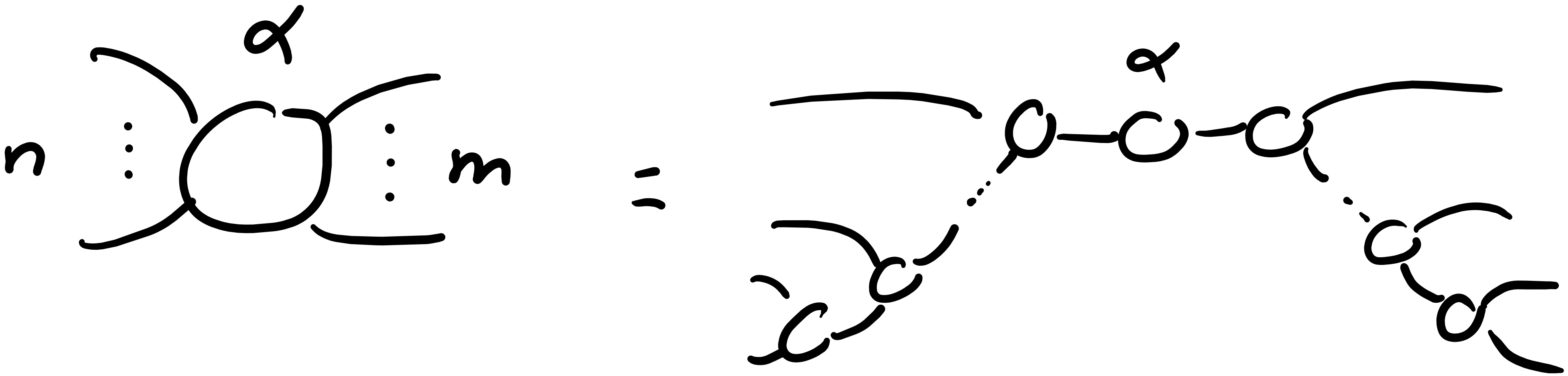
Quantum in Pictures Lecture Series

Lecturer: Stefano Gogioso

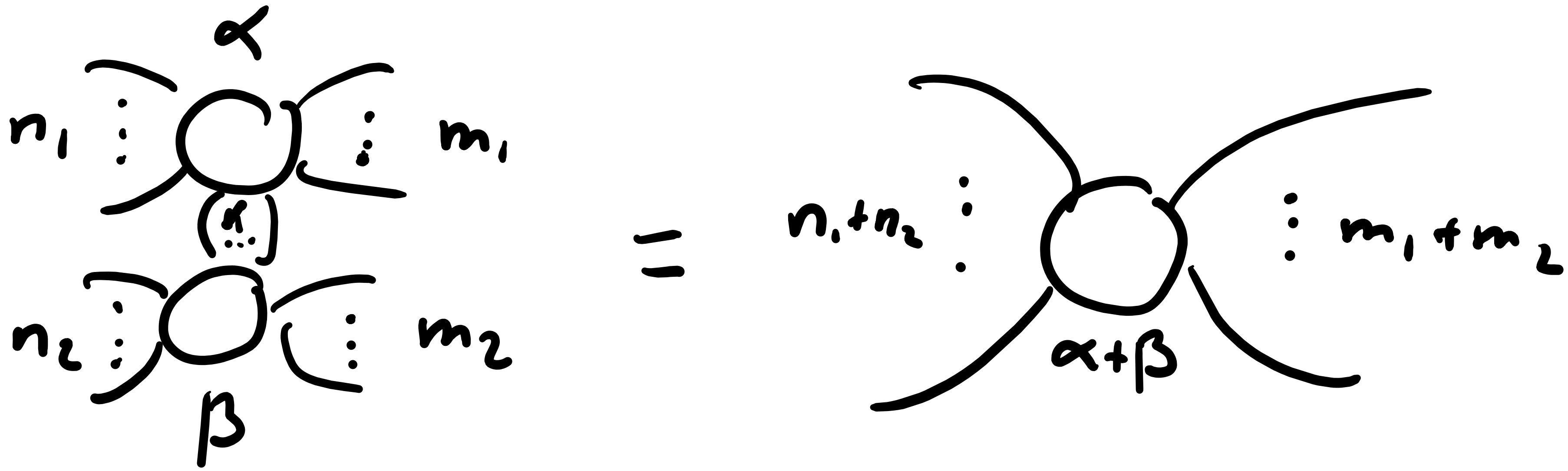
Tue 27 June 2023 – Afternoon Lecture

INDIANA UNIVERSITY BLOOMINGTON

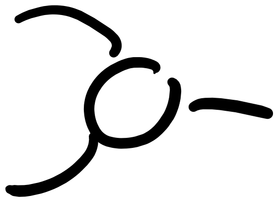


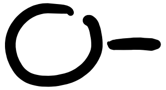
α
n
m
$=$
α

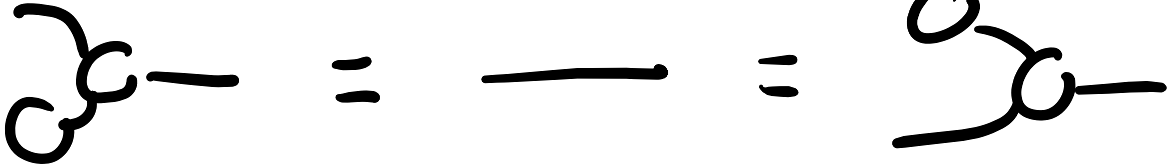
$K \geq 1$

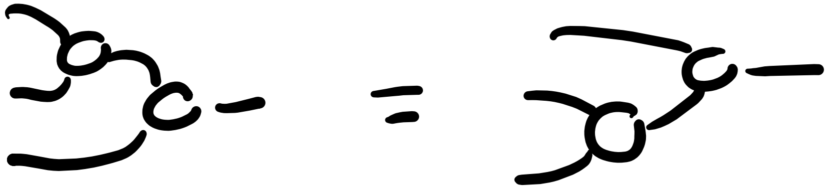


α
n_1
m_1
(α)
n_2
m_2
β
$=$
$n_1 + n_2$
$m_1 + m_2$
$\alpha + \beta$

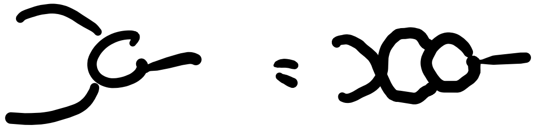
mult 

unit 



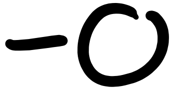


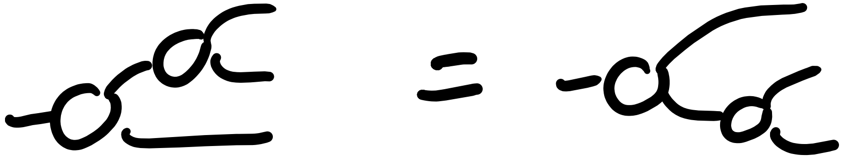
Monoid

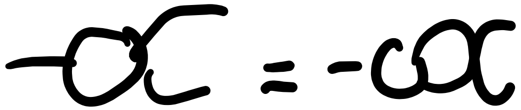


requires 

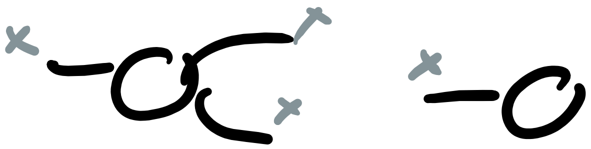
comult (copy) 

counit (delete) 

