Quantum in Pictures Lecture Series

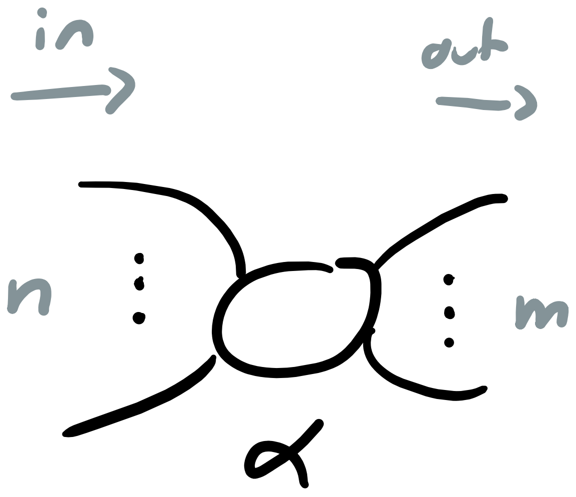
Lecturer: Stefano Gogioso

Wed 28 June 2023 – Morning Lecture

INDIANA UNIVERSITY BLOOMINGTON

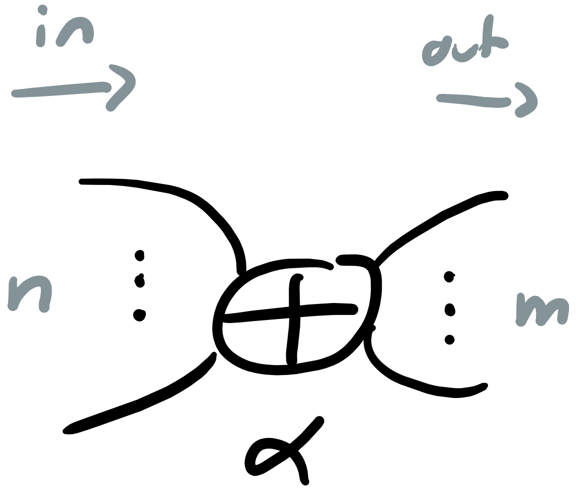
ZX Calculus

Z spiders



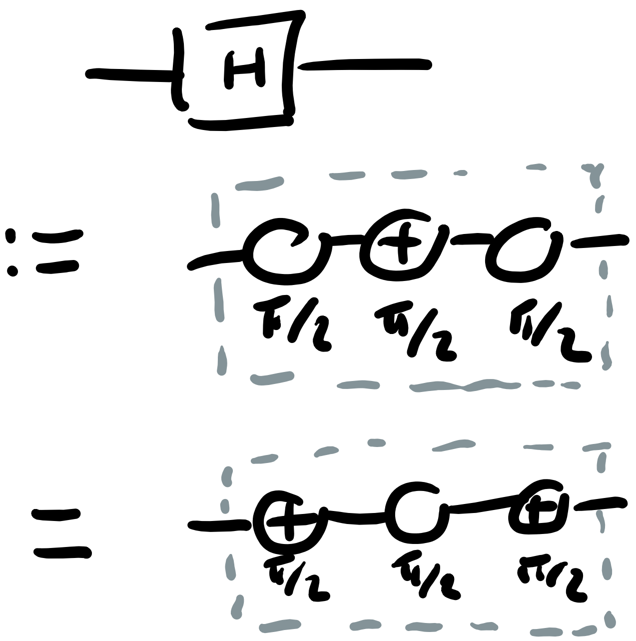
α angle

X spiders



α angle

Hadamard

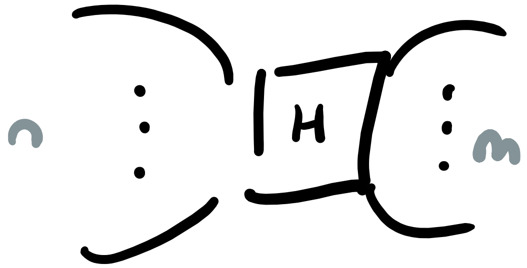


ZH Calculus

Aside!

$$= \quad \begin{array}{l} |00\rangle \mapsto |0\rangle \\ |01\rangle \mapsto |0\rangle \\ |10\rangle \mapsto |0\rangle \\ |11\rangle \mapsto |1\rangle \end{array}$$

$$H = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 1 & 1 \\ 1 & 1 & -1 \end{array}$$

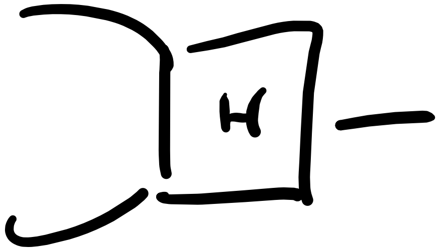


$$= \sum_{\substack{\underline{b} \in \{0,1\}^m \\ \underline{c} \in \{0,1\}^n}} |b_1 \dots b_m\rangle\langle c_1 \dots c_n| \, (-1)^{b_1 \cdot b_2 \cdot \dots \cdot b_m \cdot c_1 \cdot \dots \cdot c_n}$$

$$(-1)^0 = 1$$

$$(-1)^1 = -1$$

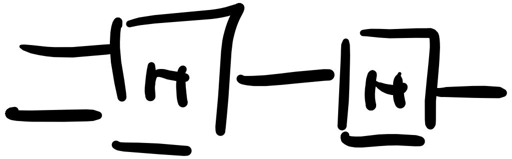
Aside

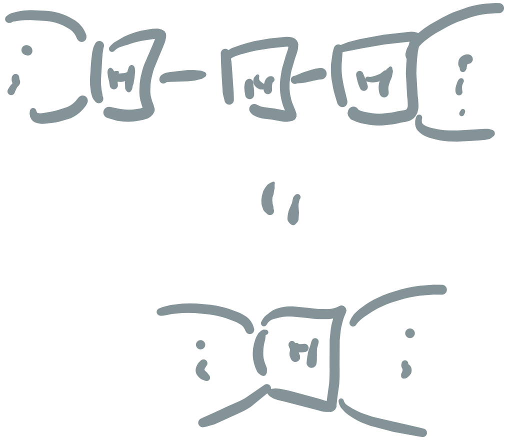


$$= \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \\ \hline \end{array}$$
$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$$

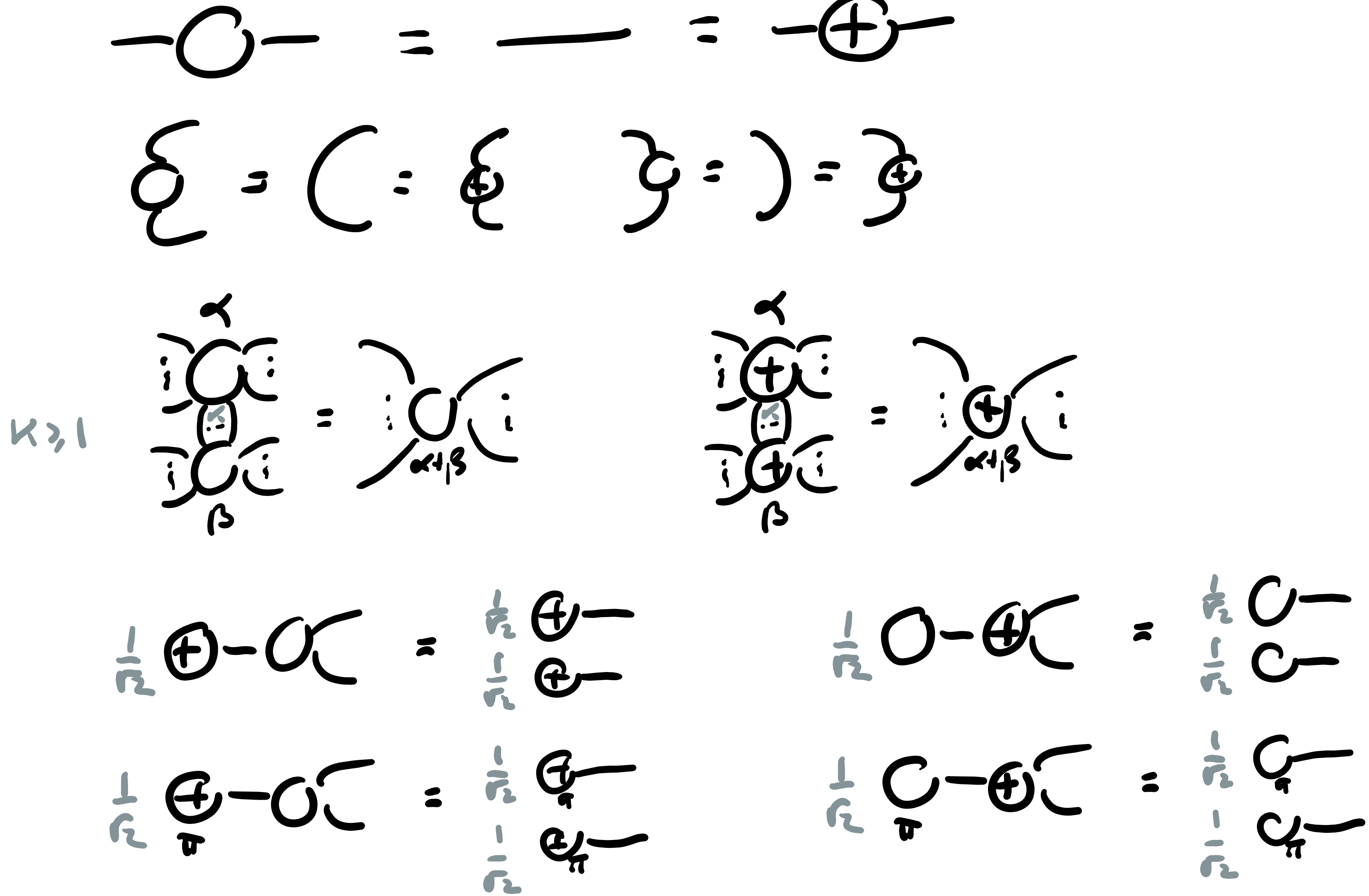
$$\left.\begin{array}{l} |00\rangle \\ |01\rangle \\ |10\rangle \end{array}\right\} \mapsto |+\rangle$$
$$|11\rangle \longmapsto |-\rangle$$

