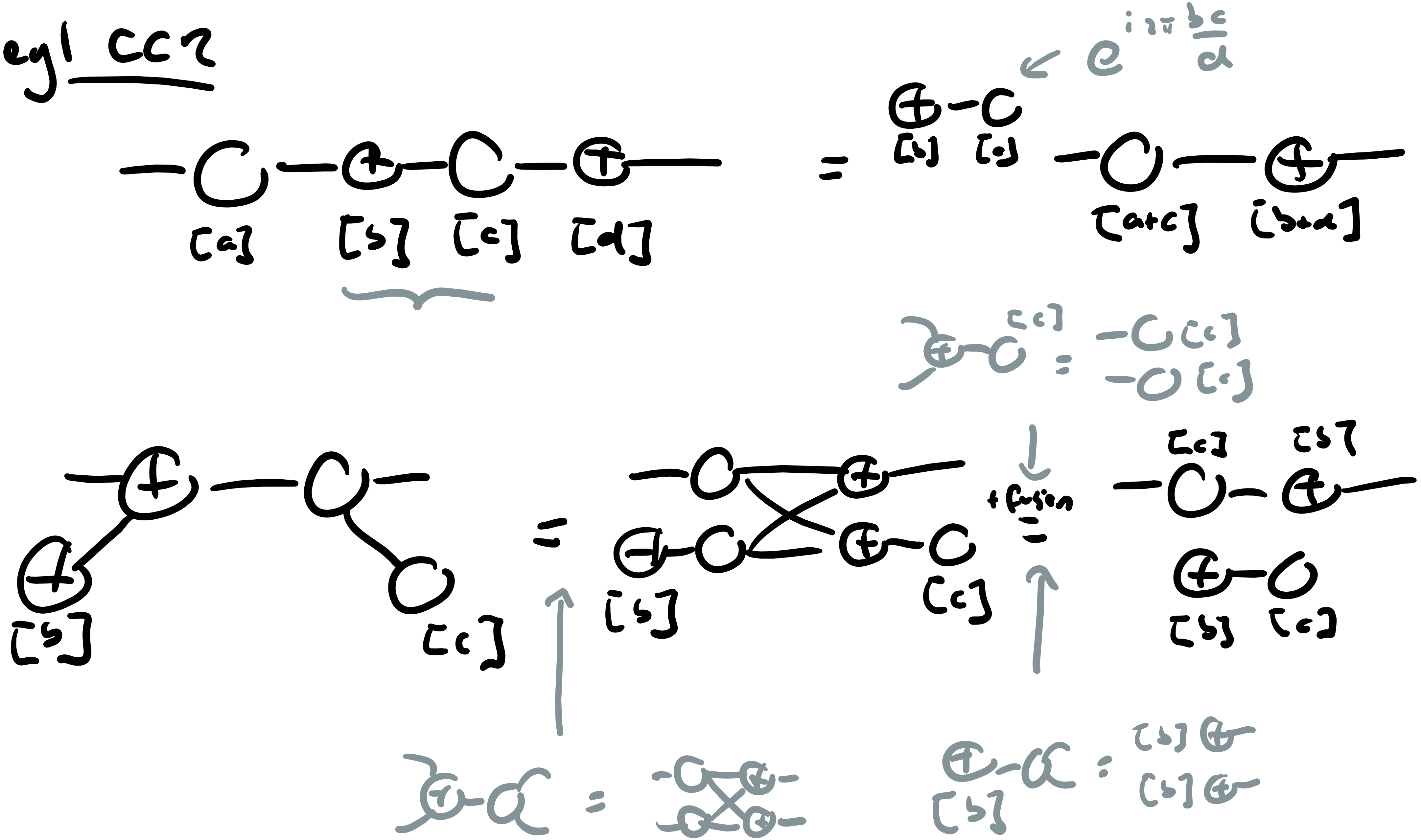


$$\chi_{-u}(j) = \chi_u(j)^\dagger$$

copy/delete/transpose for  $\longleftrightarrow$ Mult characters $\mathbb{Z}_d \xrightarrow{\chi} S^1$

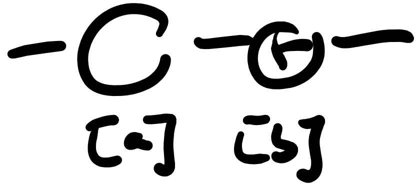

[n]
[j]
[n] [j]
ℤd
ℤd
Zn
Xj
Yn,j

Weyl CC?

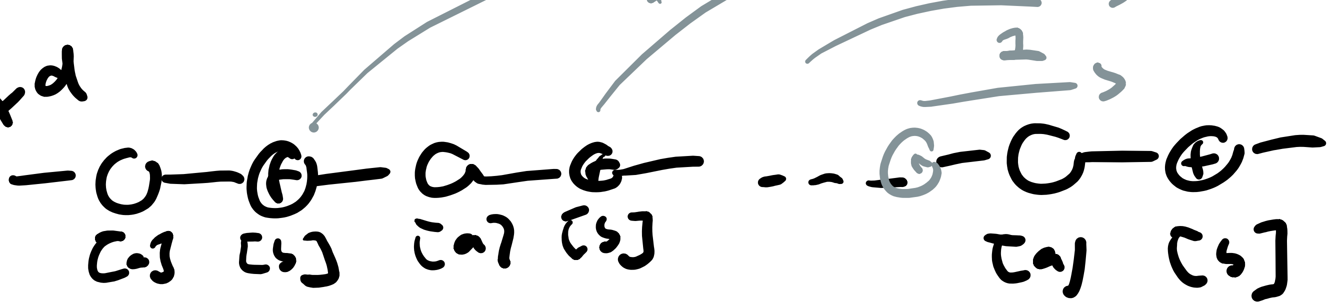


$$e^{i 2\pi \frac{ab}{d}}$$

$Y_{a,b} = \alpha$



$(Y_{a,b})^d = \alpha^d$

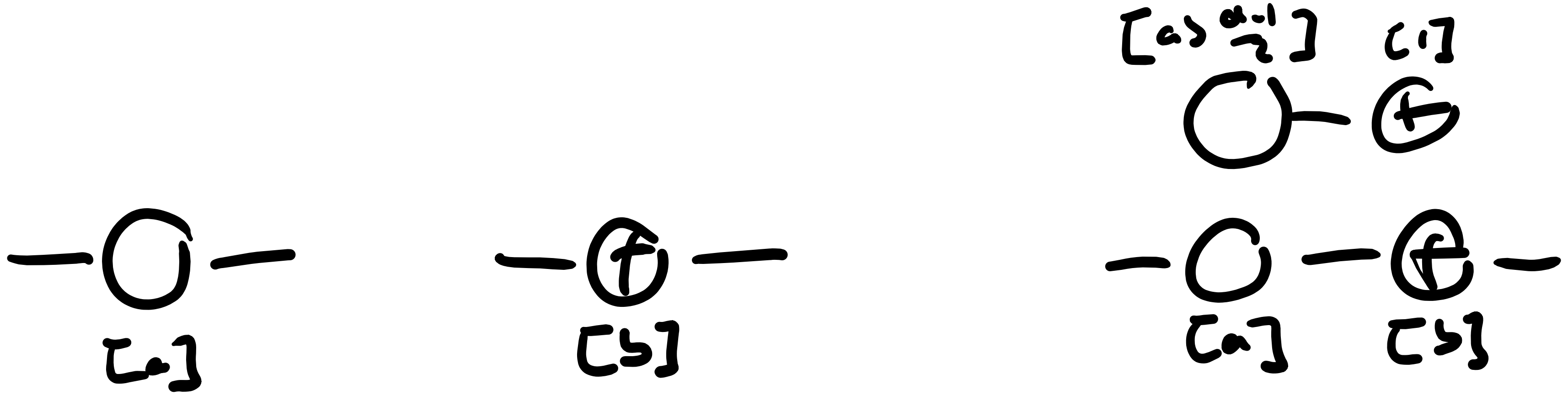


$$\underbrace{\alpha^d \left(e^{i 2\pi \frac{ab}{d}}\right)^{\frac{(d-1)d}{2}}}_{1}$$