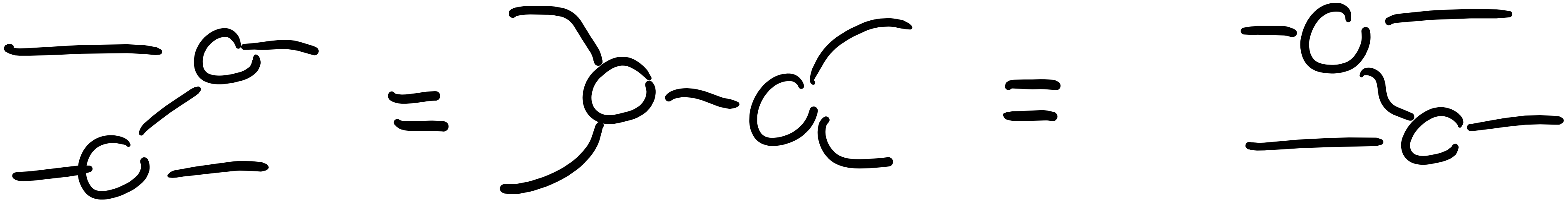




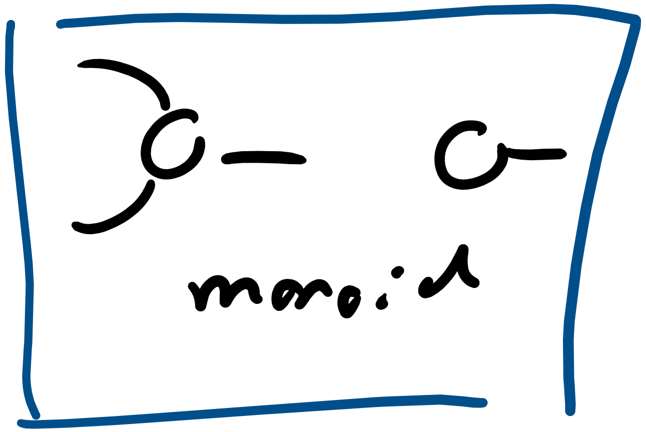
monoid

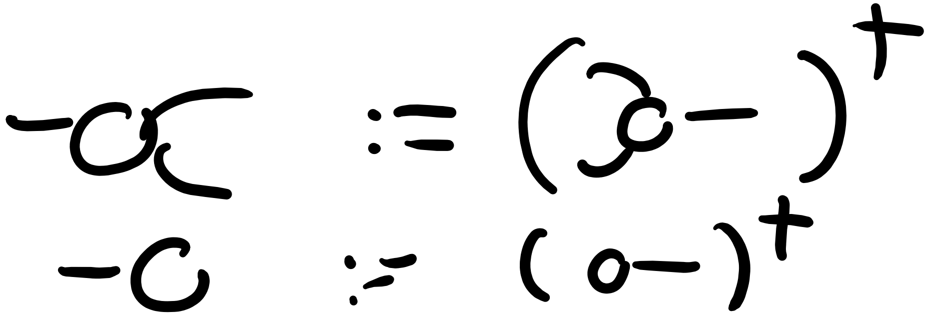


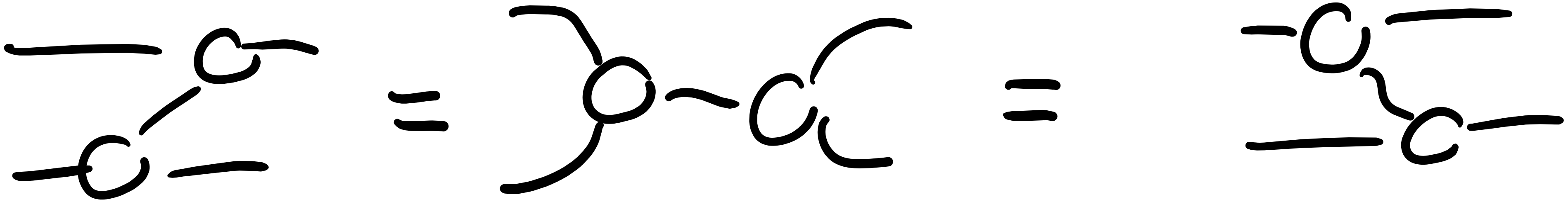
comonoid



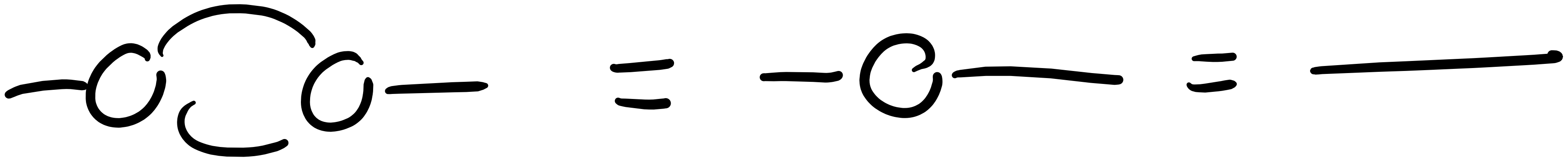
Frobenius Algebra

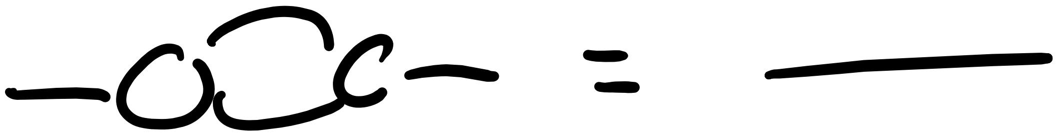




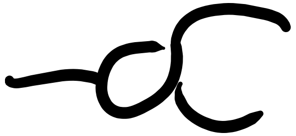


†-Frobenius Algebra





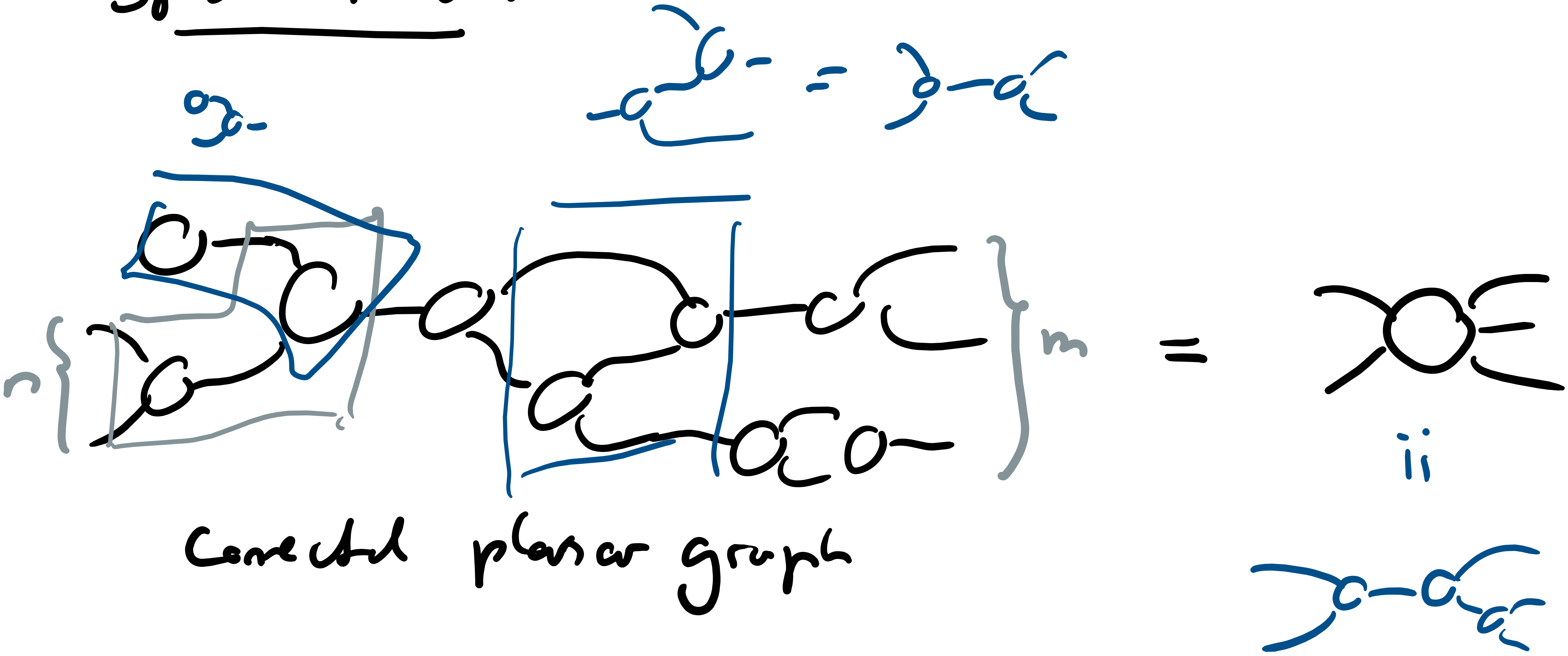
special $\dagger$-FA

 isometry

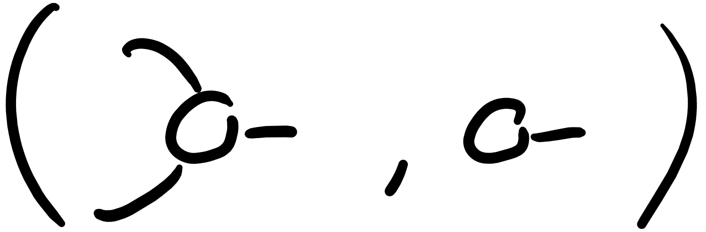
V isometry

$\Leftrightarrow V^\dagger V = \mathbb{1}$

Spider theorem

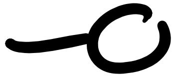


Connected planar graph

 Special Commutative †-FA in fHilb

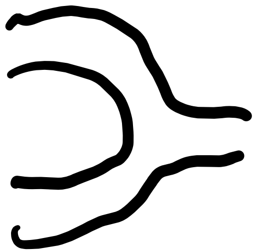
$\iff \exists! \, (|\psi_j\rangle)_j$ ONB s.t.