$:=$

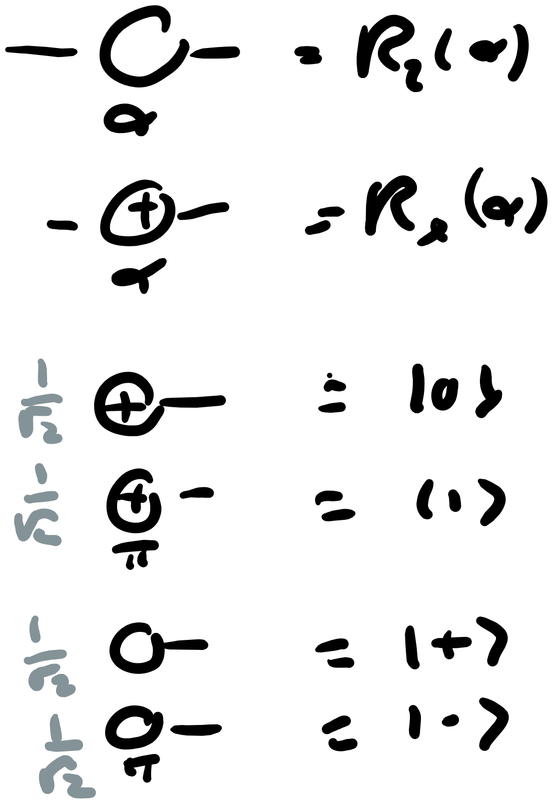




ZH Calculus



k≥1
α
β
α+β
$\frac{1}{\sqrt{2}}$
π

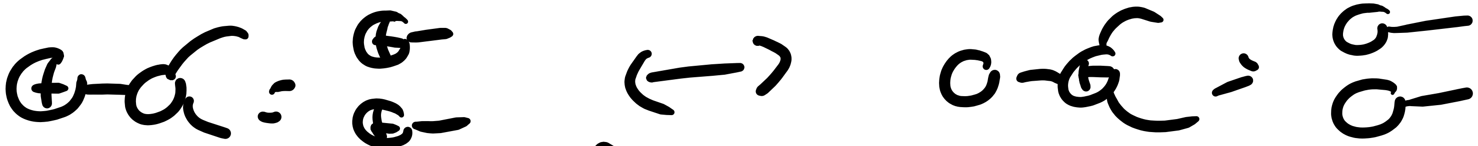


= $R_z(\alpha)$
= $R_x(\alpha)$
α
$\frac{1}{\sqrt{2}}$
= $|0\rangle$
= $|1\rangle$
= $|+\rangle$
= $|-\rangle$
π

Rule 1

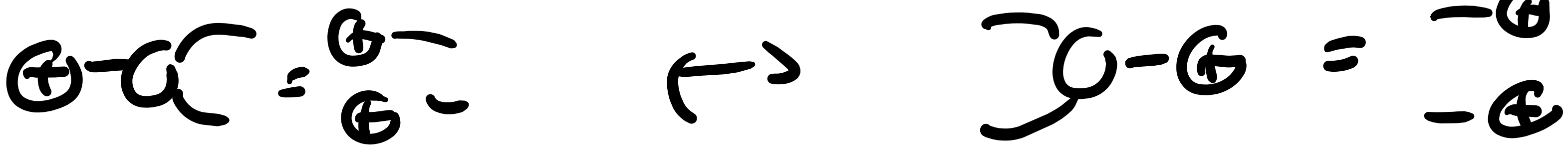
New rules from old rules:

X <-> Z

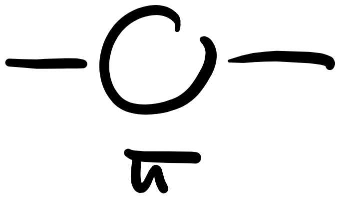


change col

Adjoints



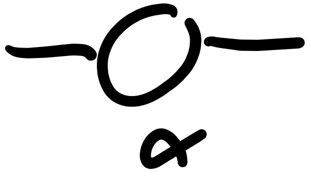
flip things



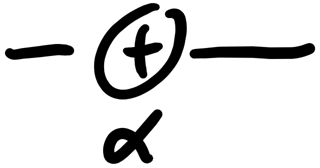
$= Z$ gate $\quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = |0\rangle\langle 0| - |1\rangle\langle 1|$

$= X$ gate $\quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = |0\rangle\langle 1| + |1\rangle\langle 0|$

$= |+\rangle\langle +| - |-\rangle\langle -|$

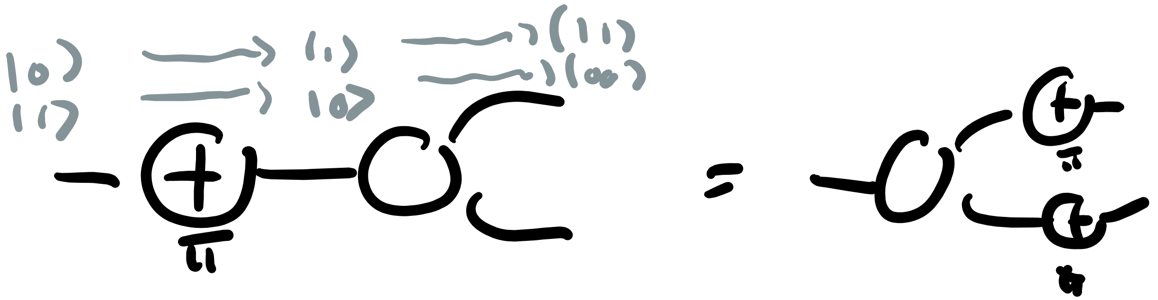


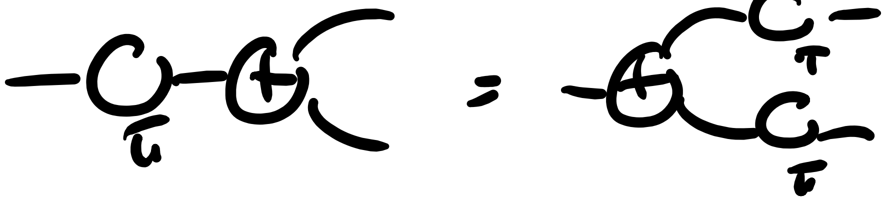
$$= \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix} = |0\rangle\langle 0| + e^{i\alpha}|1\rangle\langle 1|$$

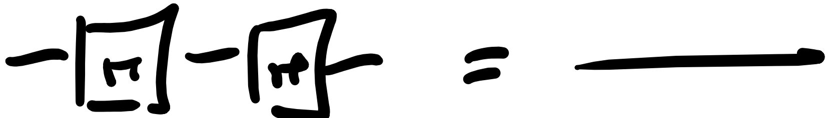


$$= \begin{pmatrix} -i\cos\frac{\alpha}{2} & \sin\frac{\alpha}{2} \\ \sin\frac{\alpha}{2} & -i\cos\frac{\alpha}{2} \end{pmatrix} = |+\rangle\langle +| + e^{i\alpha}|-\rangle\langle -|$$

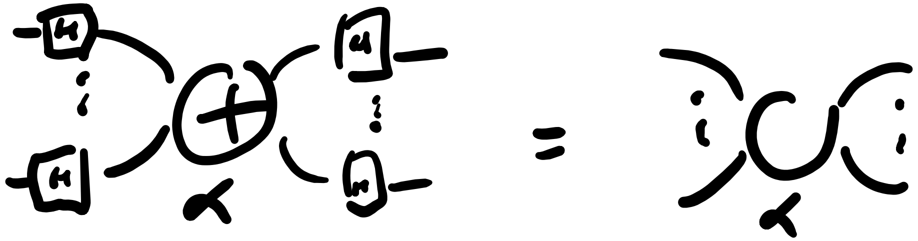
$$-\boxed{H}- = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

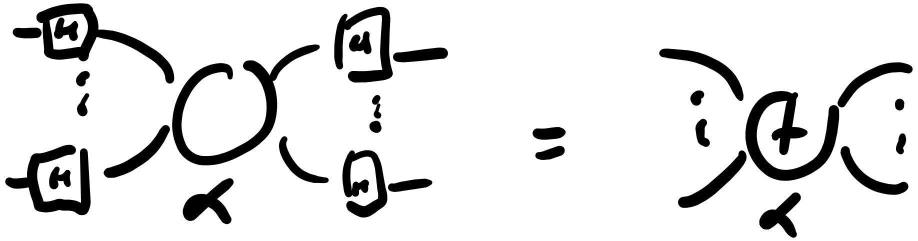






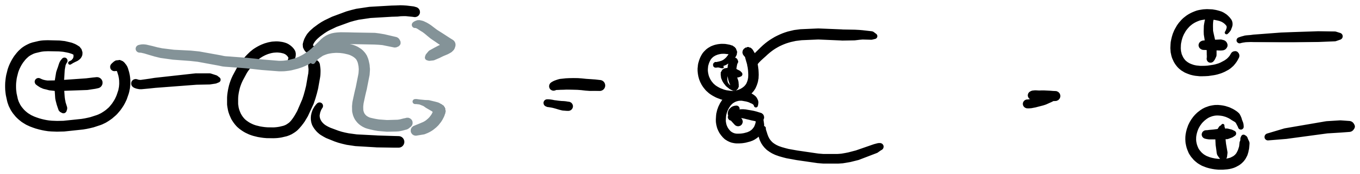
qubit



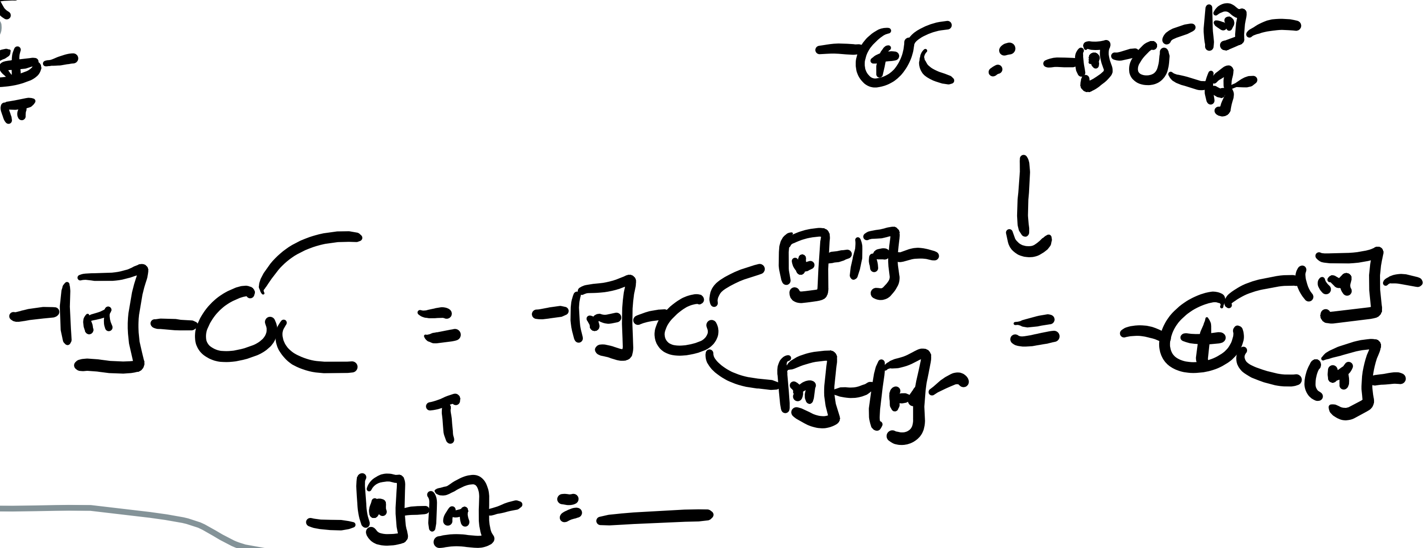


Rules?