$$\alpha = e^{-i 2\pi \frac{ab(d-1)}{2}}$$

$a = 1$
$b = 1 \Rightarrow \alpha = \pm i$
$d = 2$

$\frac{1}{2} \in \mathbb{Z}_d^{\times}$



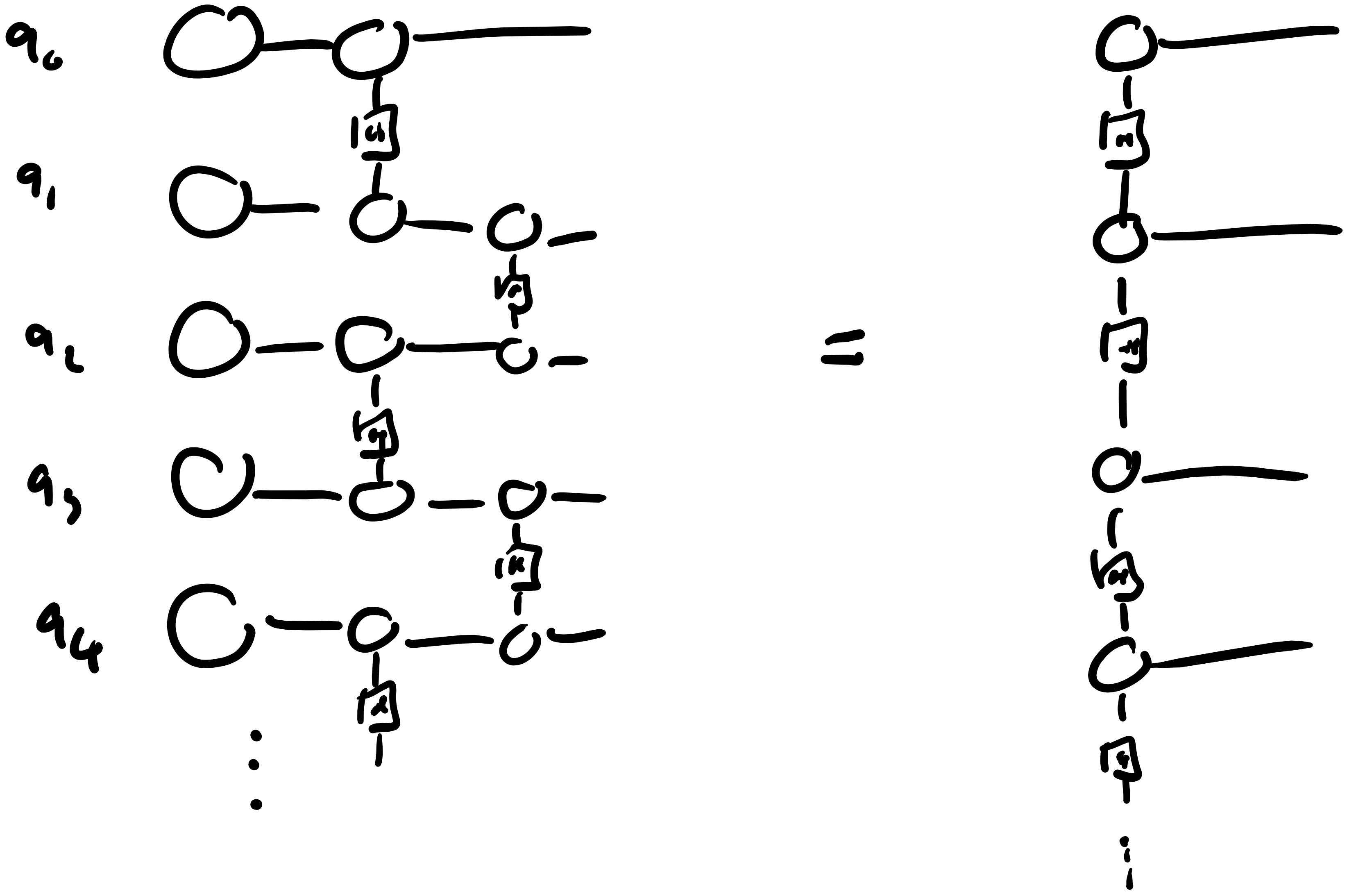
Z_a X_b $Y_{a,b}$

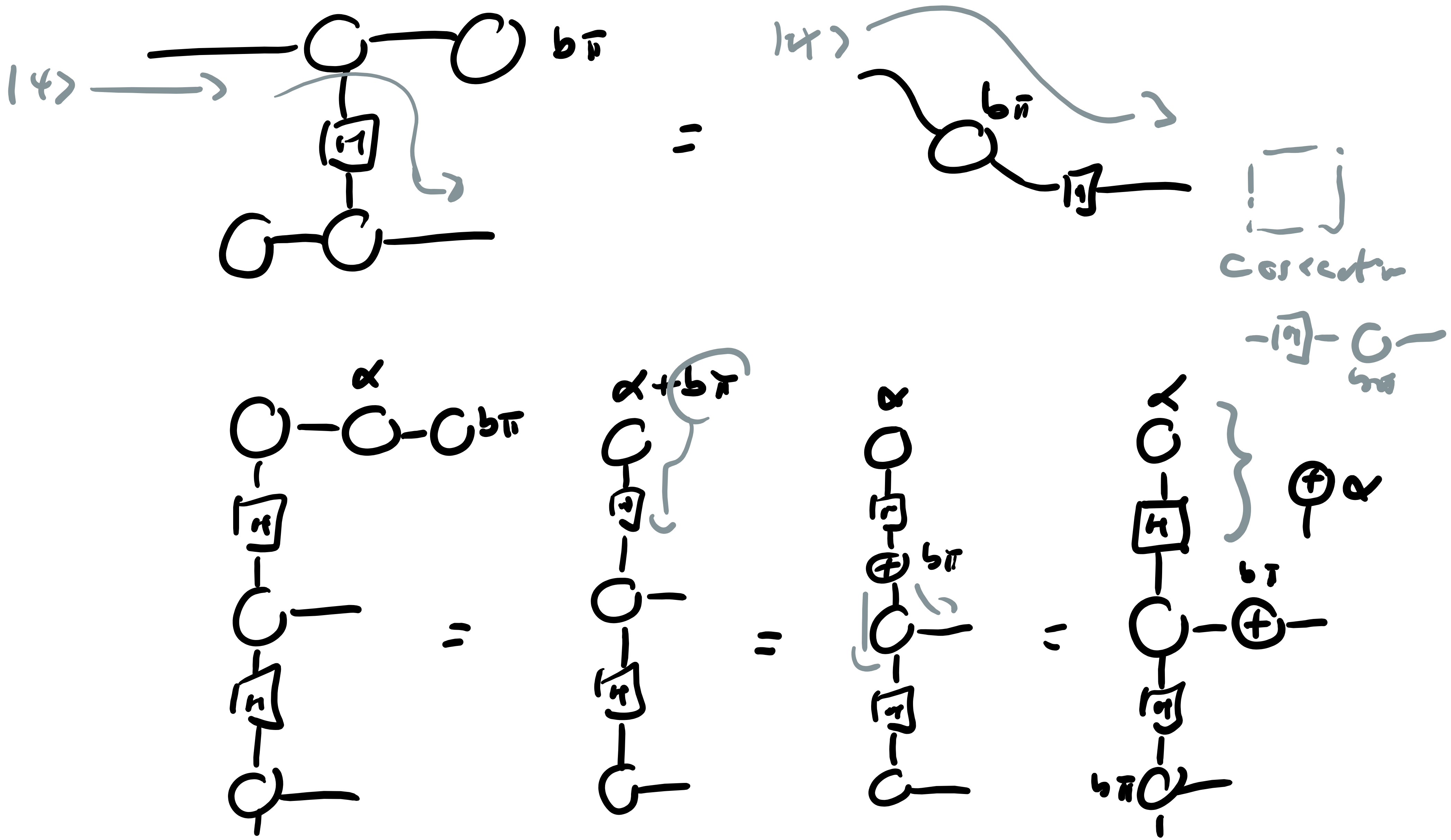
Pauli Grp in $\mathbb{C}[\mathbb{Z}_d]$

ZX for qudit stabilisers:

https://arxiv.org/abs/2304.10584

1D Cluster State




|ψ⟩
bπ
=
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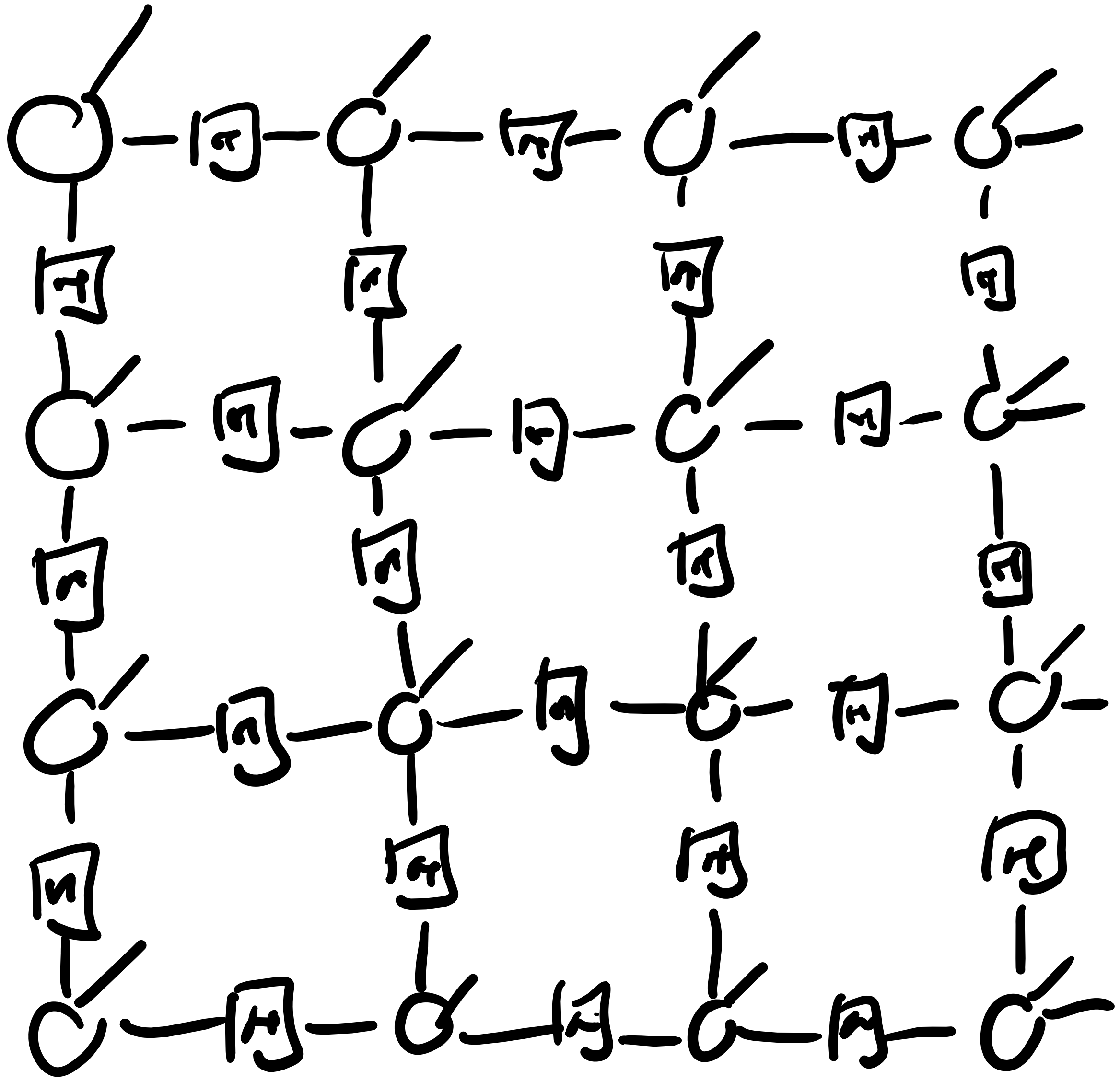
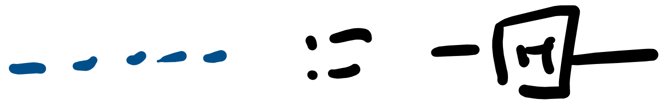
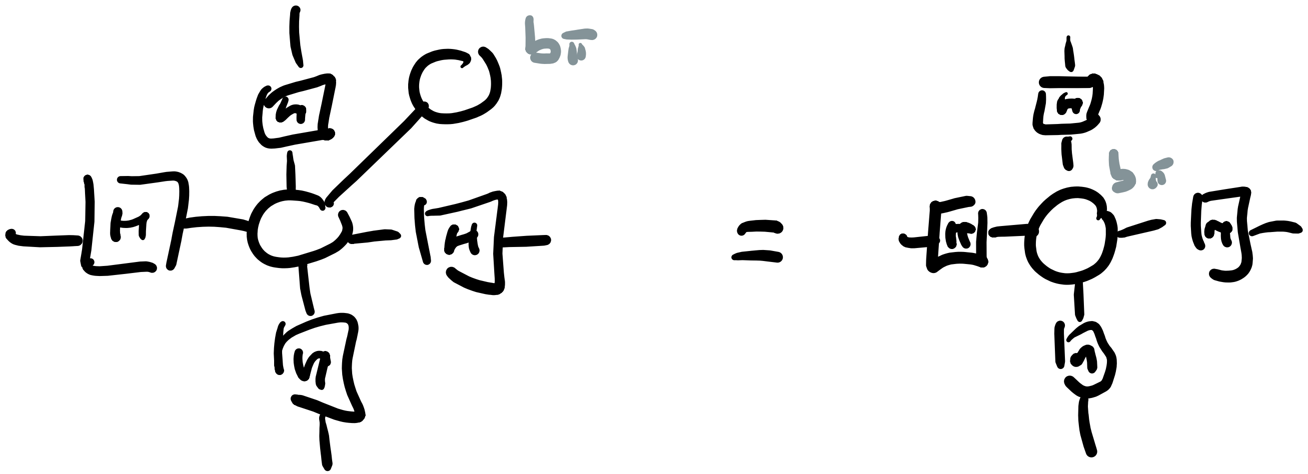
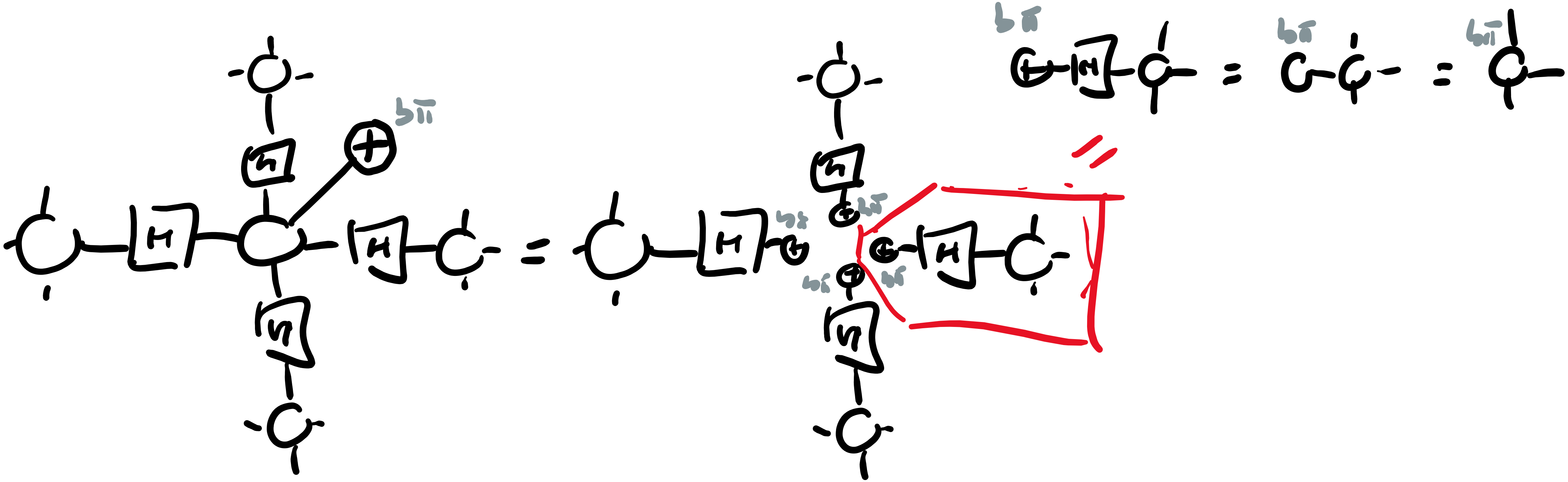
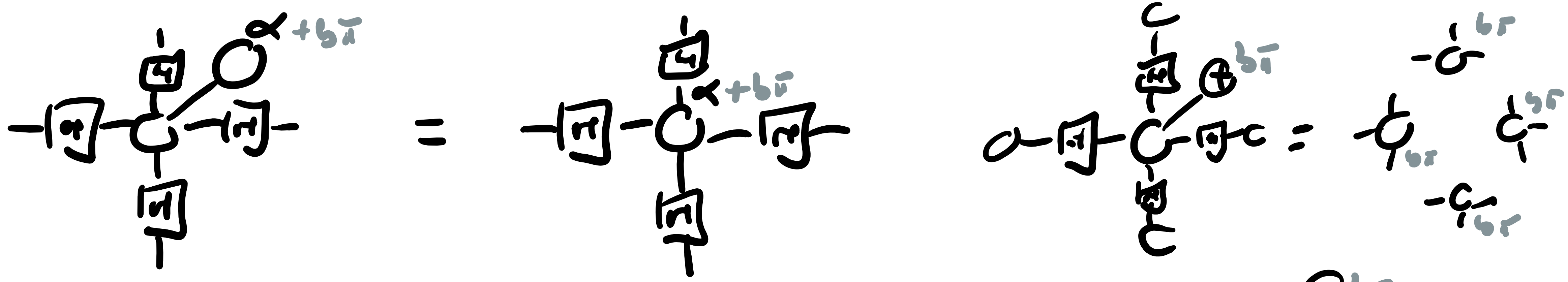
bπ
α
bπ
=
α+bπ
=
α
bπ
=
α
α
bπ
bπ