 $:= \sum_j |\psi_j \psi_j\rangle\langle\psi_j|$

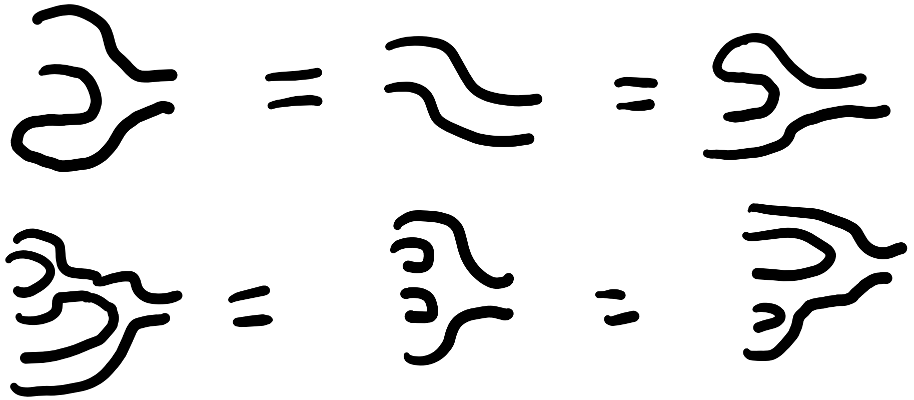
 $:= \sum_j \langle\psi_j|$

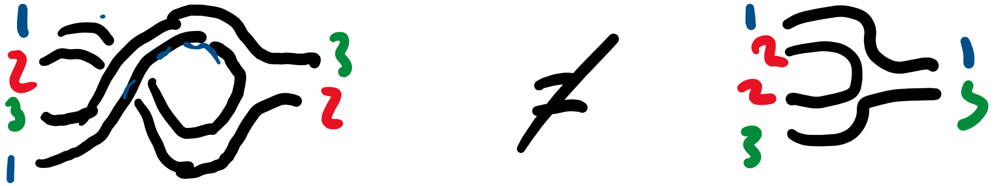
— H

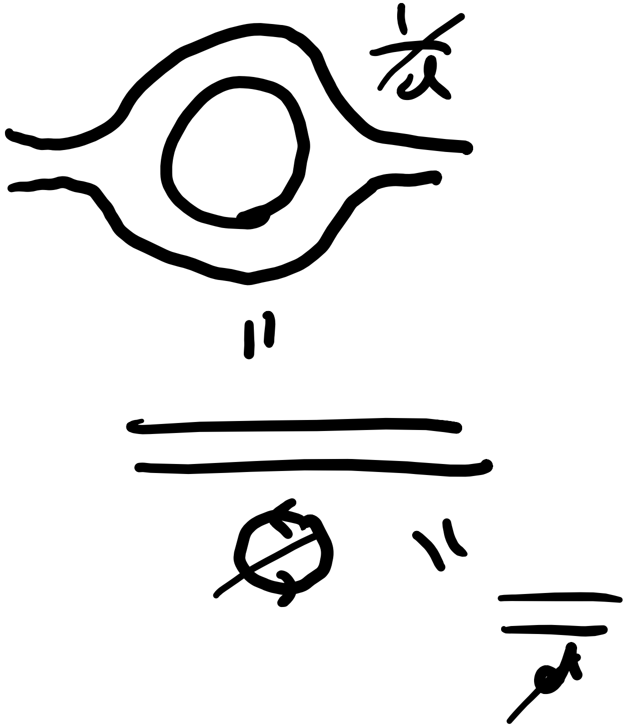
dim H = d

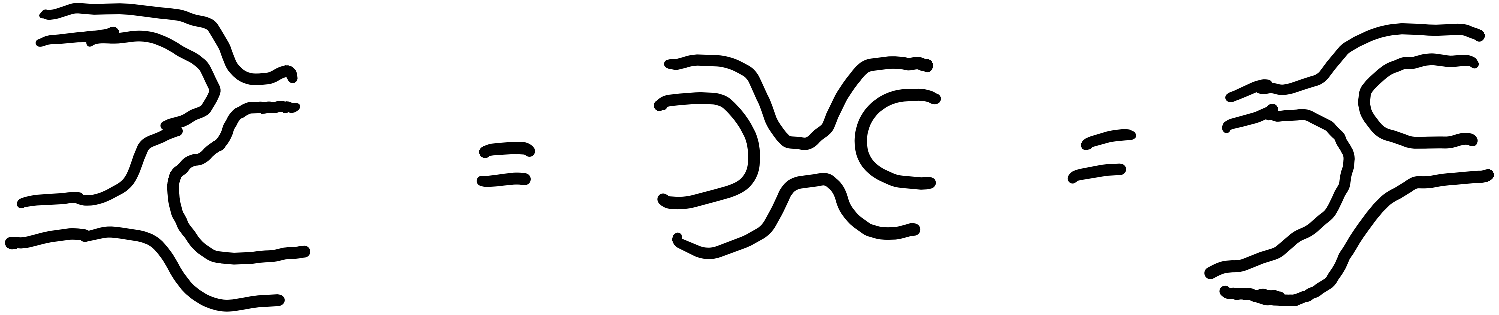
$\frac{1}{\sqrt{d}}$ 

$\sqrt{d}$ 



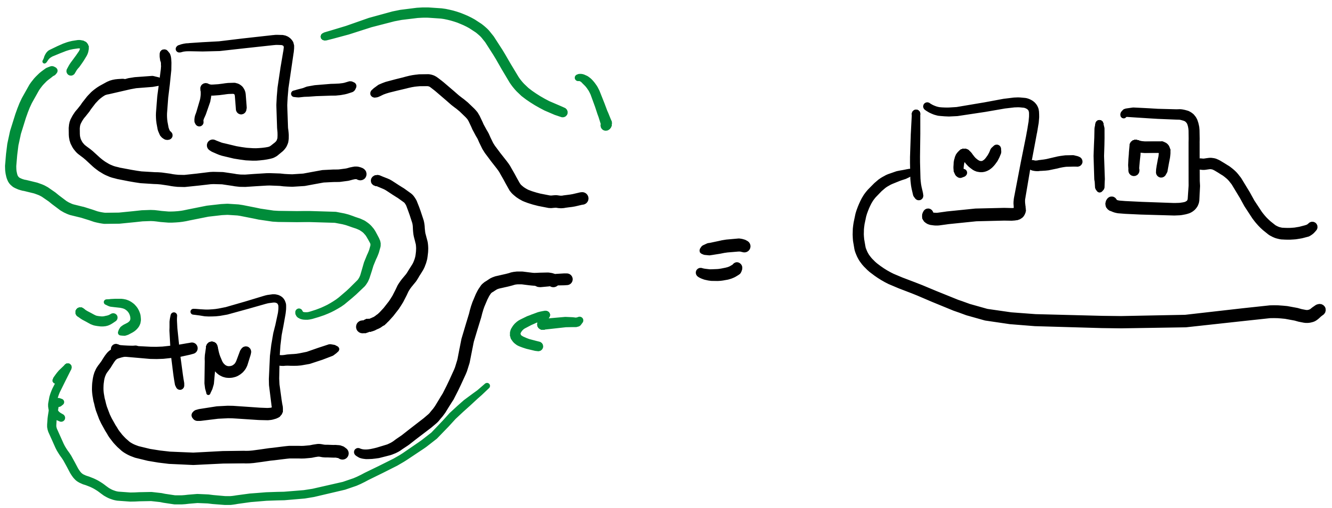




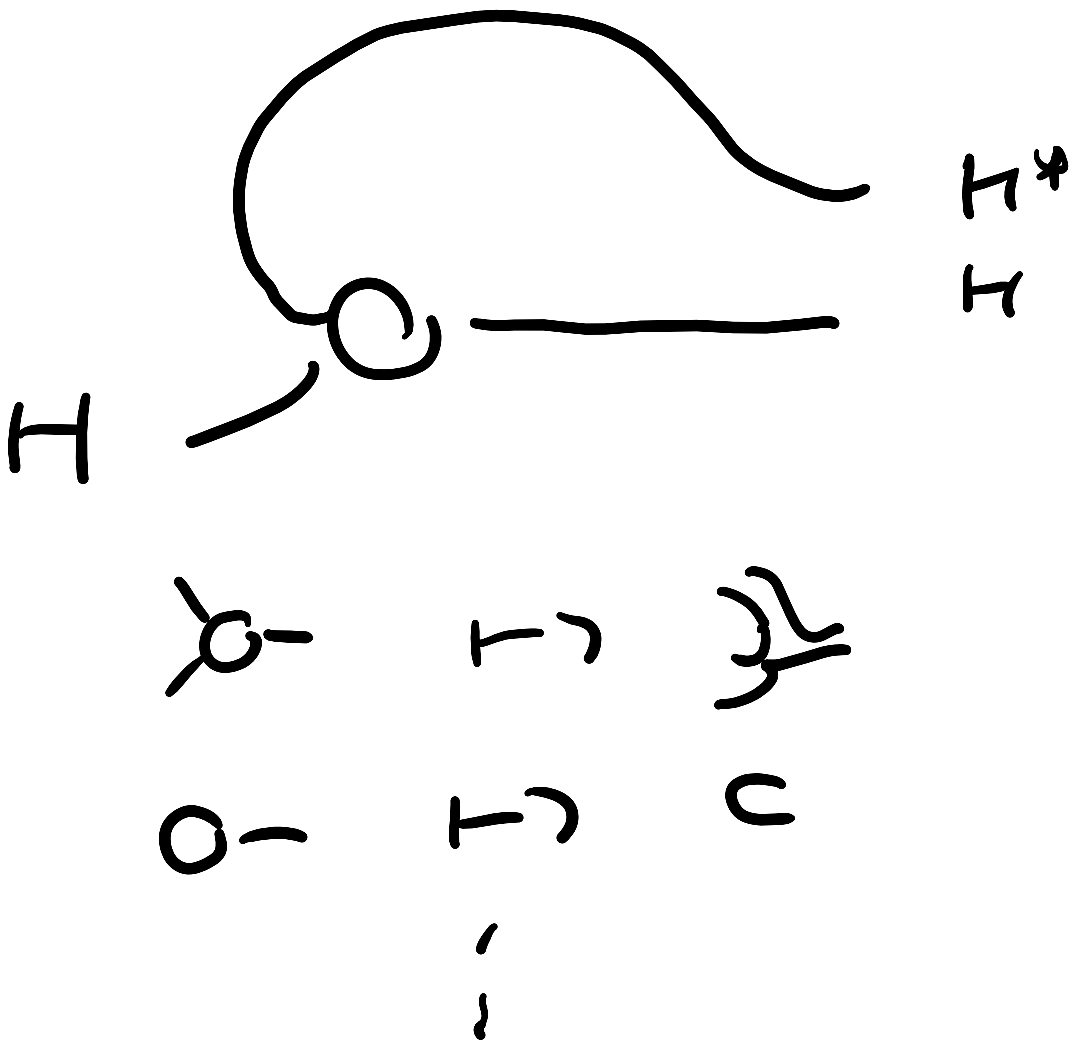


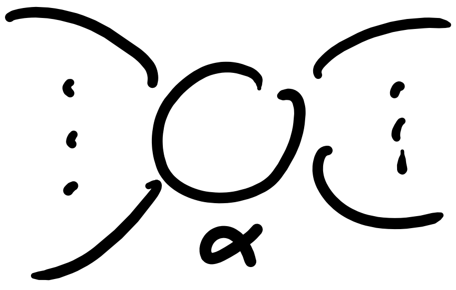
Special †-Frobenius Algebra (in fHilb)