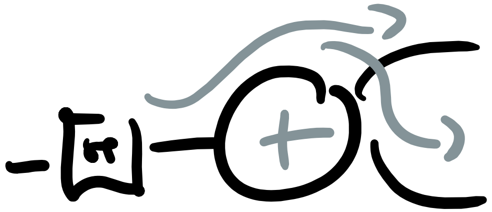
(1)



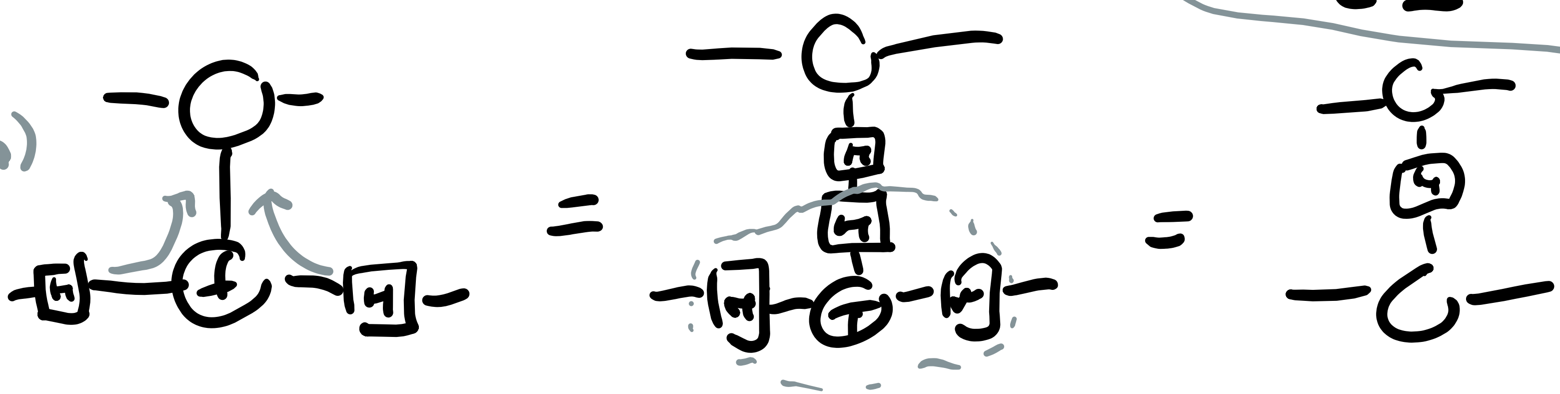
(2)

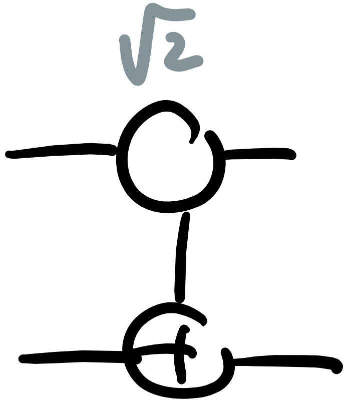


(3)



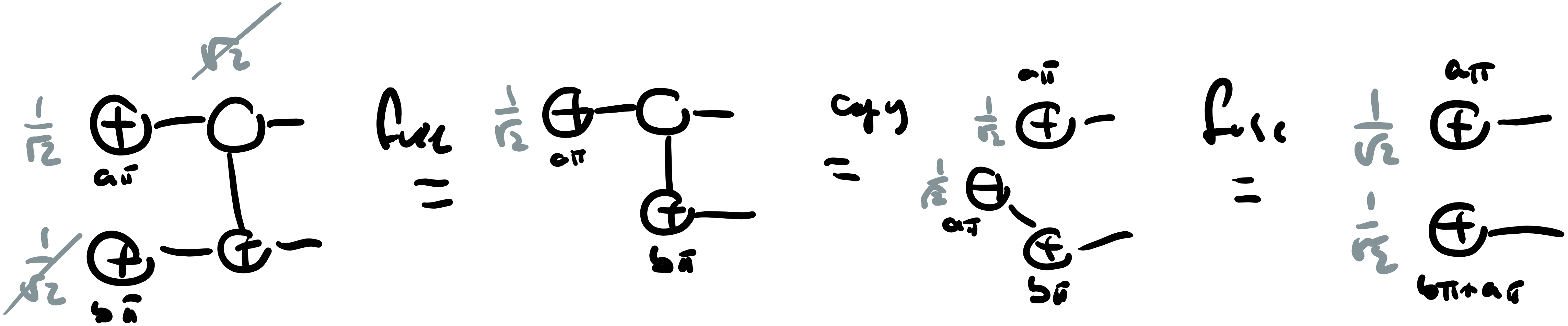
(4)