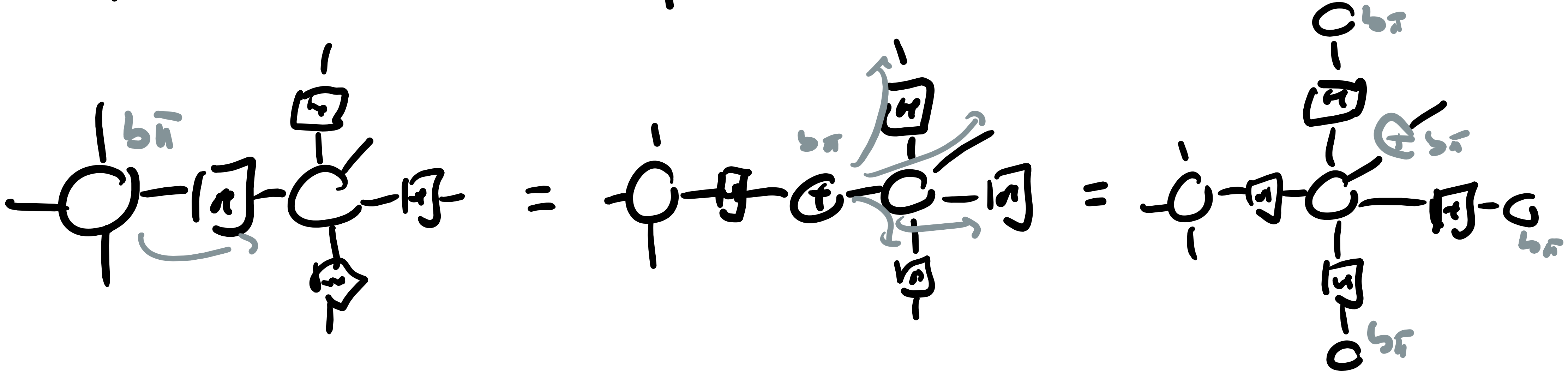


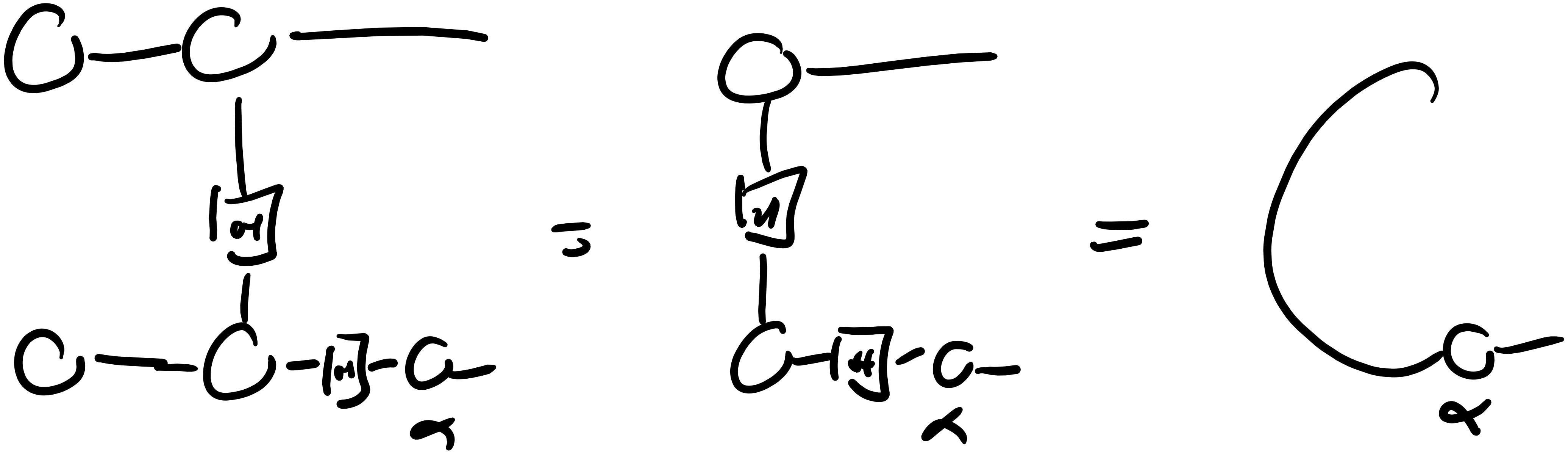


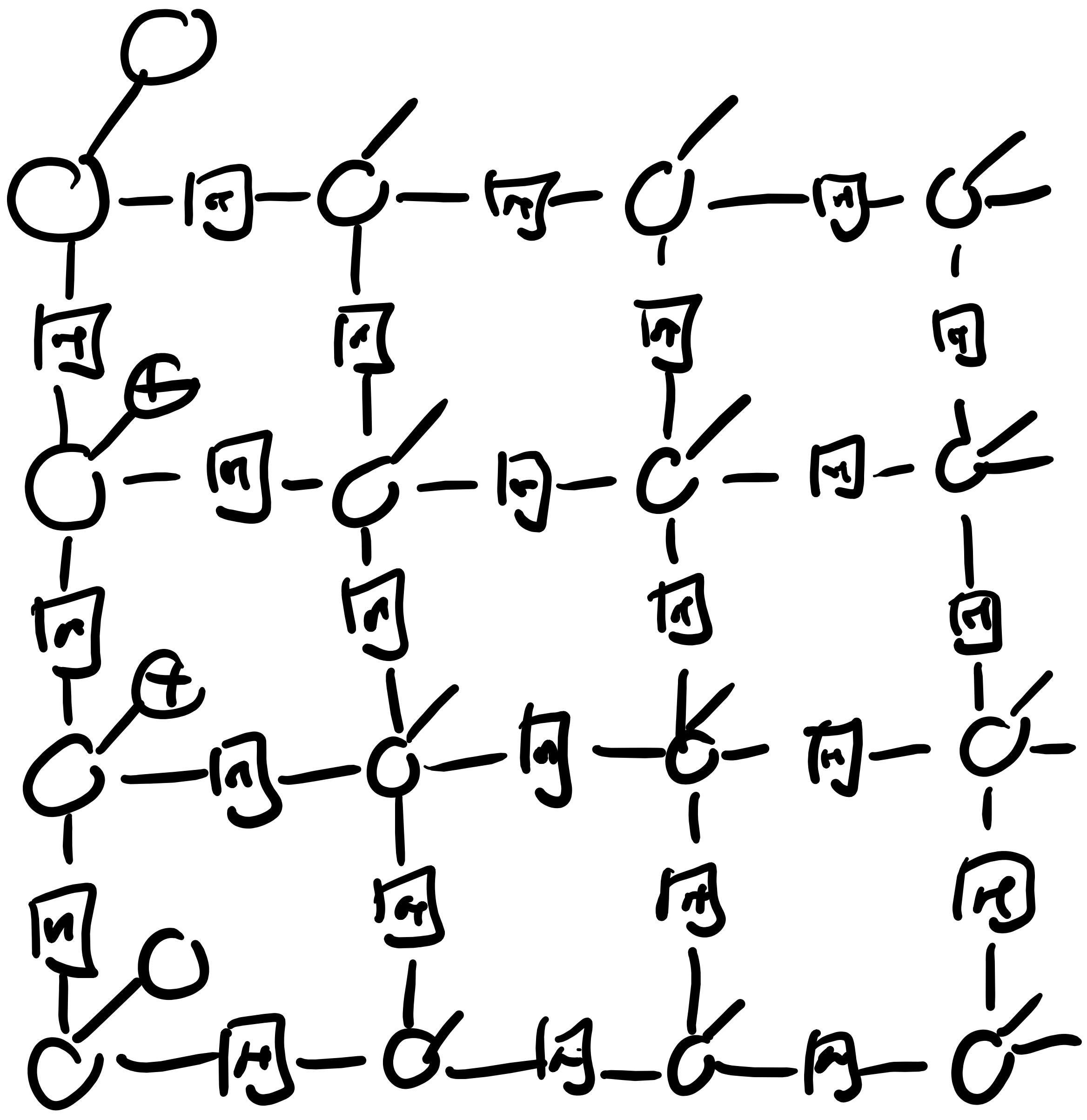


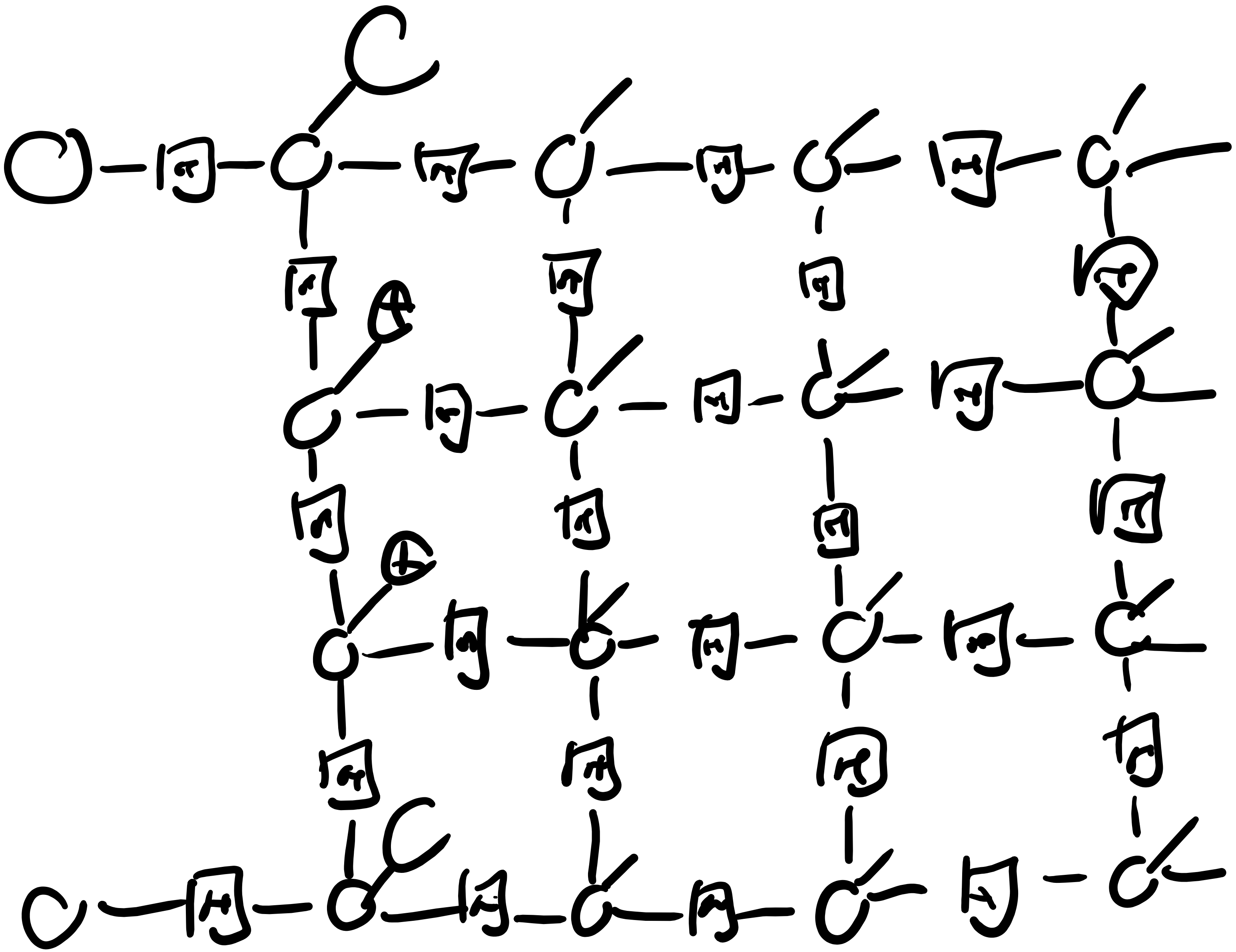