$\longleftrightarrow$ fin-dim C^*-Algebras

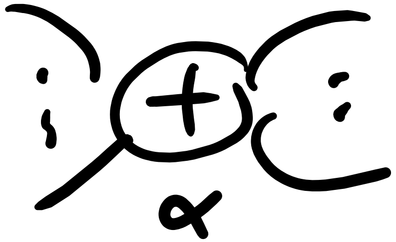


Matrix Algebra

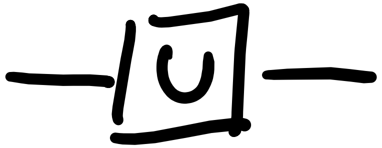

H*
H
H



Z basis

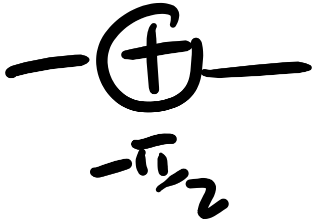


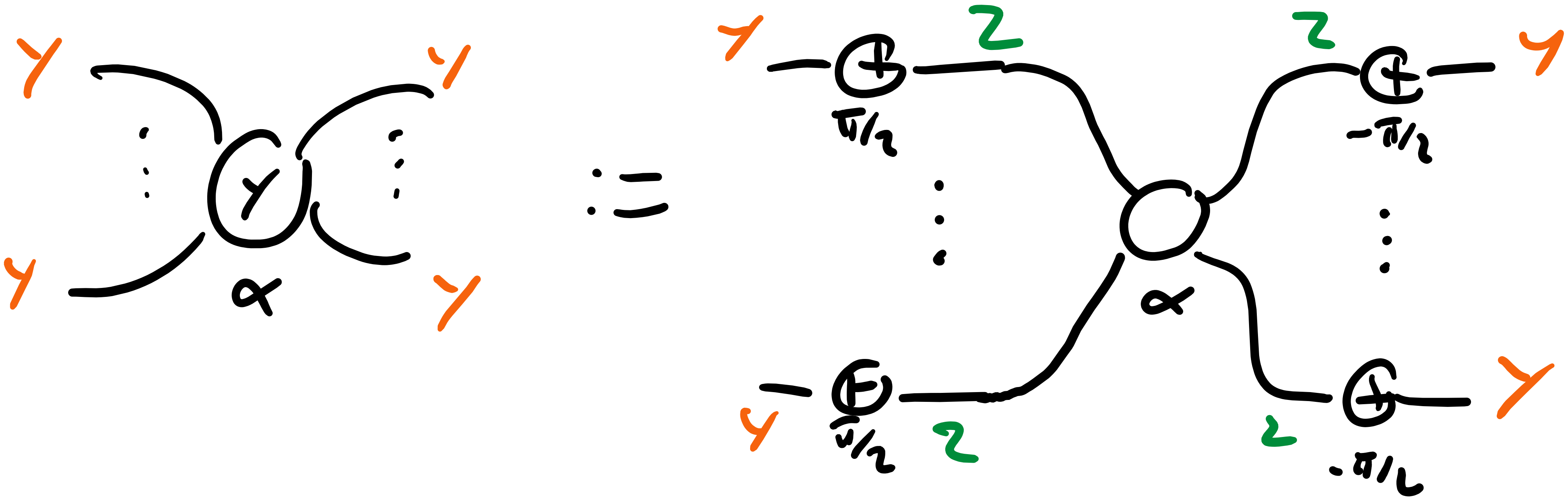
X basis

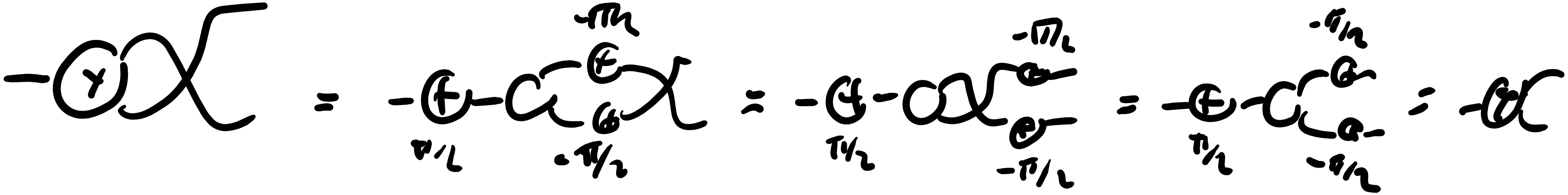


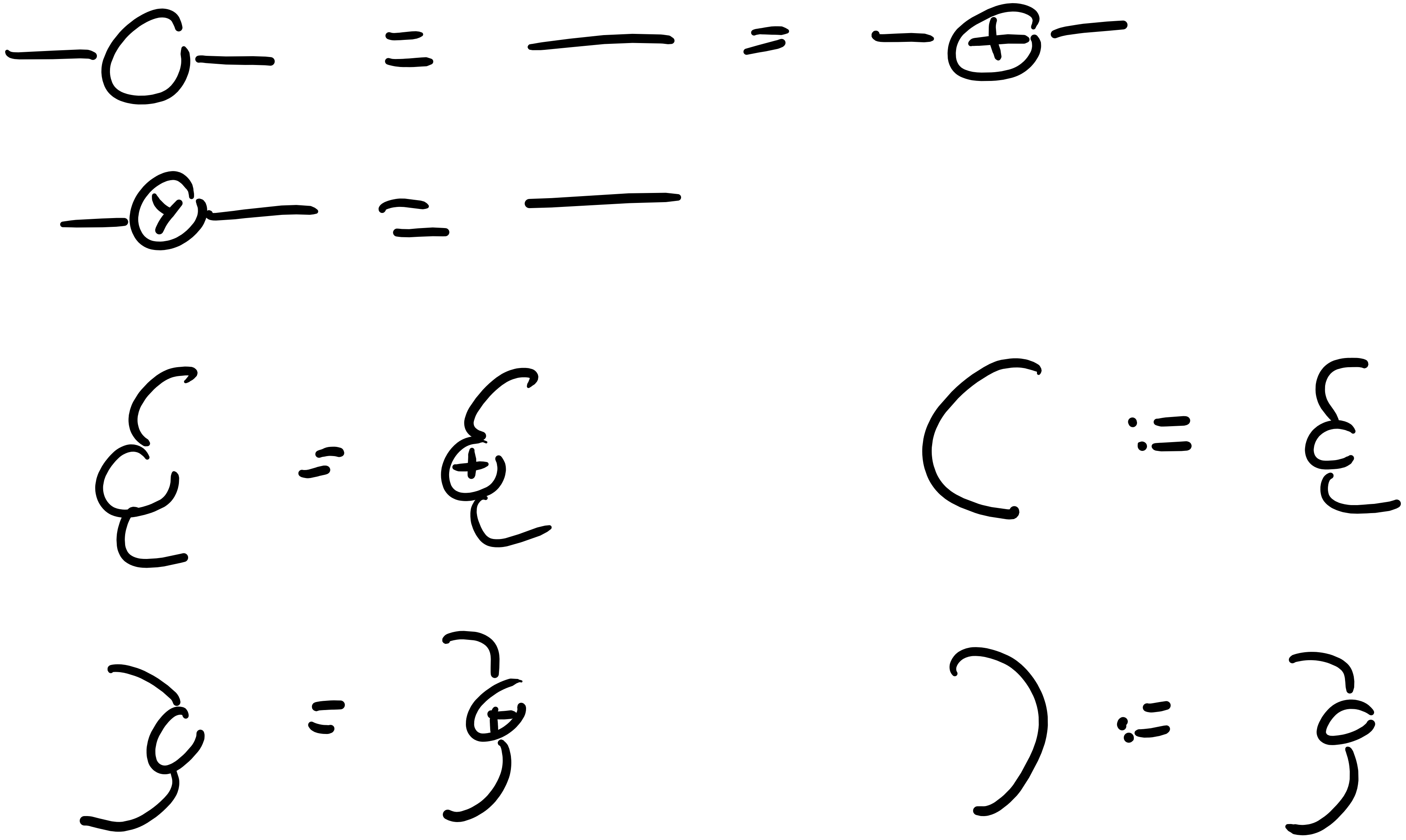
Z basis → Y basis

=

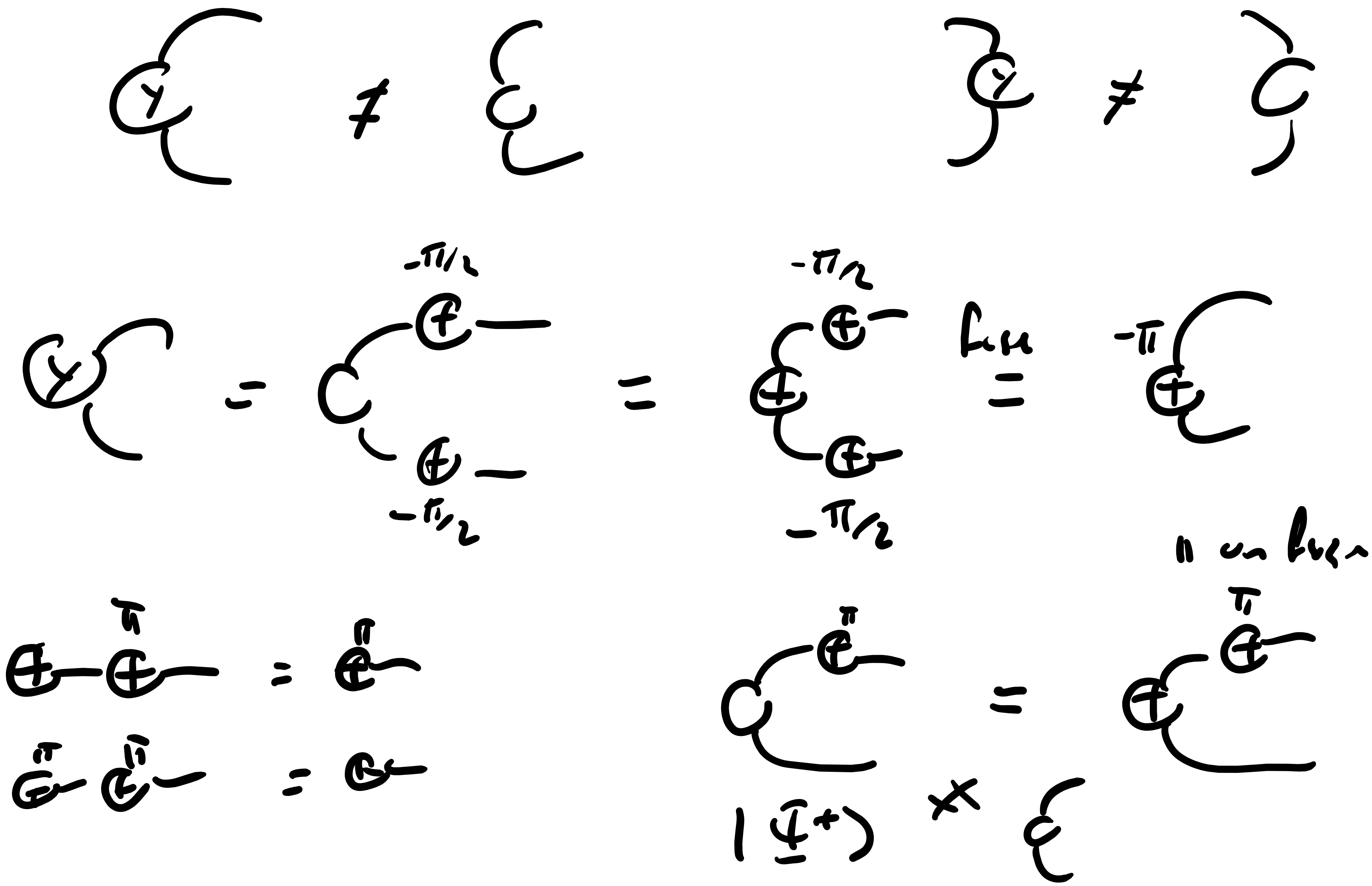