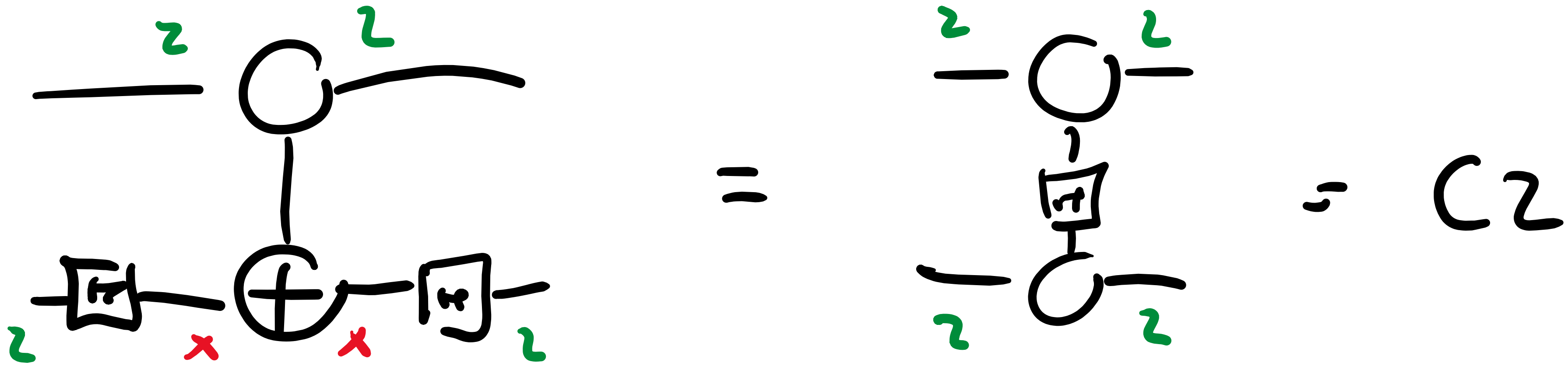


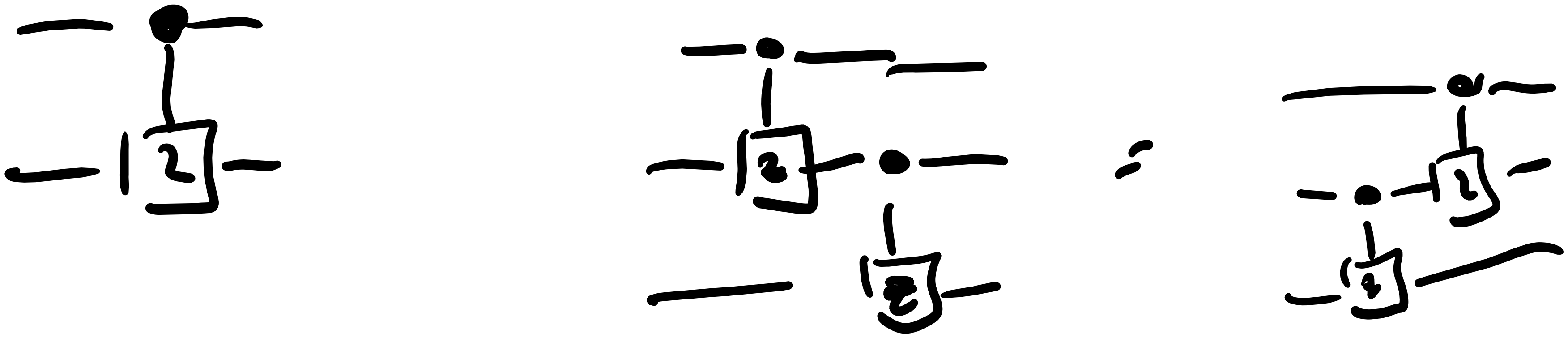


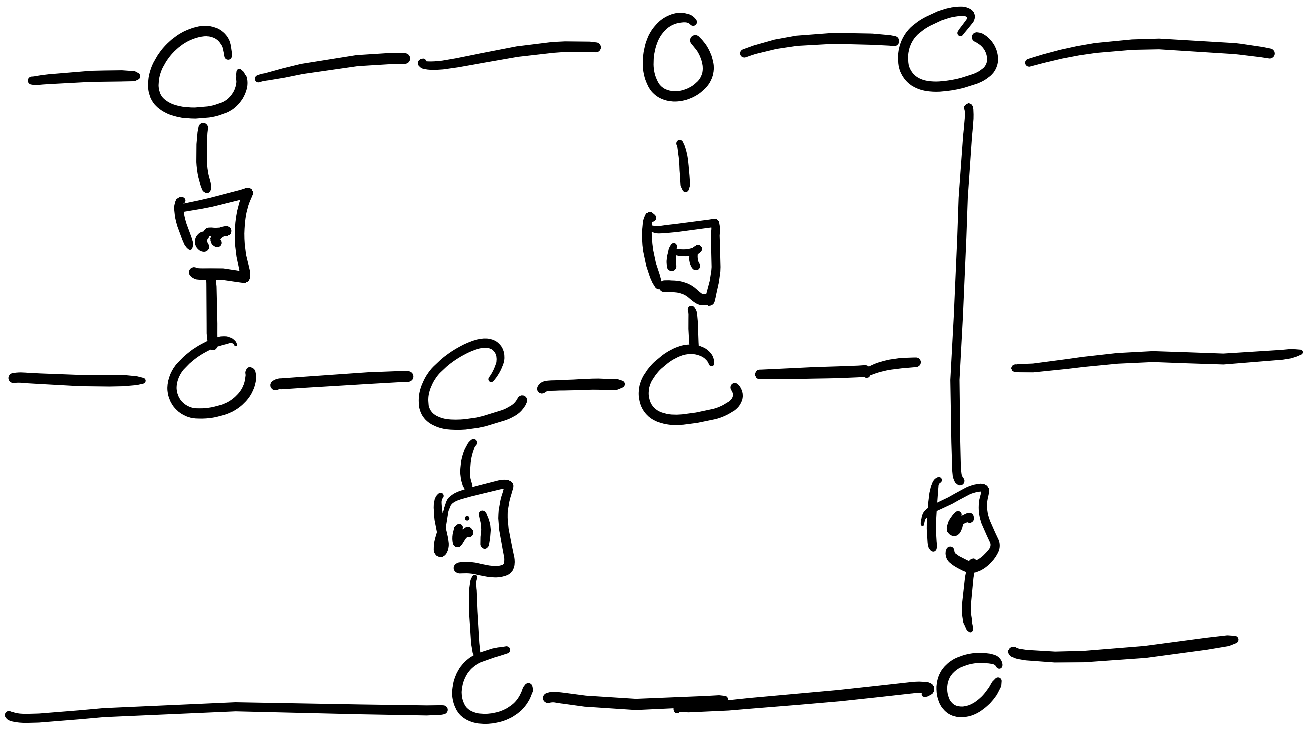
$= \text{CNOT}$

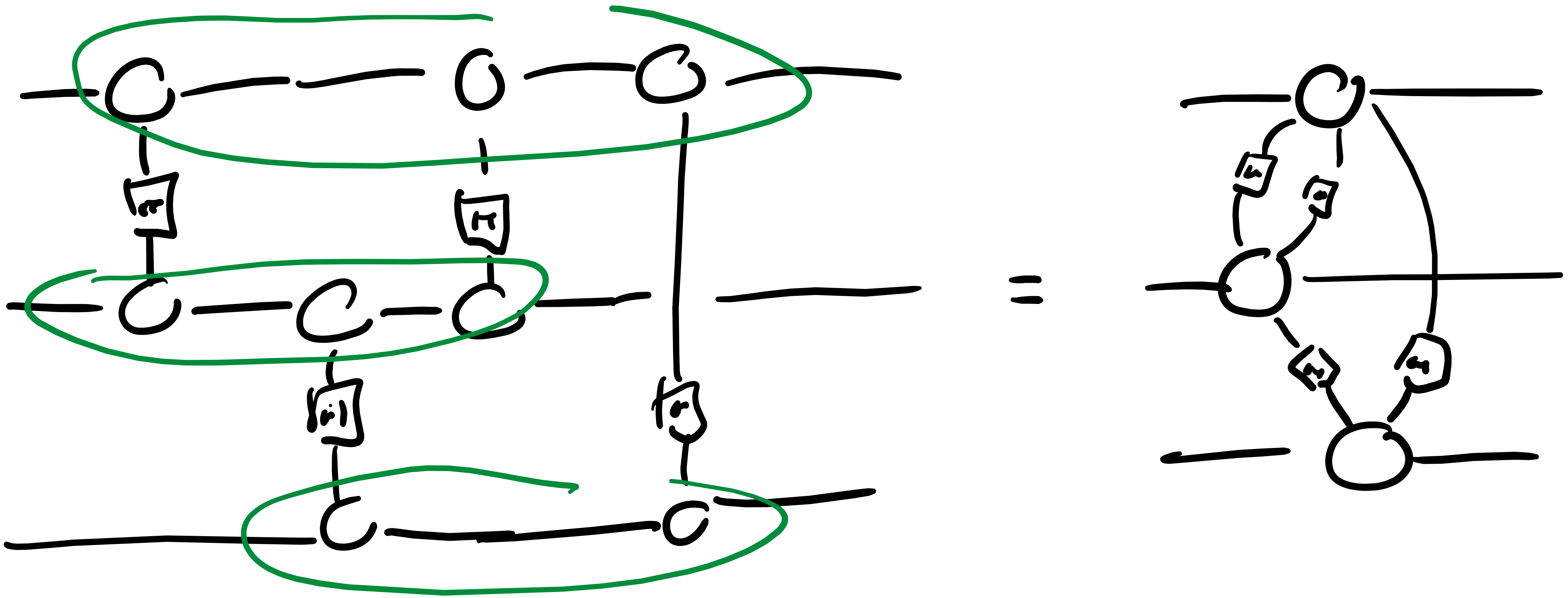
$$\begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 00 & & & & \\ 01 & & & & \\ 10 & & & & \\ 11 & & & & \end{array} \quad \begin{pmatrix} 1 & 0 & \multicolumn{2}{c}{\emptyset} \\ 0 & 1 & 0 & 1 \\ \multicolumn{2}{c}{\emptyset} & 1 & 0 \end{pmatrix}$$




= CZ



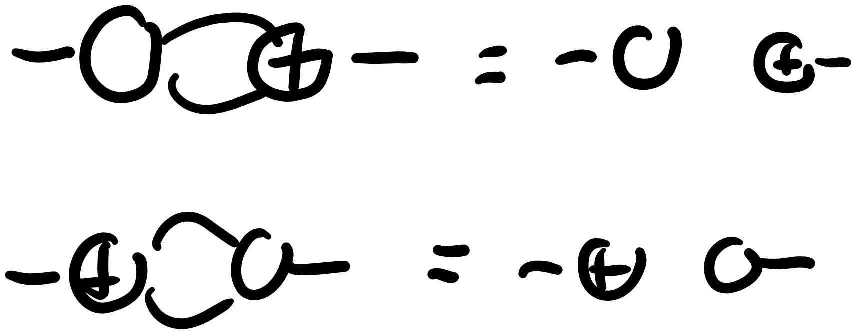




New Rulez!

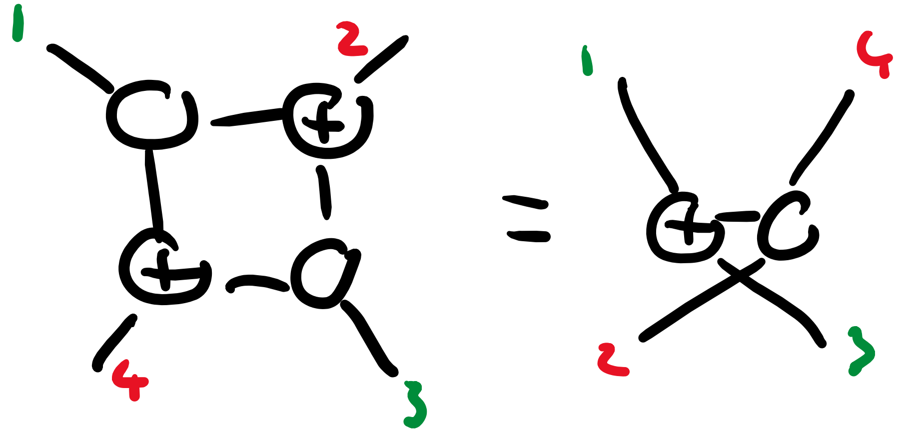
(Hopf law)