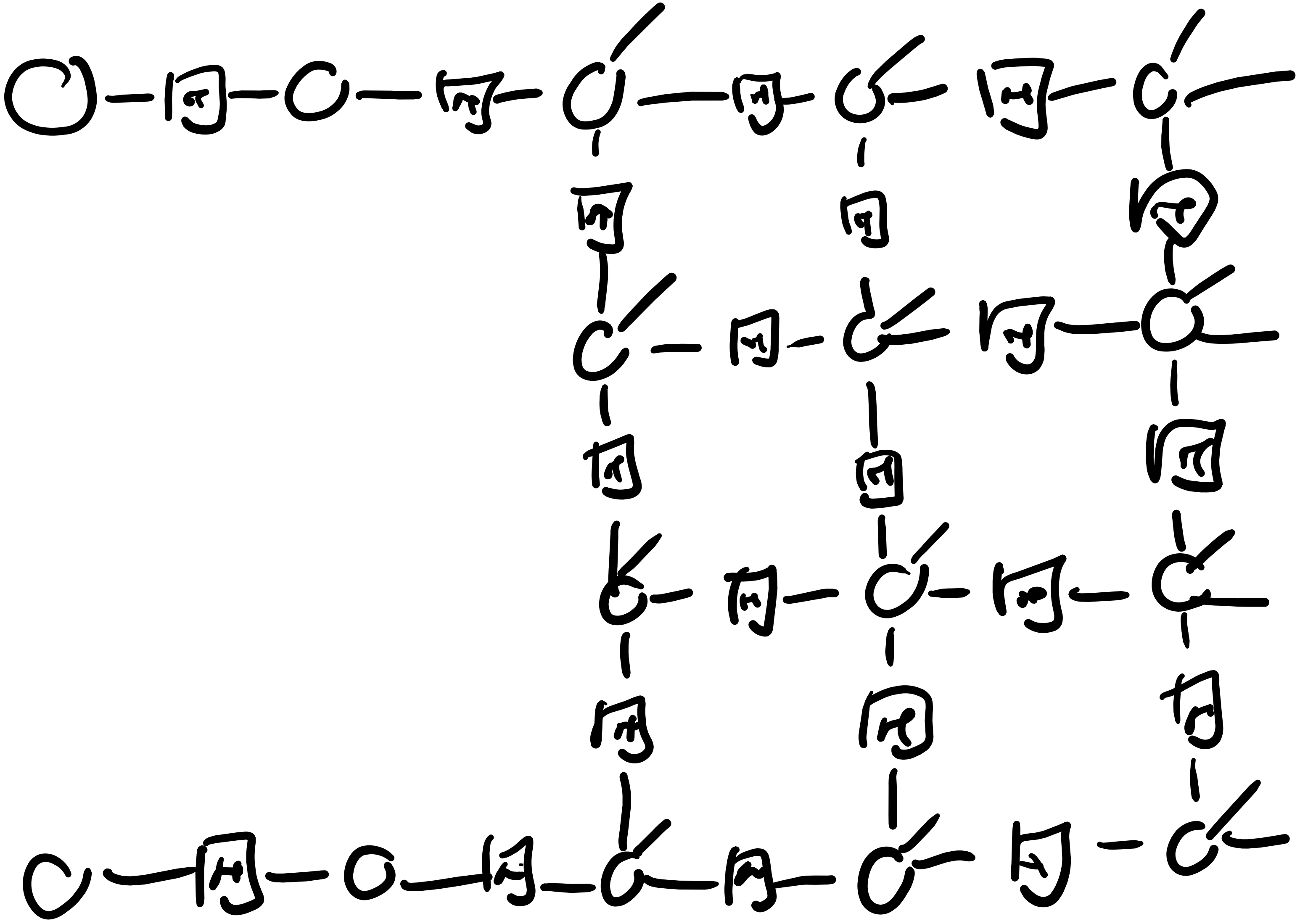


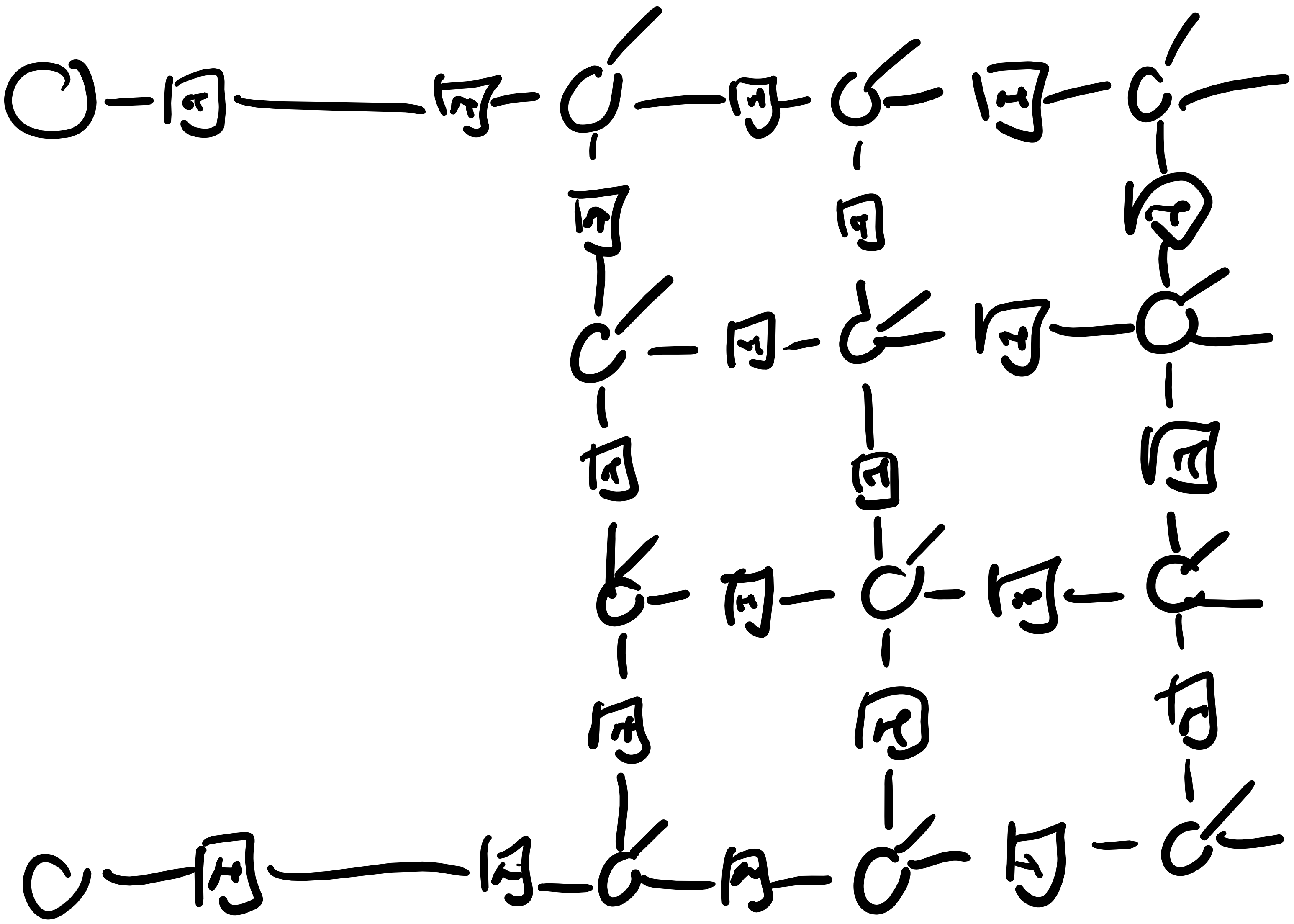


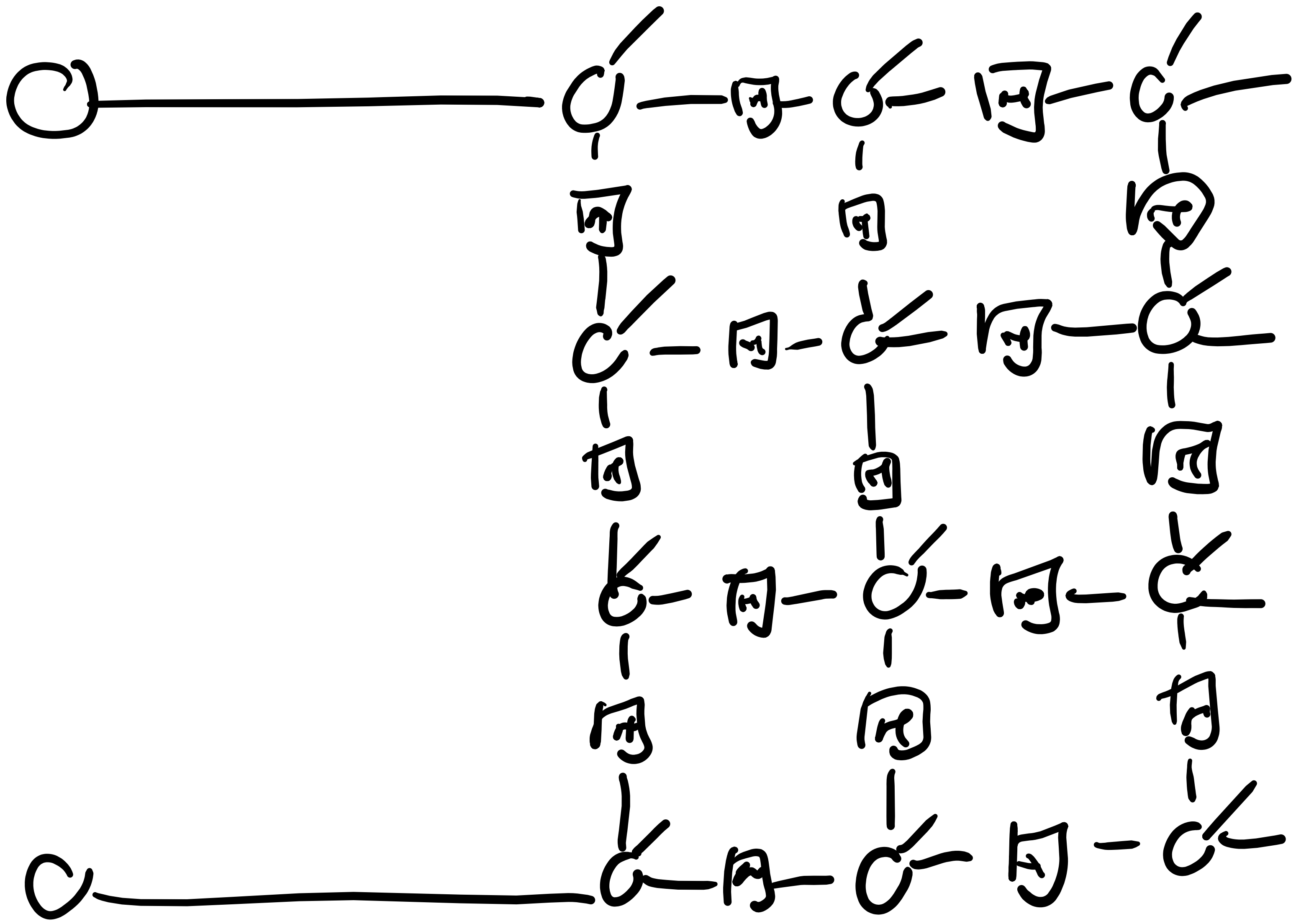