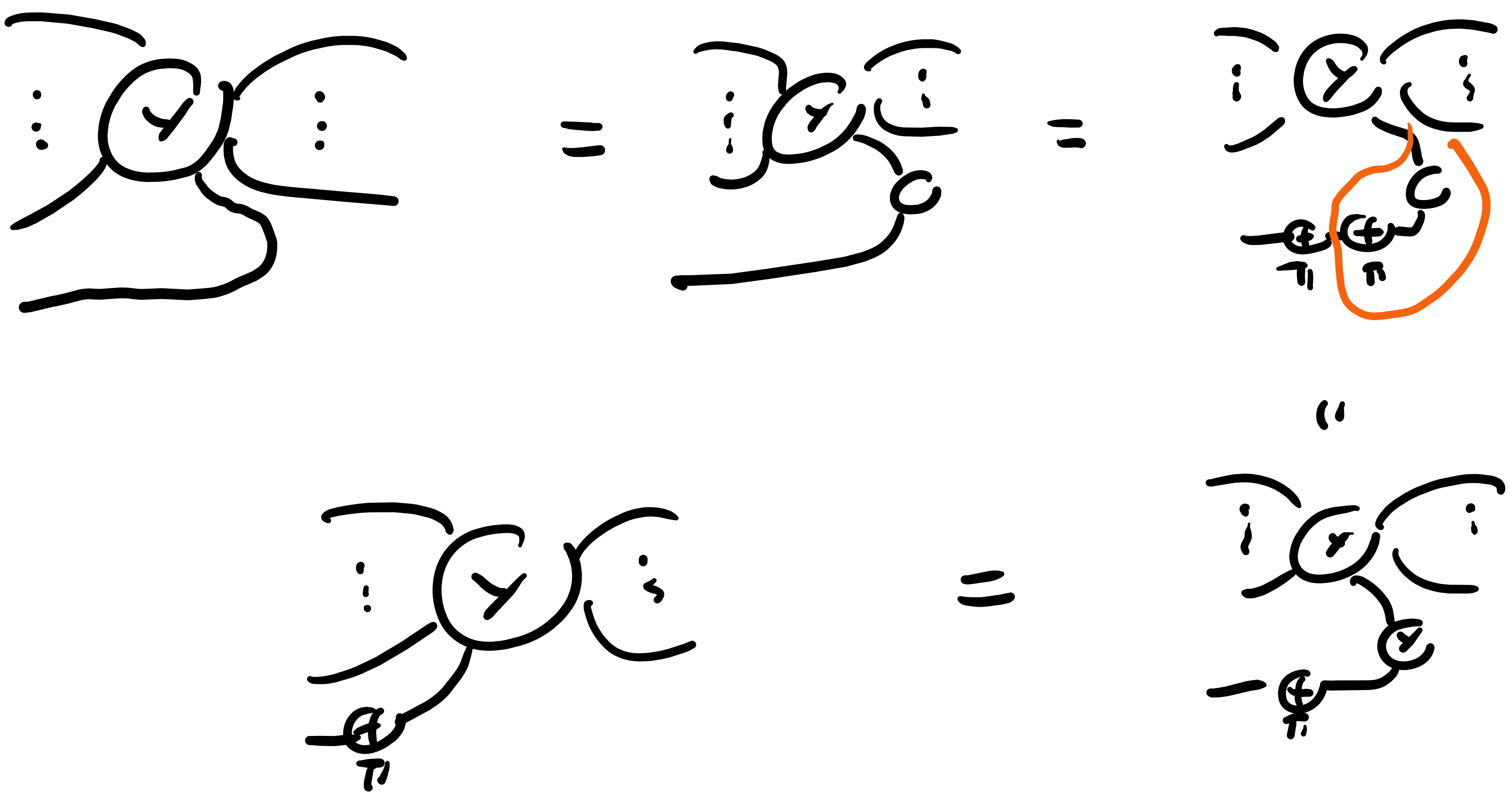

y
Y
α
:=
z
π/2
-π/2

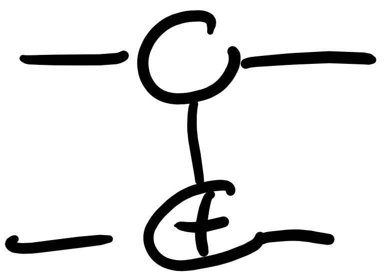

Y
=
π/2
-π/2

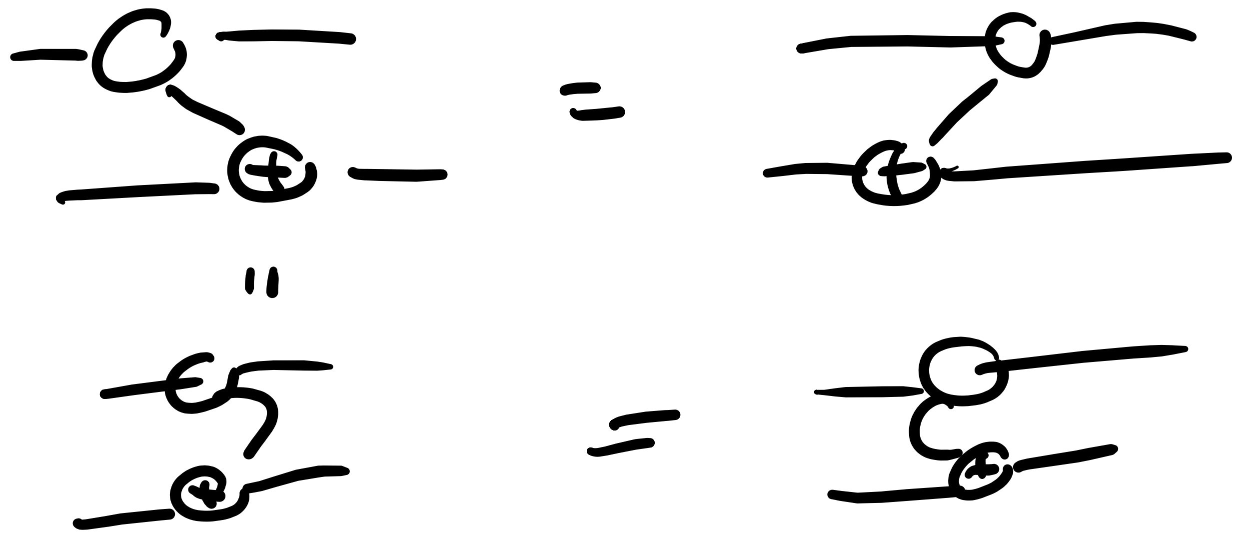


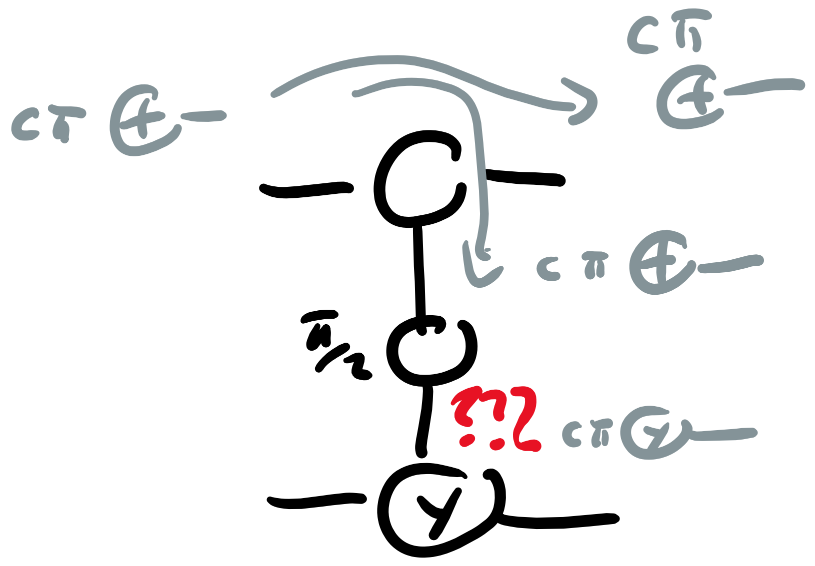
$\mathbb{C}[x_i^n]$

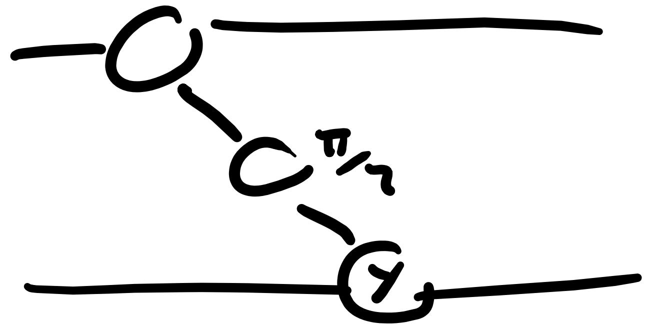

-π/2
-π/2
-π/2
-π/2
-π
π
π
π
π
|Φ+⟩



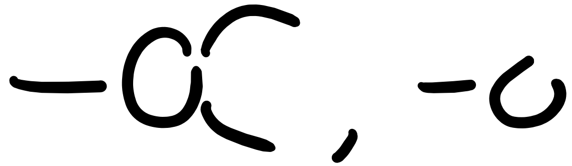


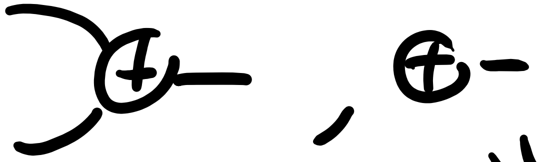



cπ
cπ
cπ
π/2