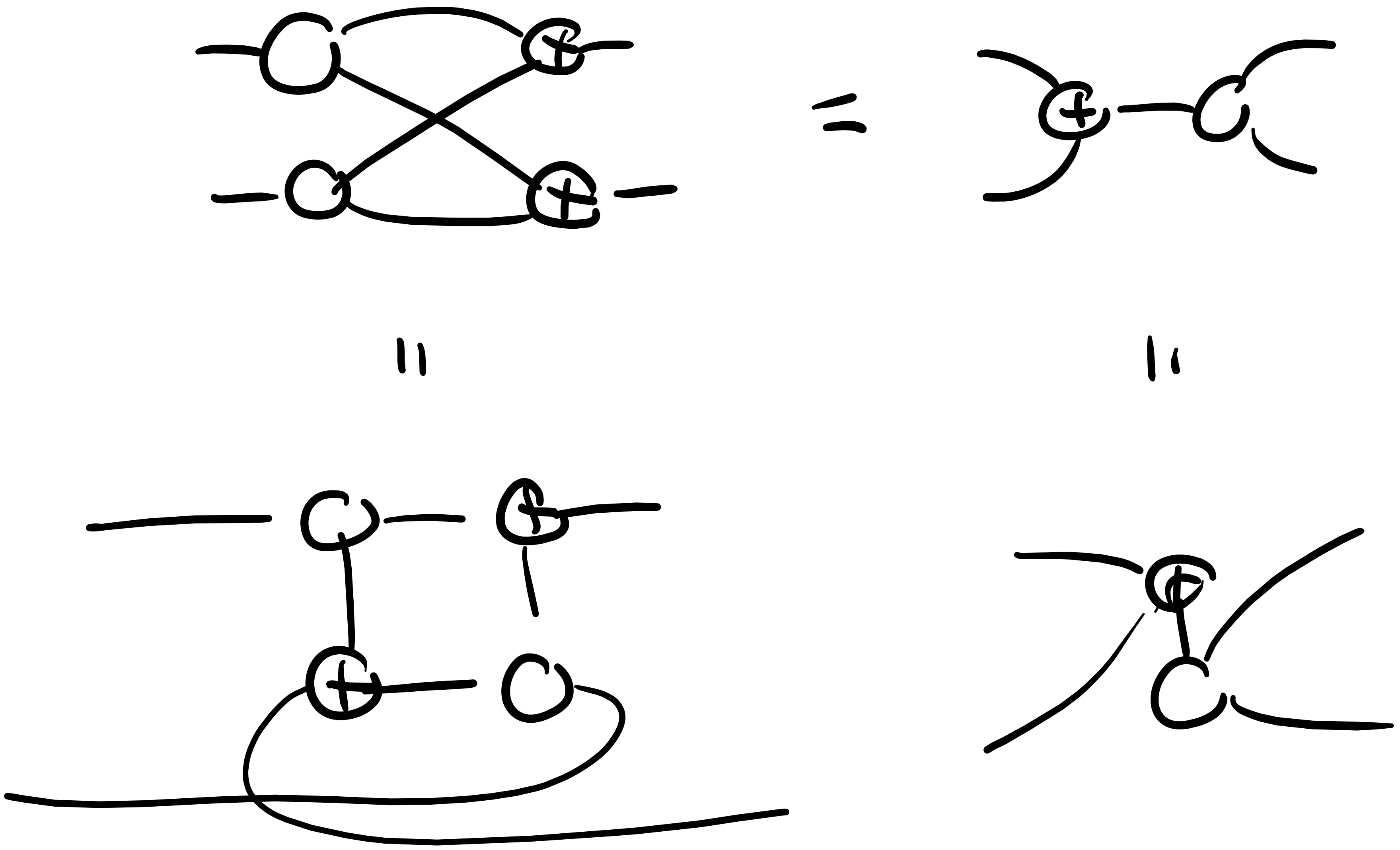
Leg chopping



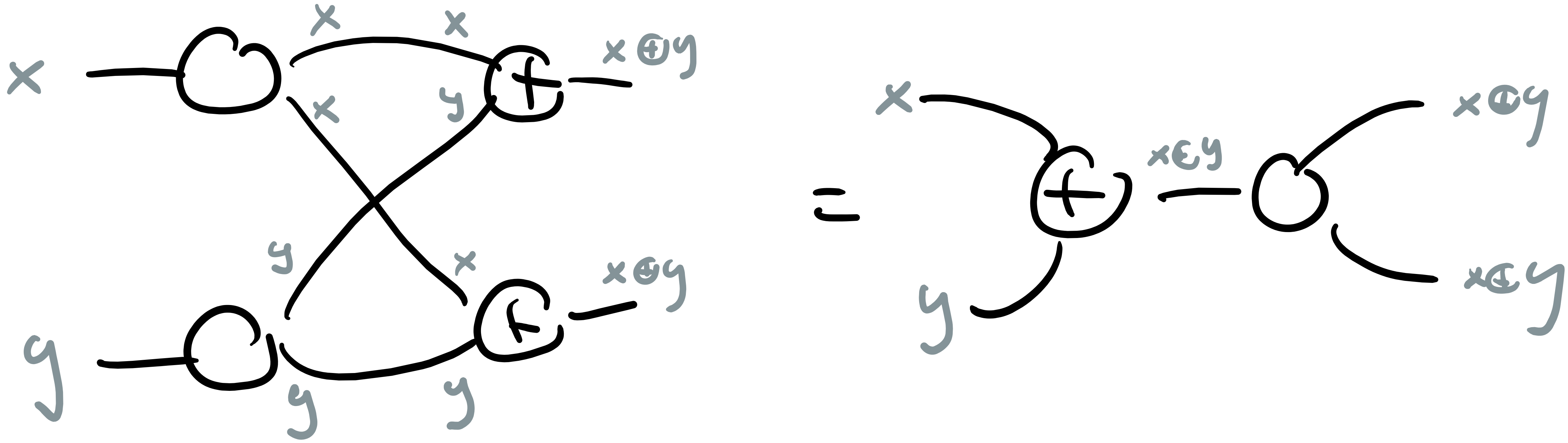
(Bialgebra law)

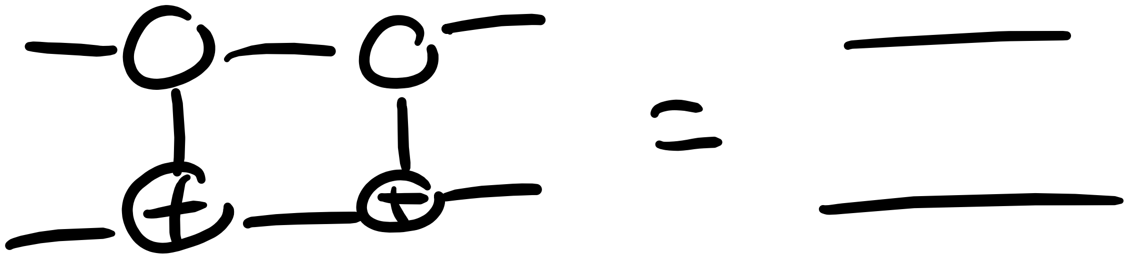
Square Popping



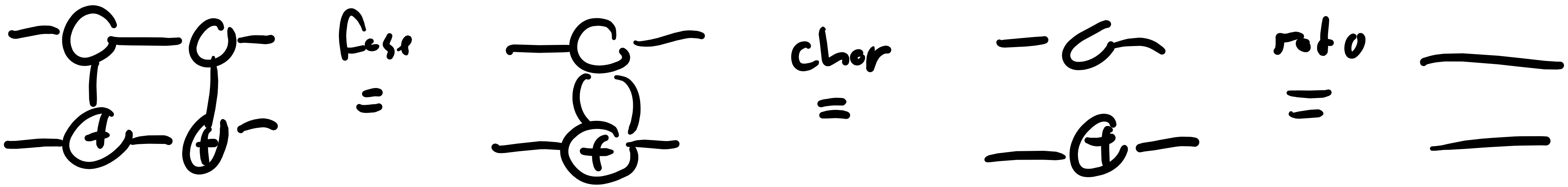


Square Popping = copy interacts with add (xor)

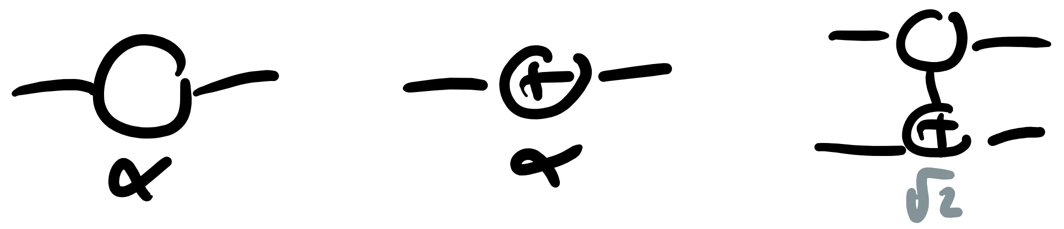


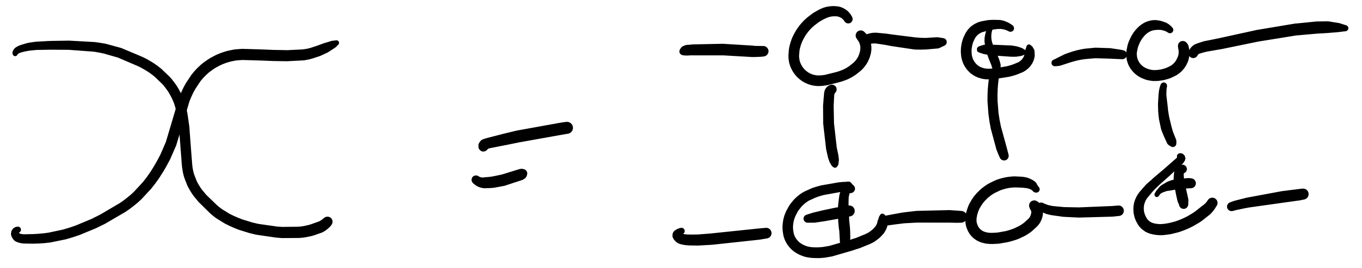


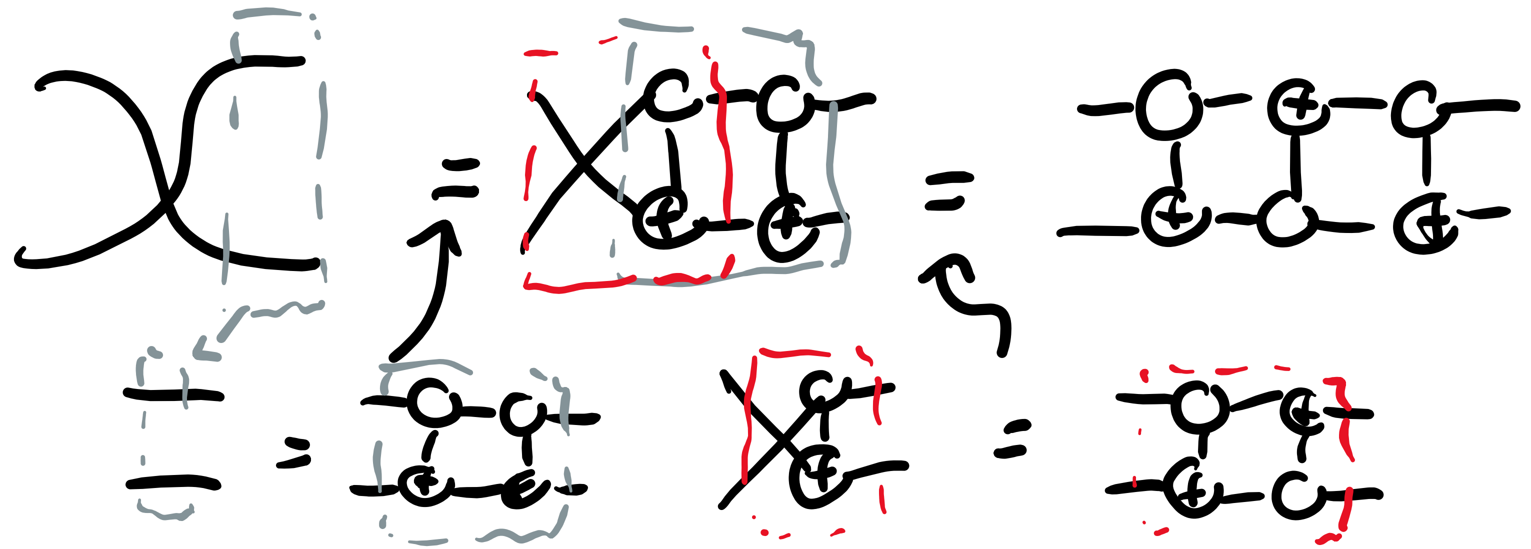
proof:

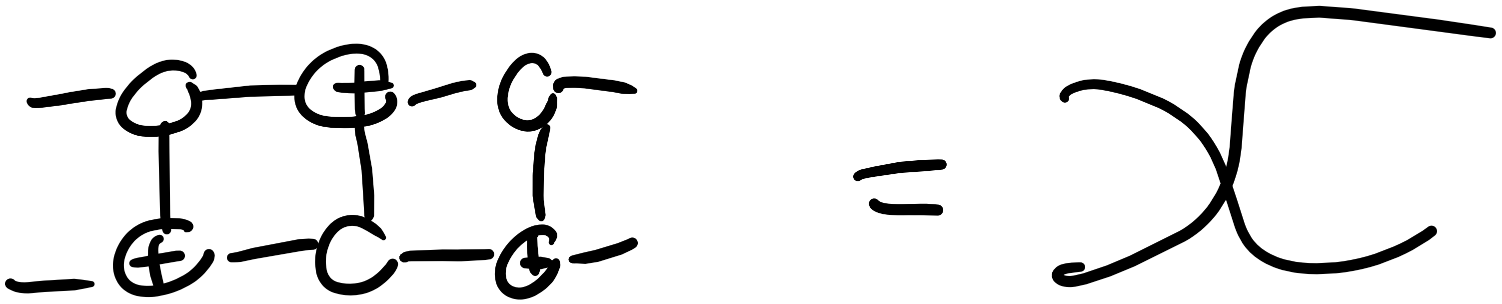


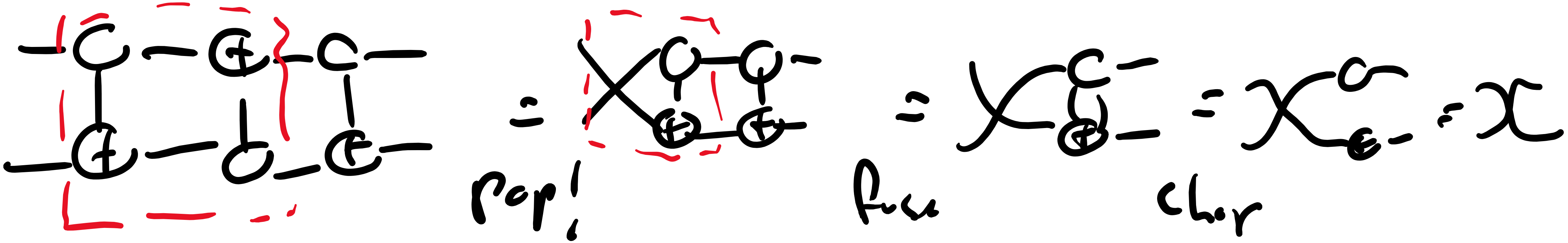
□


α
α
√2

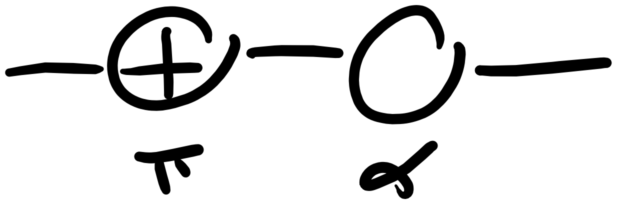




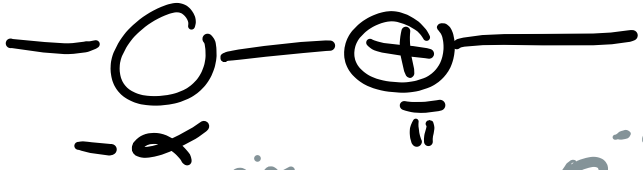



pop!
fuse
chor

$|0\rangle \rightsquigarrow |1\rangle \rightsquigarrow e^{i\alpha}|1\rangle$ $\qquad\qquad$ $|0\rangle \rightsquigarrow |0\rangle \rightsquigarrow |1\rangle$



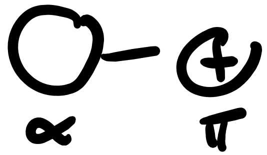
$\propto$

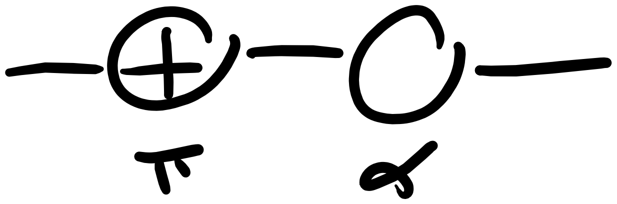


$|1\rangle \rightsquigarrow |0\rangle \rightsquigarrow |0\rangle$ $\qquad\qquad$ $|1\rangle \rightsquigarrow e^{-i\alpha}|1\rangle \rightsquigarrow e^{-i\alpha}|0\rangle$

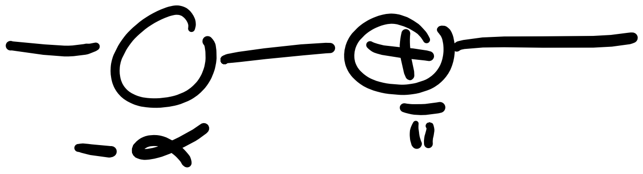
$$\begin{pmatrix} & 1 \\ e^{i\alpha} & \end{pmatrix} \qquad\qquad \begin{pmatrix} & e^{-i\alpha} \\ 1 & \end{pmatrix}$$

$$= e^{i\alpha}\begin{pmatrix} & e^{-i\alpha} \\ 1 & \end{pmatrix}$$

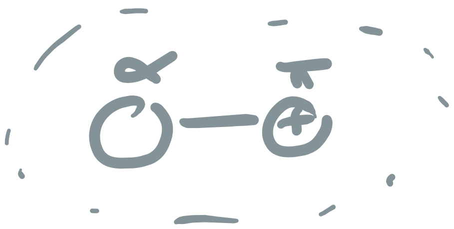




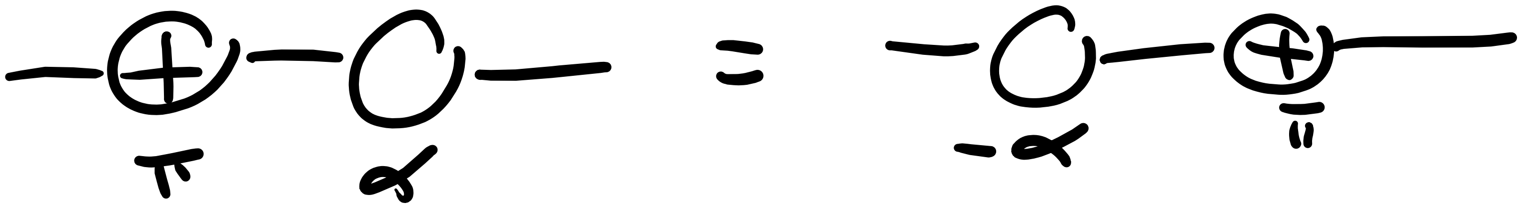
$= e^{i\alpha}$

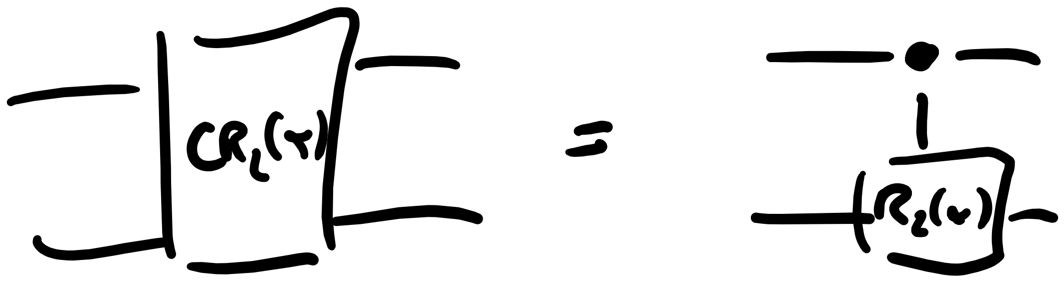


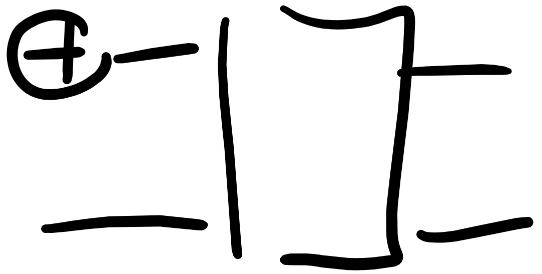
$$\langle 1|\big(|0\rangle + e^{i\alpha}|1\rangle\big) = \cancel{\langle 1|0\rangle}^{0} + e^{i\alpha}\langle 1|1\rangle = e^{i\alpha}$$