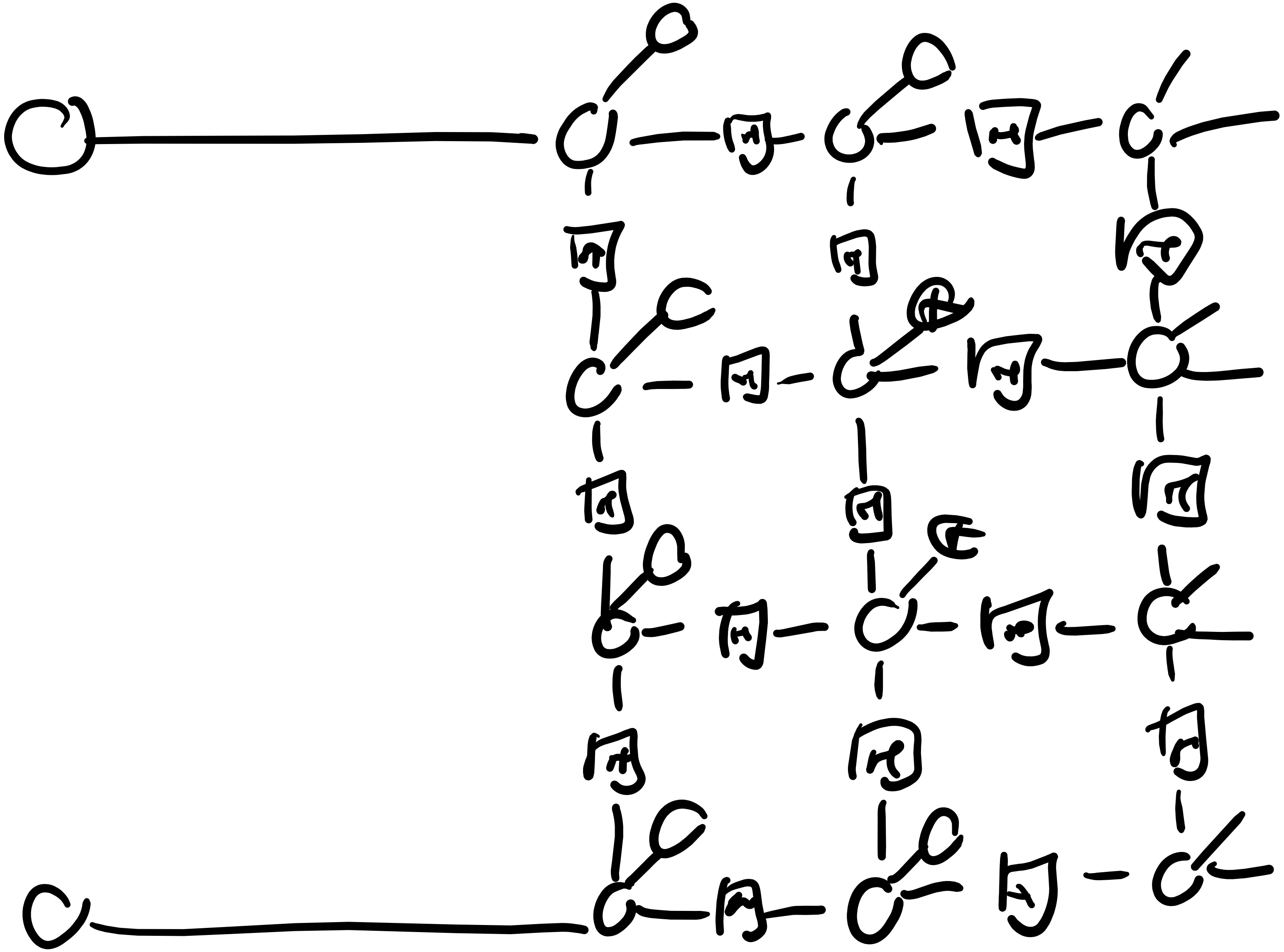


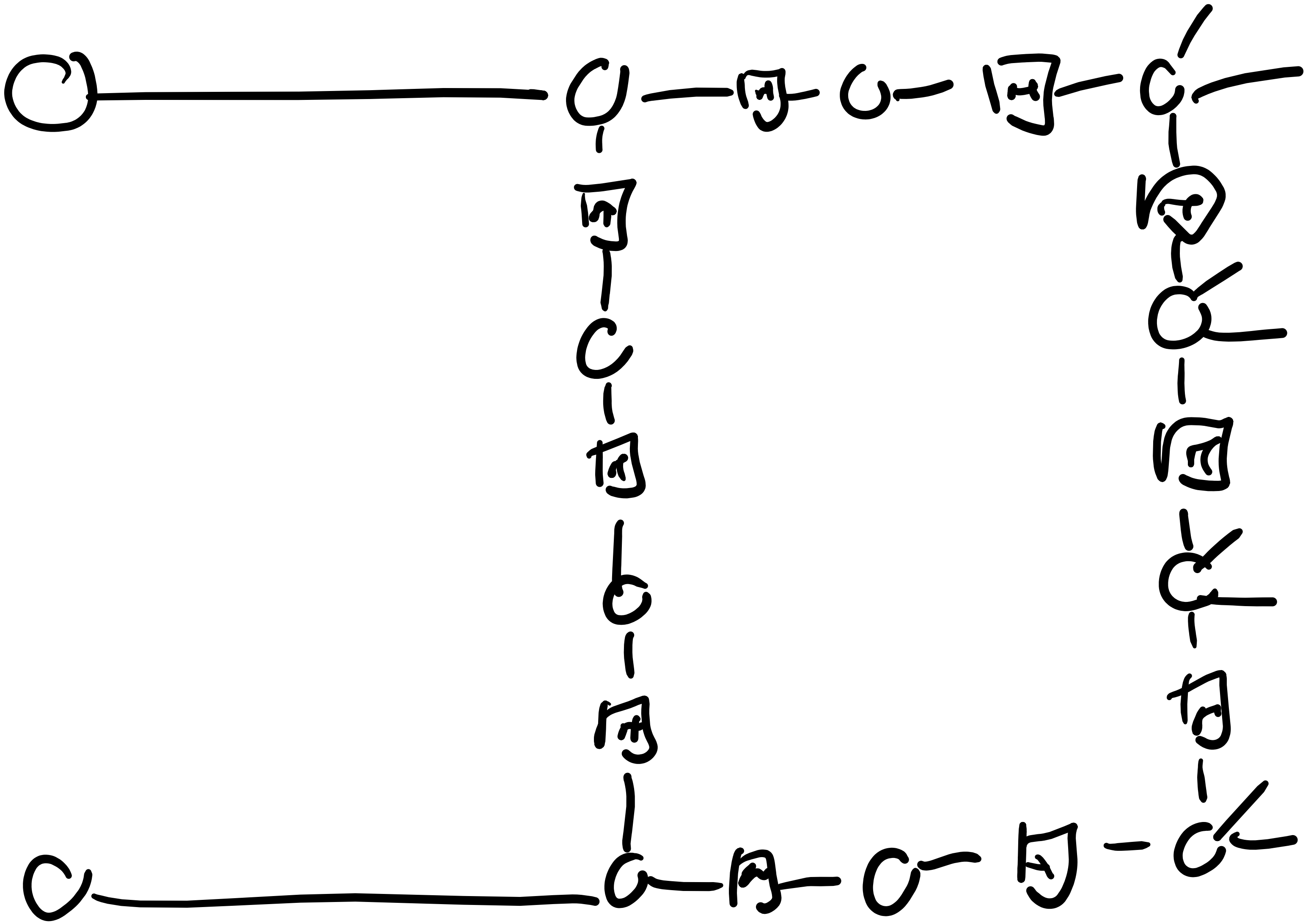


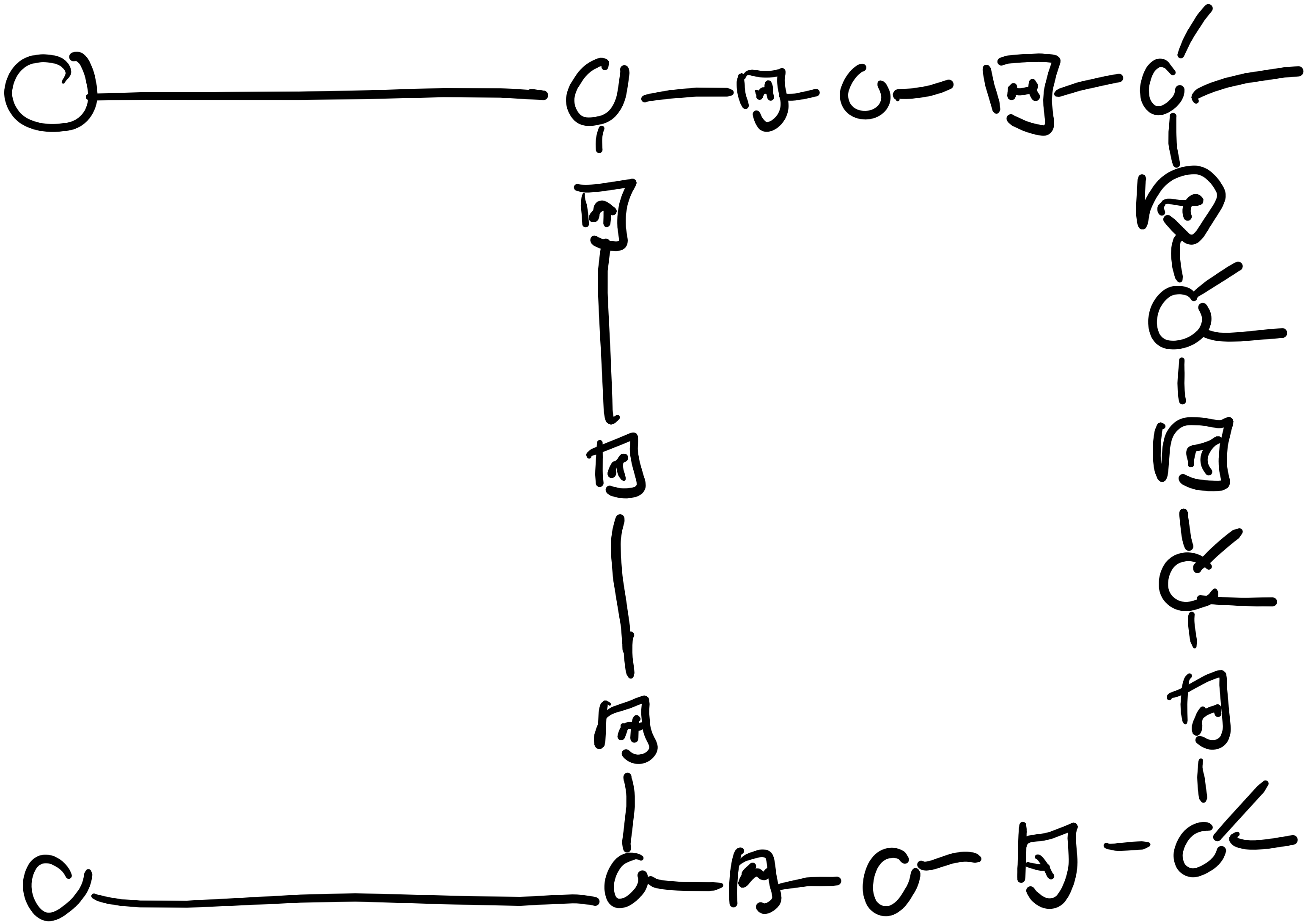


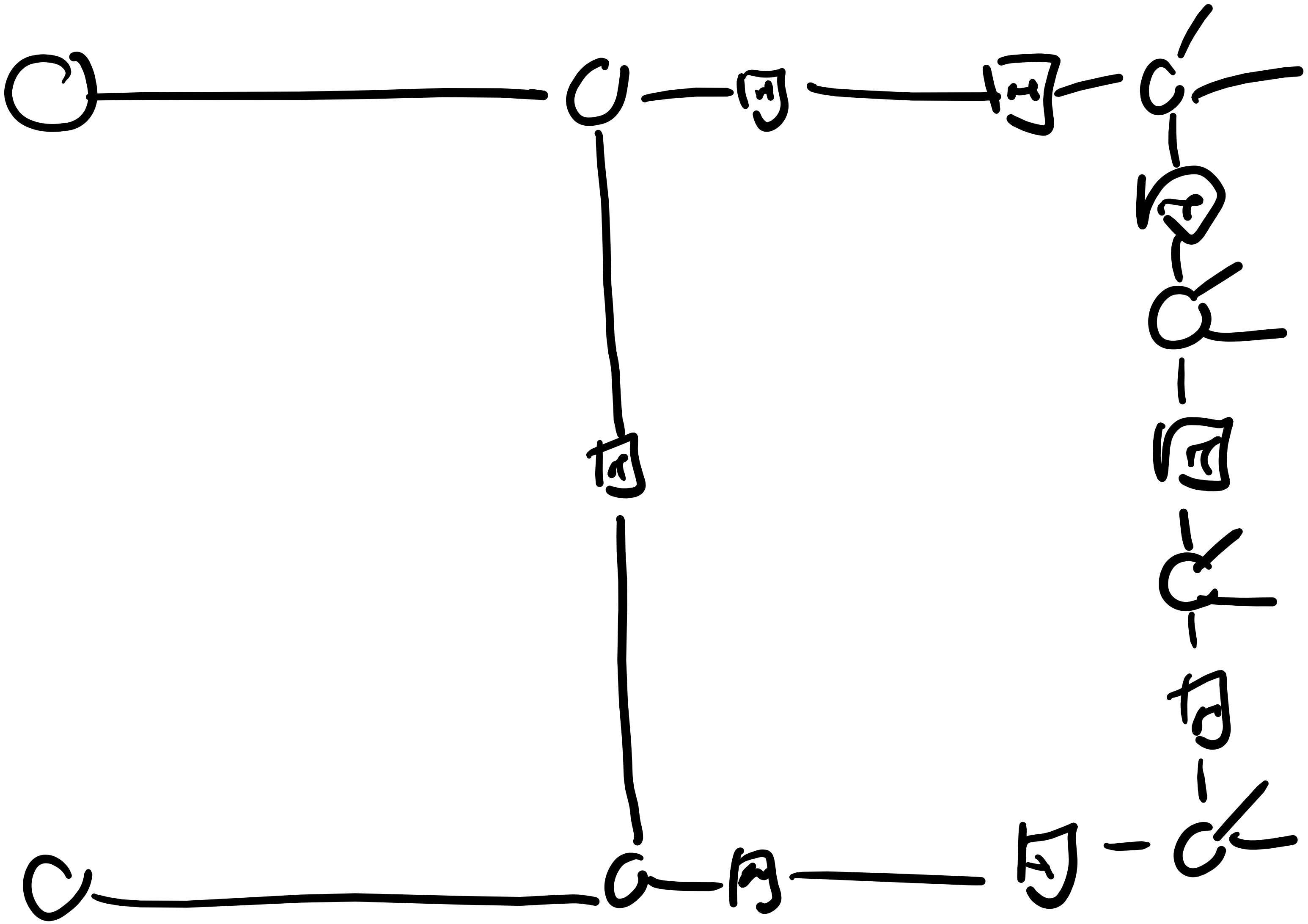