??
cπ
Y


π/2
Y

$\mathbb{C}[G]$ G finite Group S_4

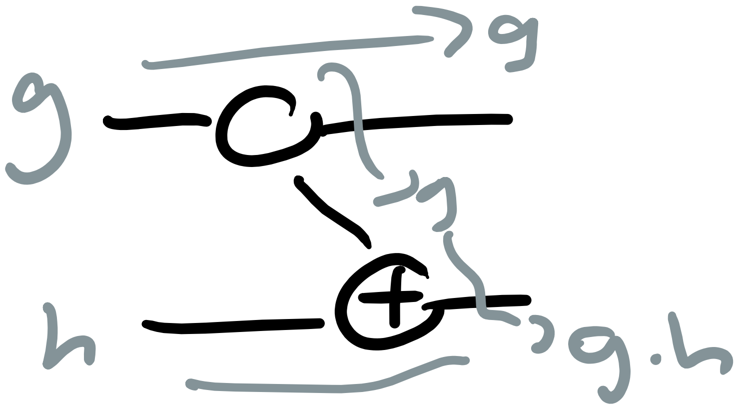
 $(|g\rangle)_{g\in G}$ + scFA

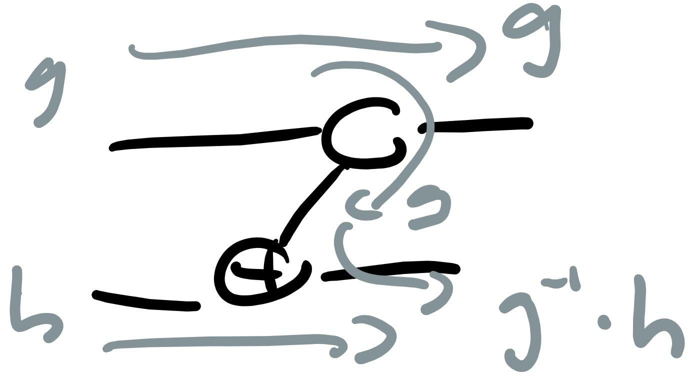
   + sFA   commutative $\Leftrightarrow$ G abelian

$= |1_G\rangle$

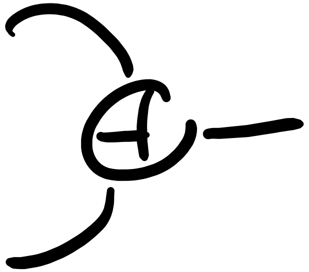
$= \frac{1}{\sqrt{|G|}} \sum_{g,h} |g\cdot h\rangle\langle g h|$

$(\text{if } G \neq \mathbb{Z}_2^n)$

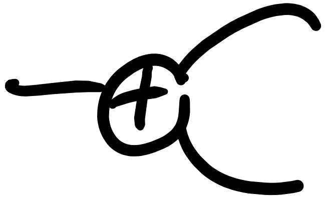