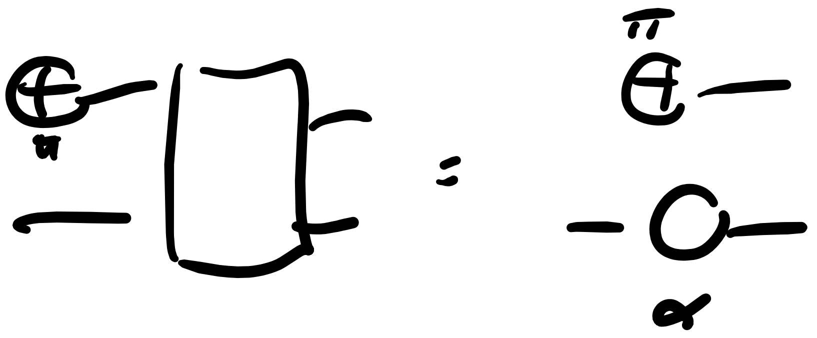


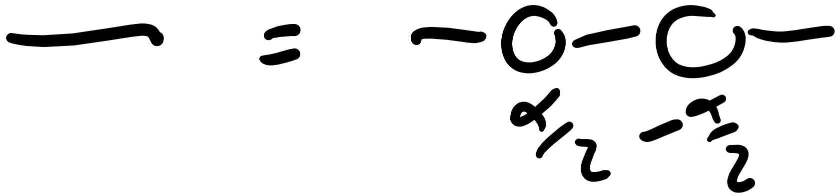
$e^{i\alpha} \in \mathbb{C}$



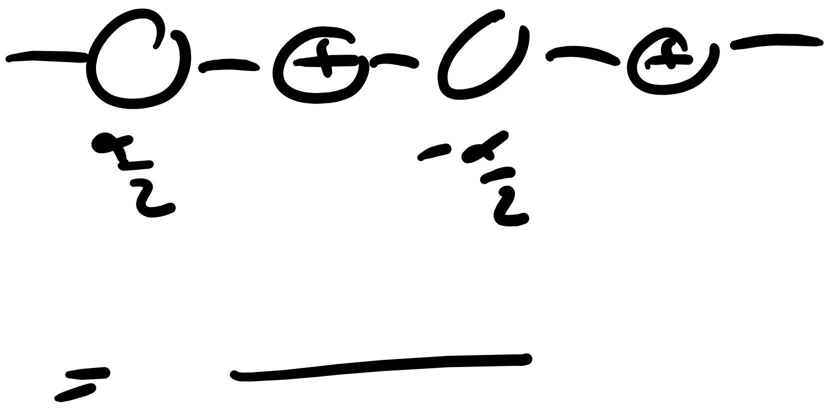

CRz(γ)
=
Rz(γ)

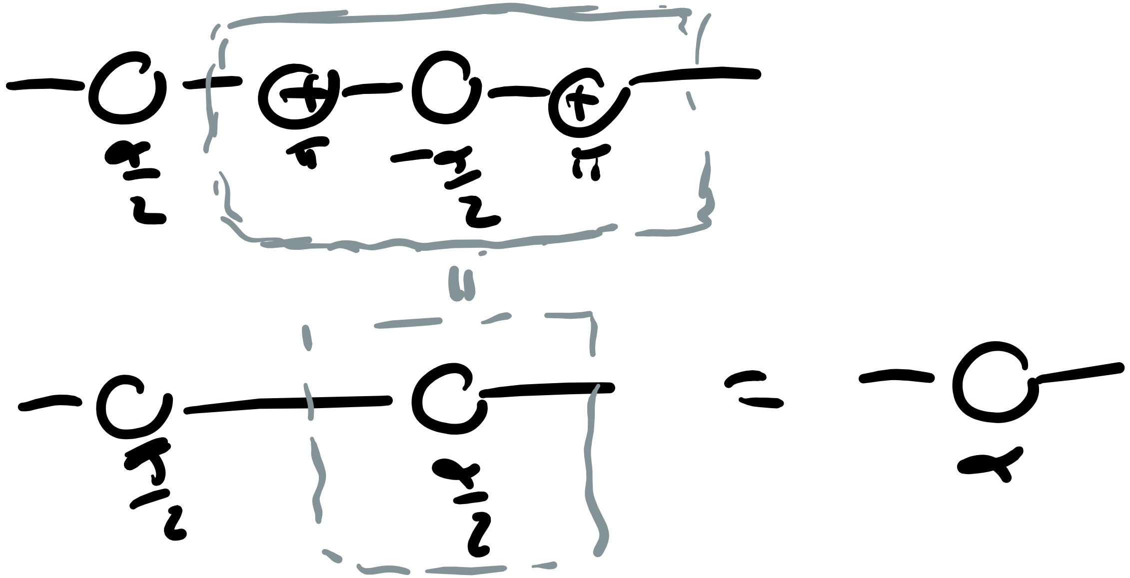

=

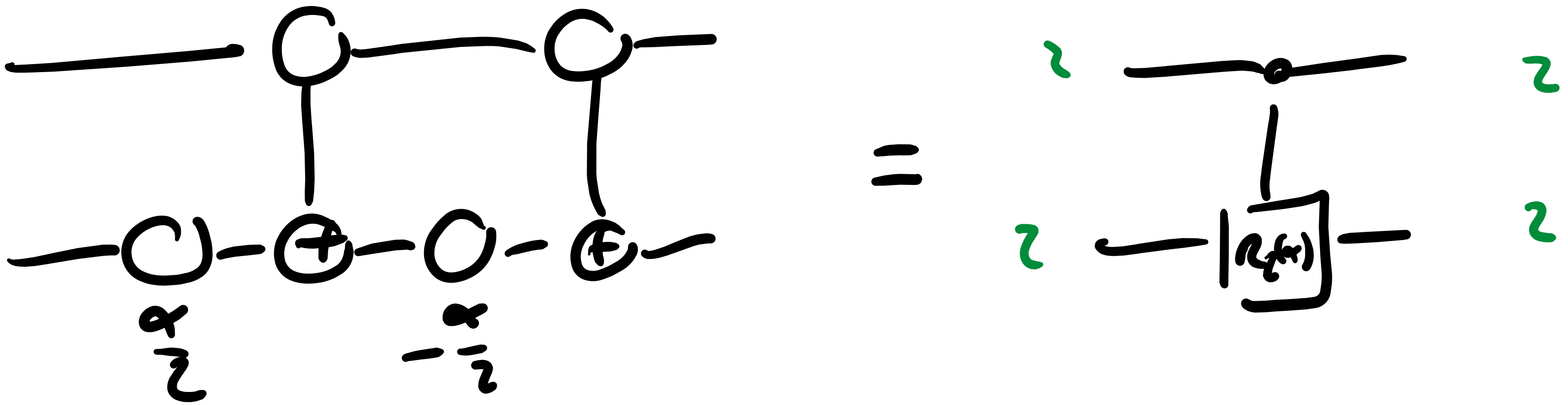

π
=
π
α

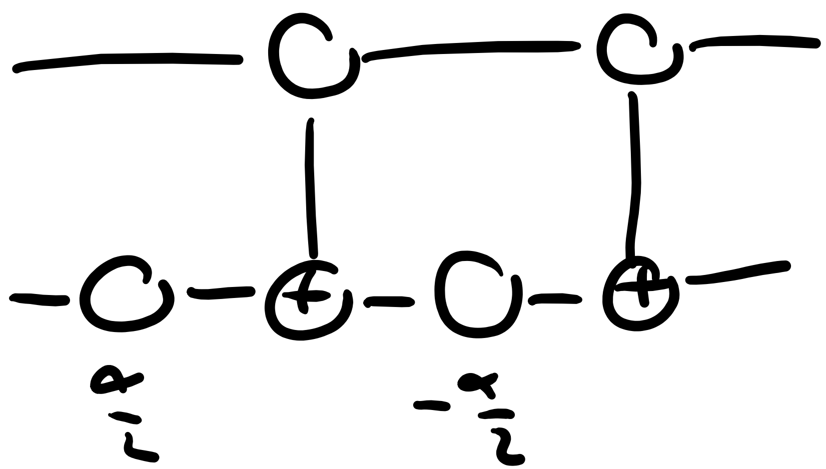

=
α/2
-α/2


α
=
α/2
α/2


α/2
-α/2
=

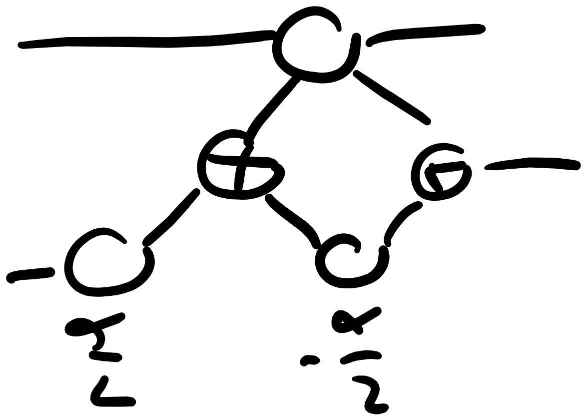

α/2
π
-α/2
π
α/2
α/2
=
α


α/2
-α/2
=
z
z
z
z

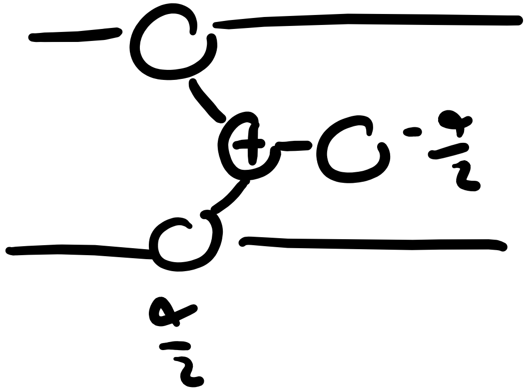


fuse

=



|| unfuse



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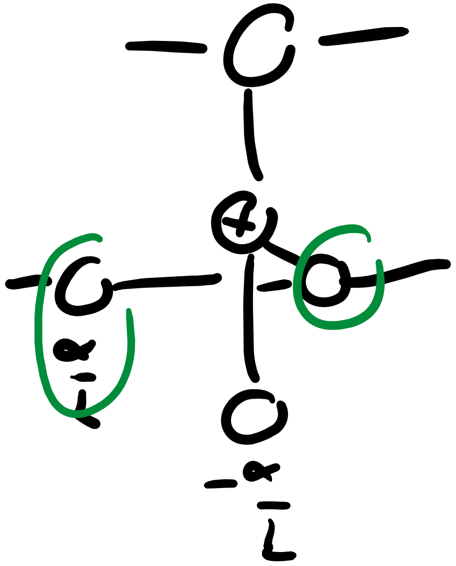