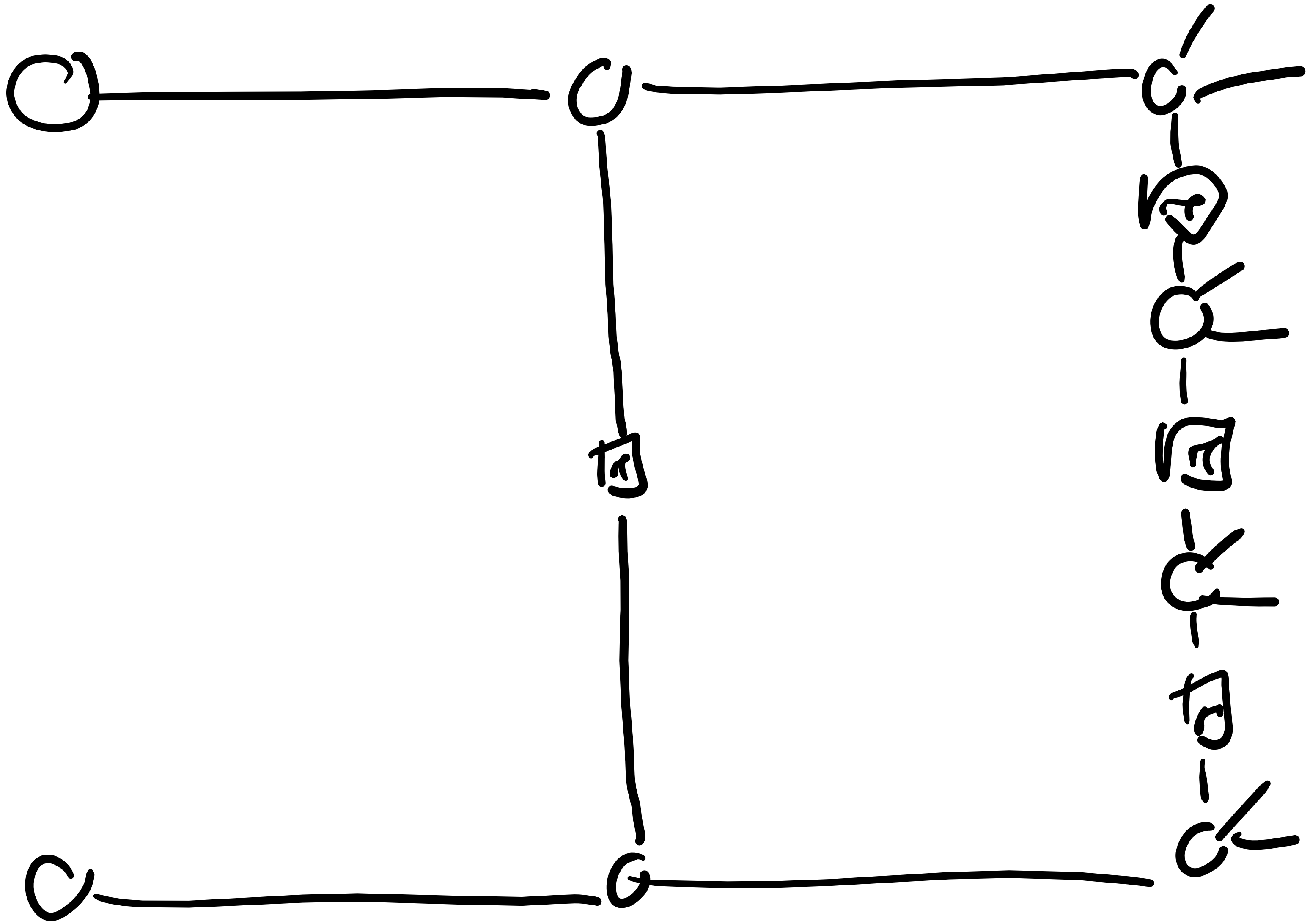


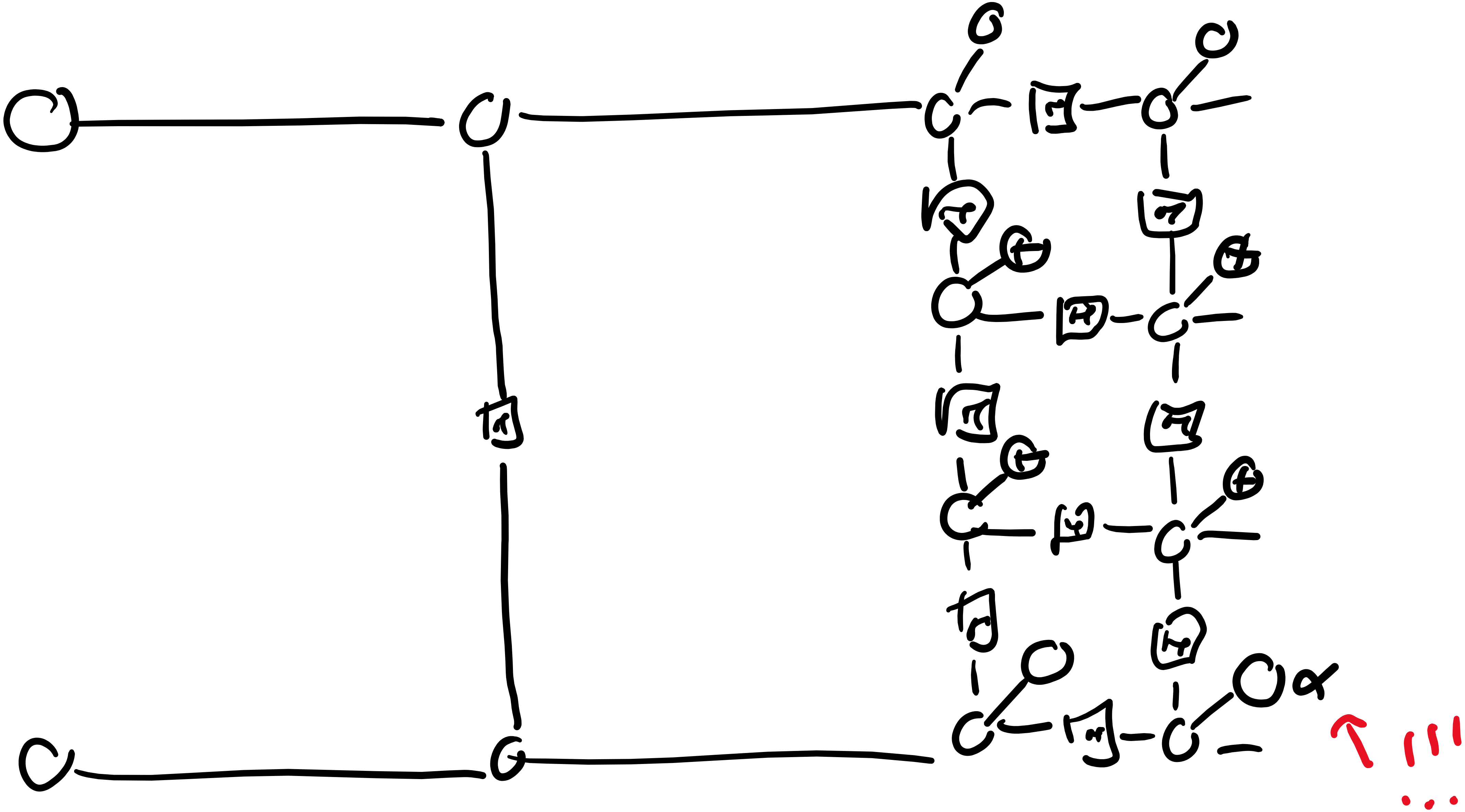








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