CMULT / CADD



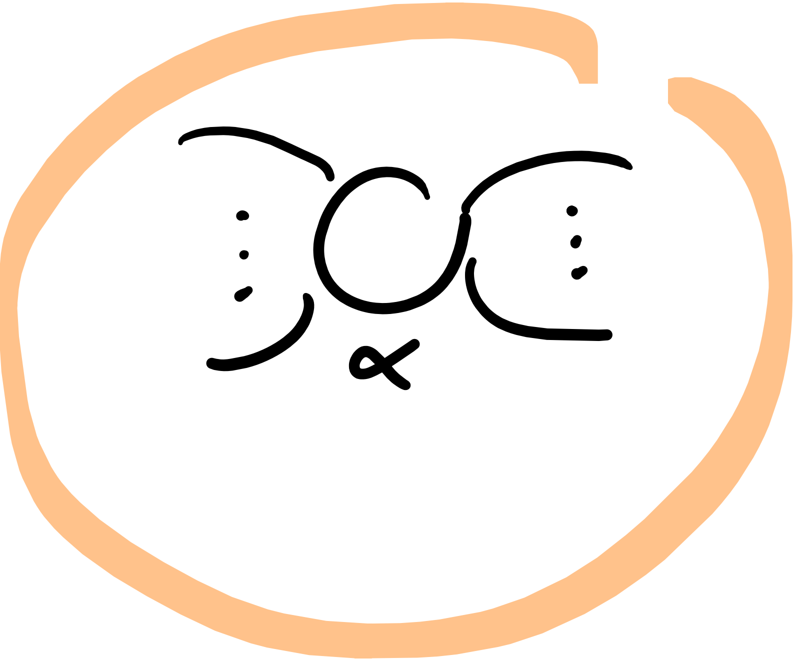
$$= \sum_{g,h} |g\cdot h\rangle\langle g|\langle h|$$

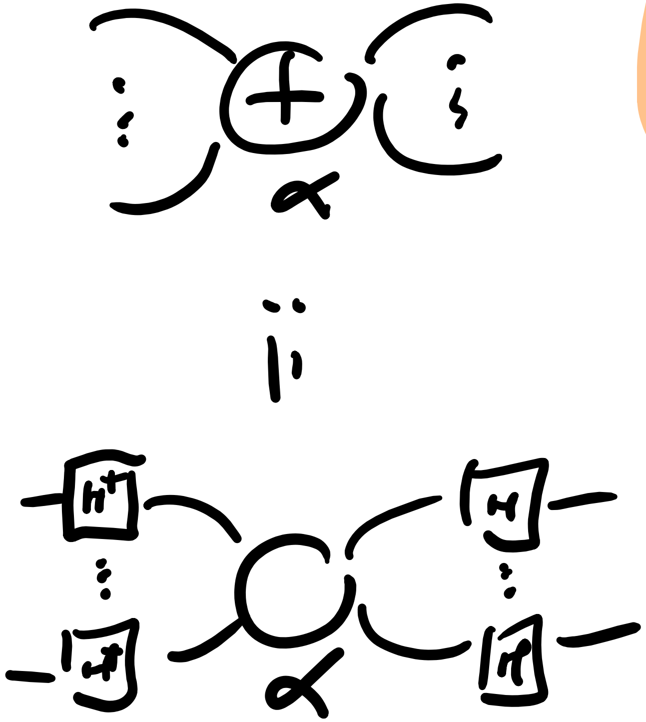


$$= \sum_{g,h} |g\rangle|h\rangle\langle g\cdot h|$$

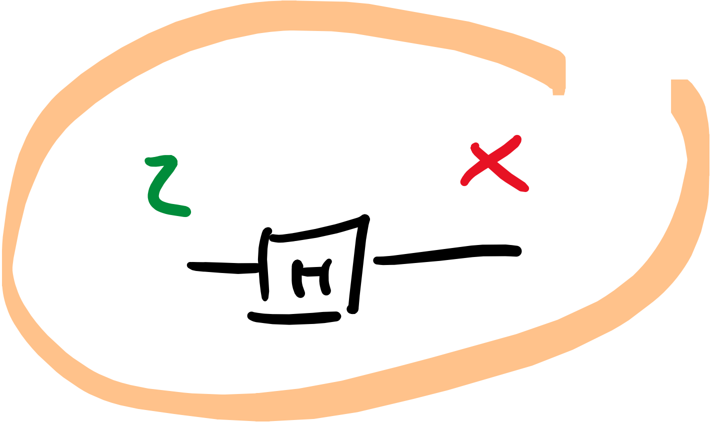
$$g\cdot h = k$$
$$h = g^{-1}k$$

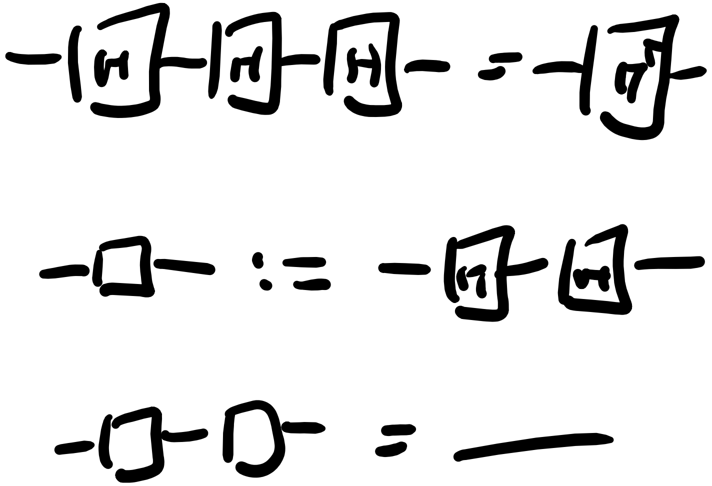
$$= \sum_{g,k} |g\rangle|g^{-1}\cdot k\rangle\langle k|$$