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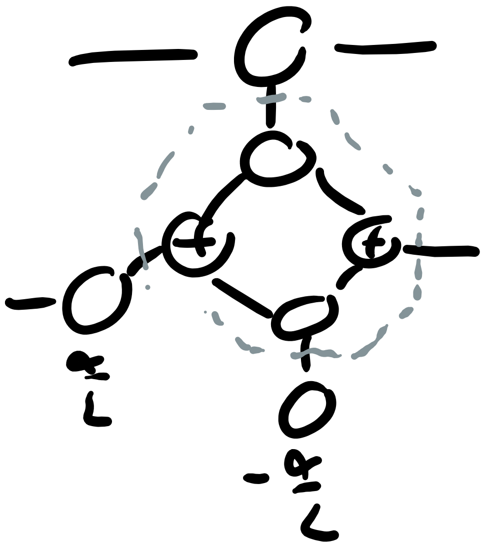
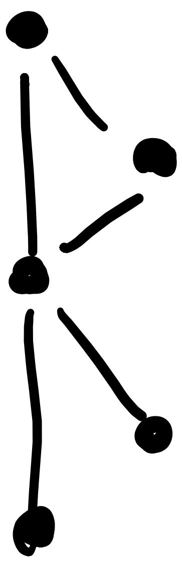
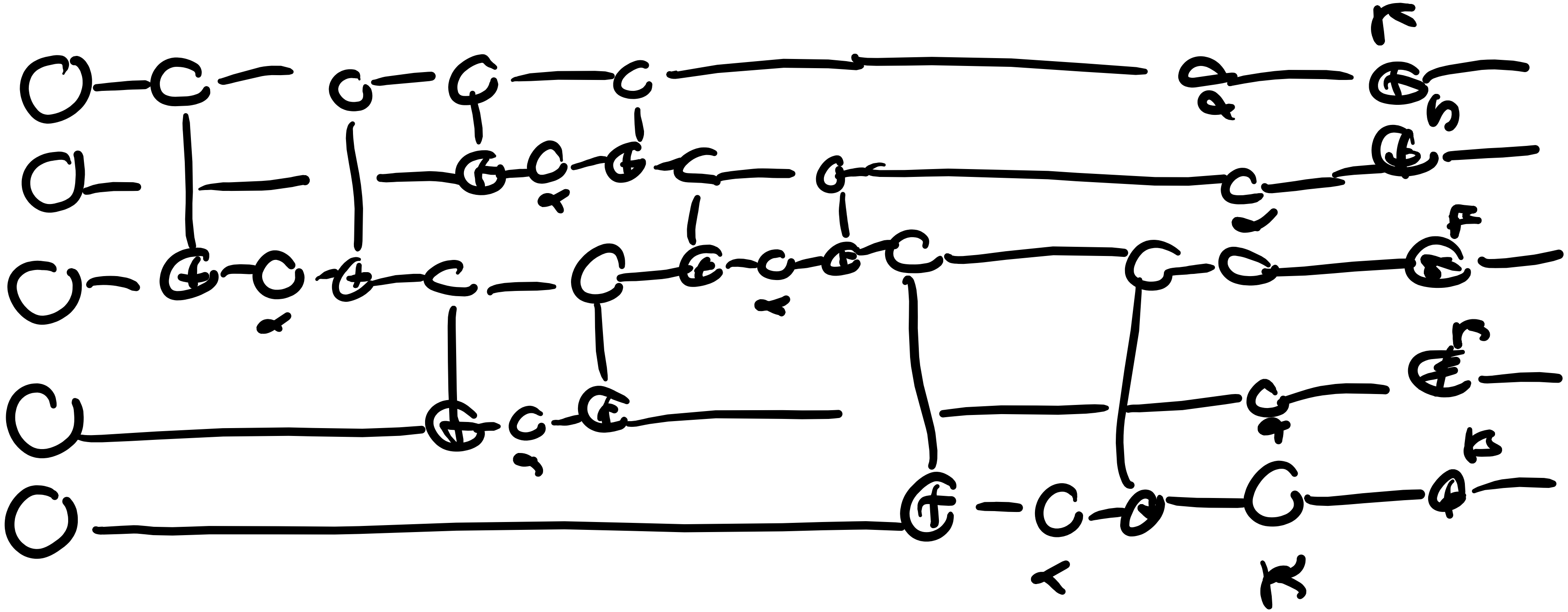
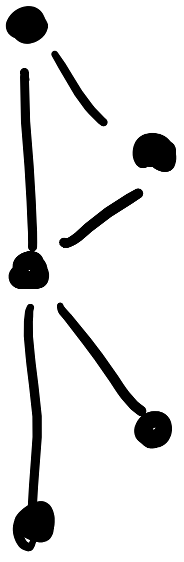
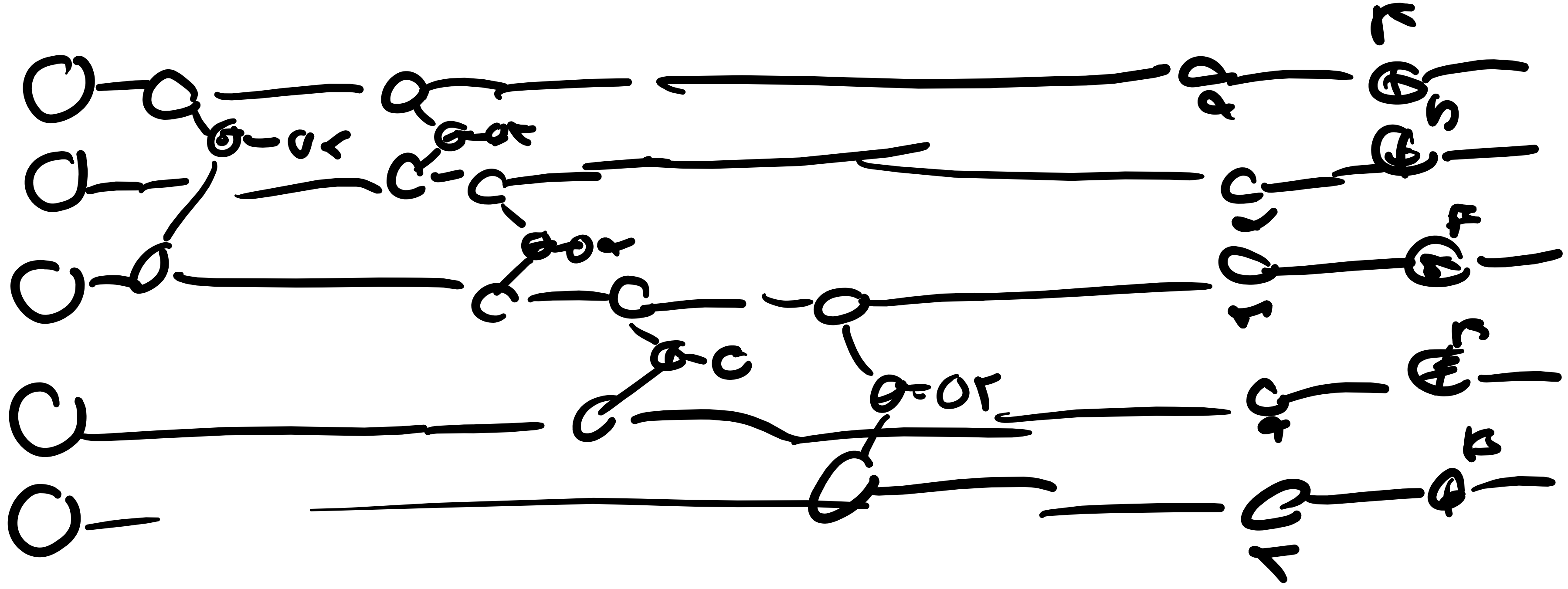


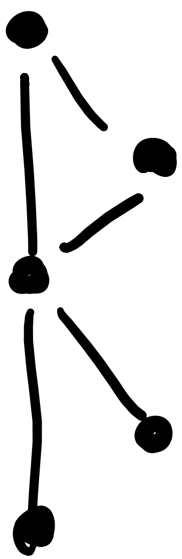


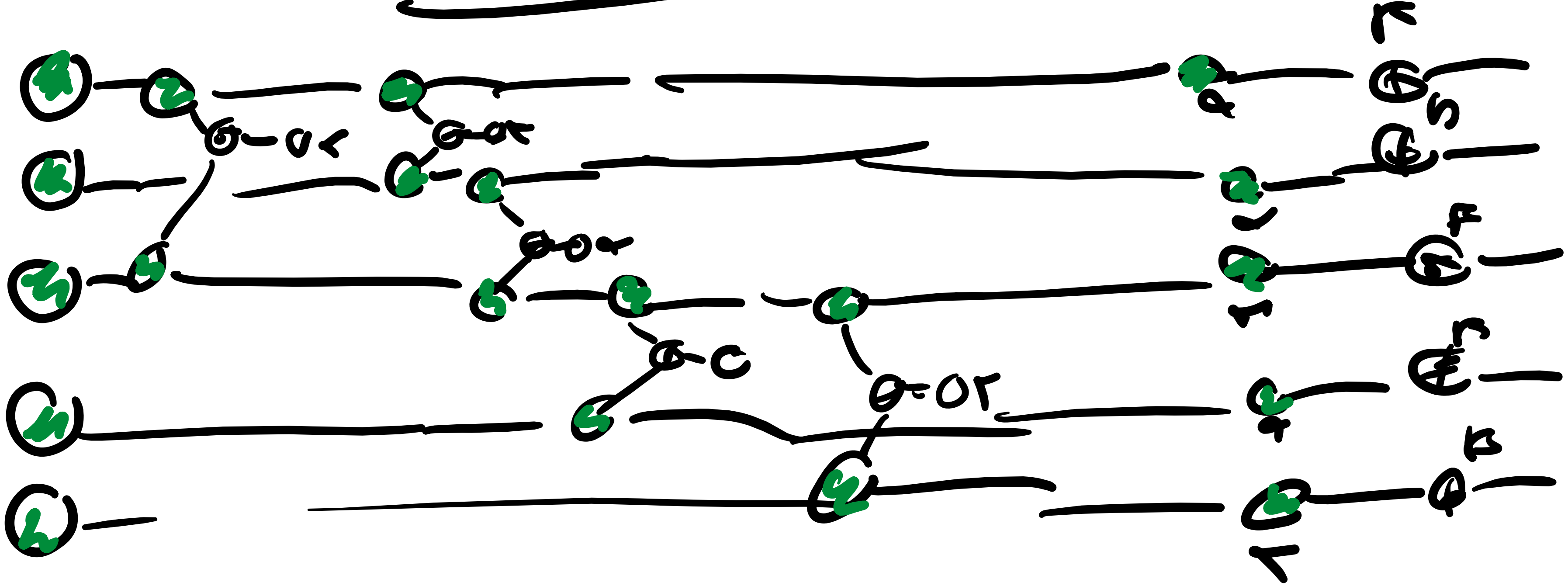






MASS FUSE!





MASS FUSE!