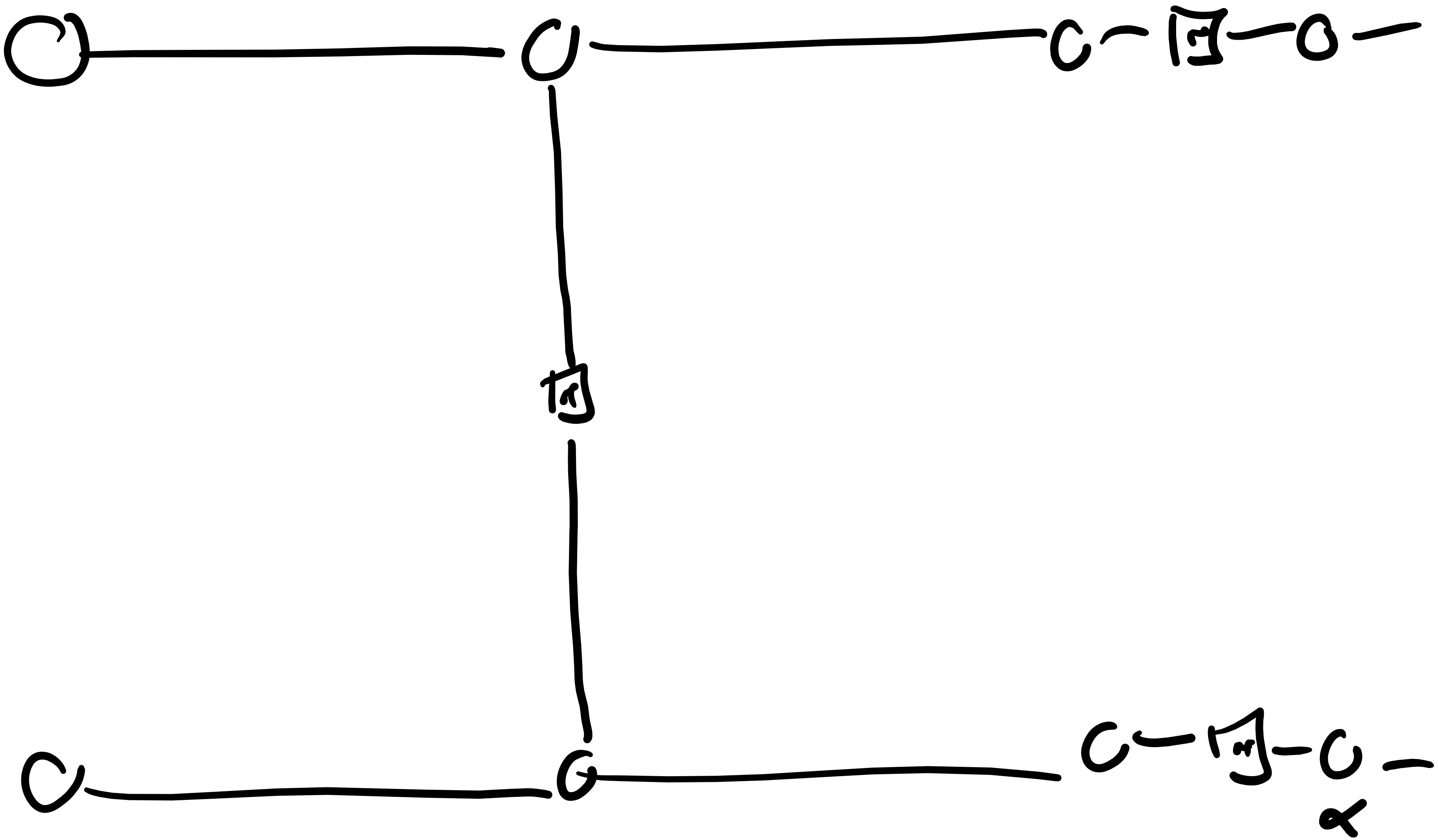

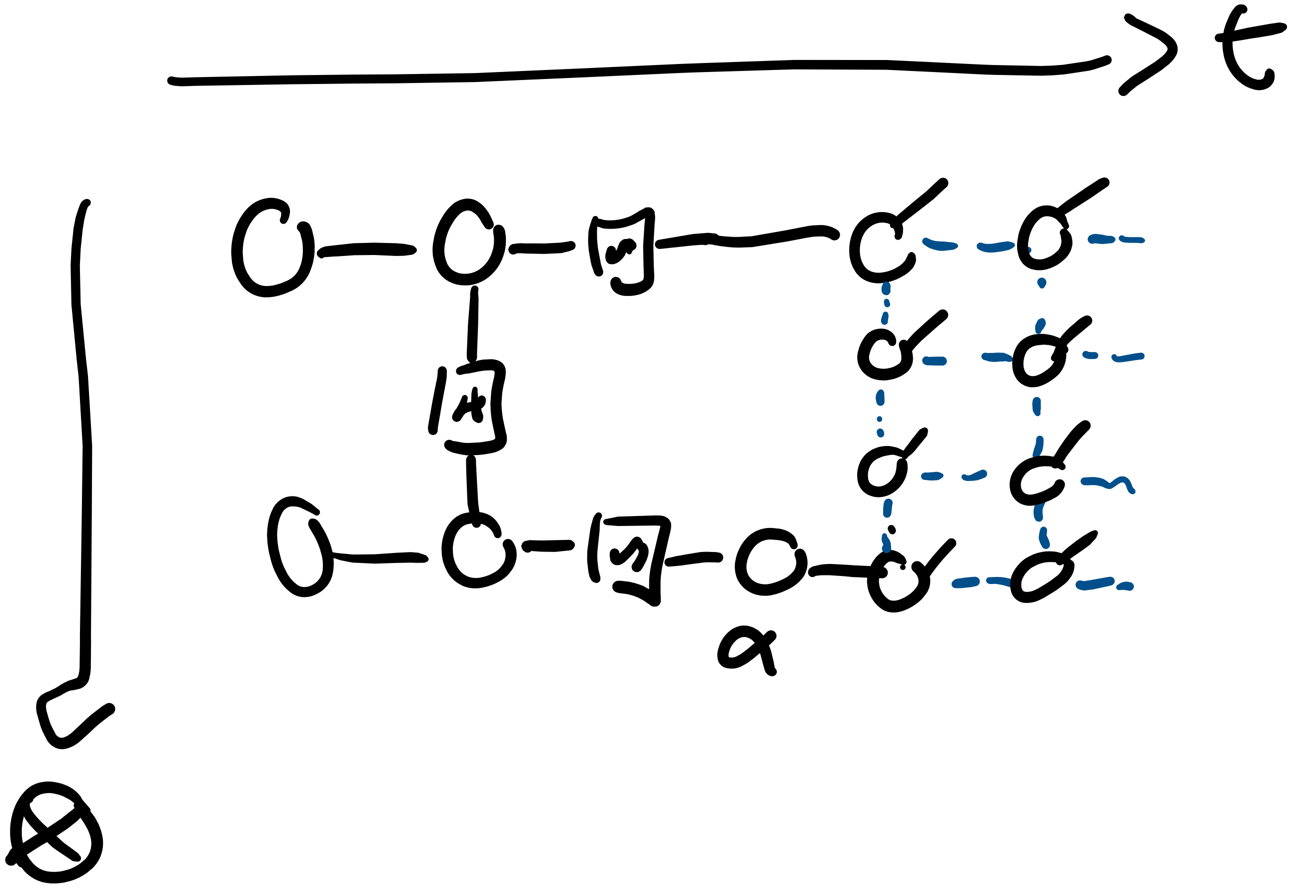

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