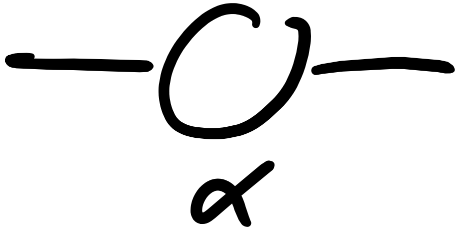

α


α
α


g ⟼ g⁻¹

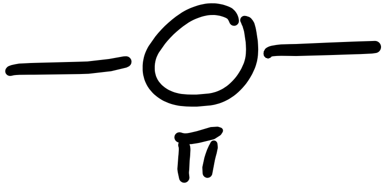

Z
X
H


:=



$$= \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

1 degree of freedom

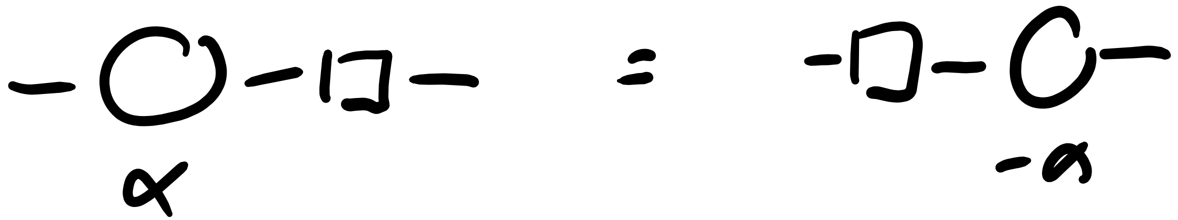


$$= \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & & & \\ & e^{i\alpha_1} & & \\ & & \ddots & \\ & & & e^{i\alpha_{d-1}} \end{pmatrix} \qquad \alpha \in \left(\mathbb{R}/_{2\pi\mathbb{Z}}\right)^{d-1}$$

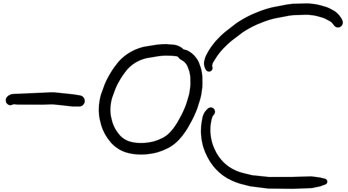
$$\alpha_1, \dots, \alpha_{d-1} \in [0, 2\pi) \mod 2\pi$$


=
α
β
α+β

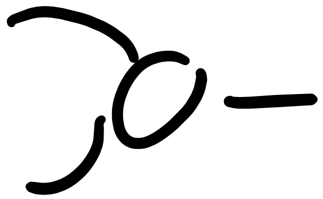
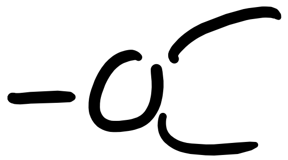

=
α
-α

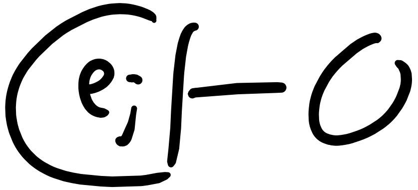
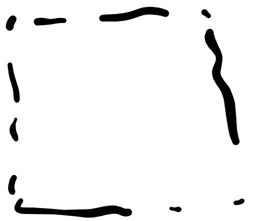
$\ell^2(\mathbb{N})$ $\quad |e_0\rangle, |e_1\rangle, \dots$

ONB $\left(|e_j\rangle\right)_{j\in\mathbb{N}}$

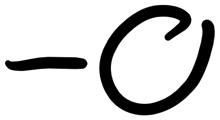
 $:= \sum_{j\in\mathbb{N}} |e_j\rangle|e_j\rangle\langle e_j|$: something

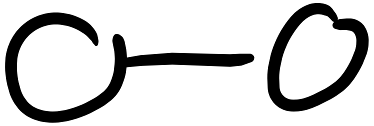
co commutative

 $:= \Big($  $\Big)^{\dagger}$

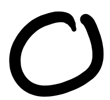
 $= $  $\langle \varphi | e_j \rangle = 1$

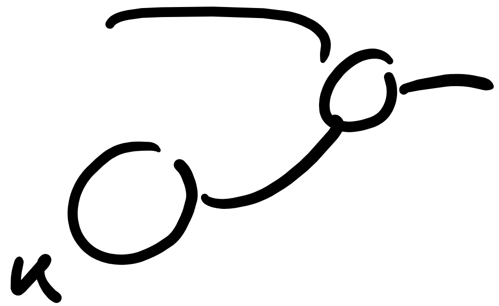
$\langle \varphi |$

 "=" $\sum_{j \in \mathbb{N}} \langle e_j |$

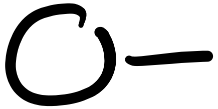
 $= \sum_j \sum_k \langle e_j | e_k \rangle = \sum_k 1 = \dim H$

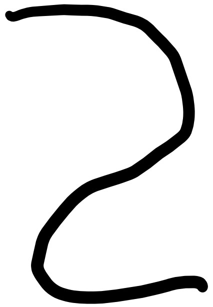
$\dim H = \infty = |\mathbb{N}|$

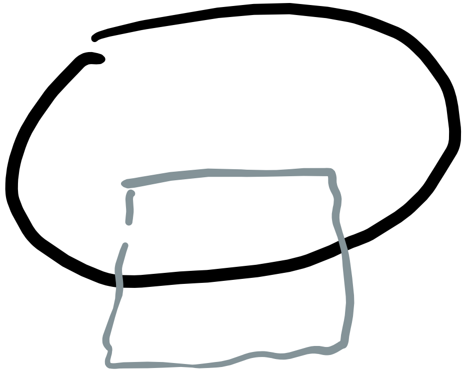
$\sum_k 1 =$ 

$\lim_{n \to \infty}$  $=$ 

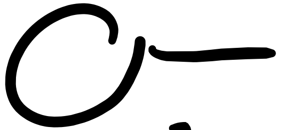
H^*-algebras

 $:= \sum_{j=0}^{n} |e_j\rangle$

 $=$ 

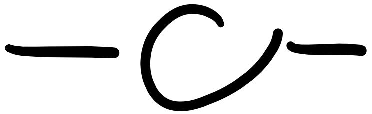
 $= \mathrm{Tr}[\mathbb{1}] = \infty$

$L^2(\mathbb{R})$



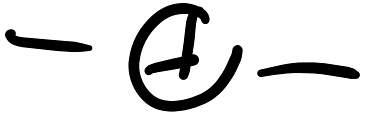
$[p \in \mathbb{R}]$

$[x \in \mathbb{R}]$



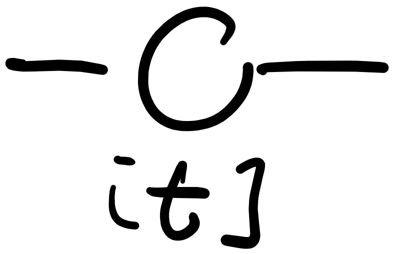
$[p]$

translation by p in momentum

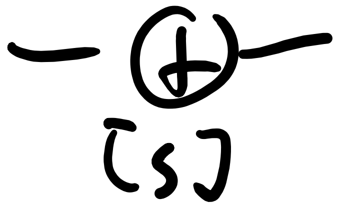


$[x]$

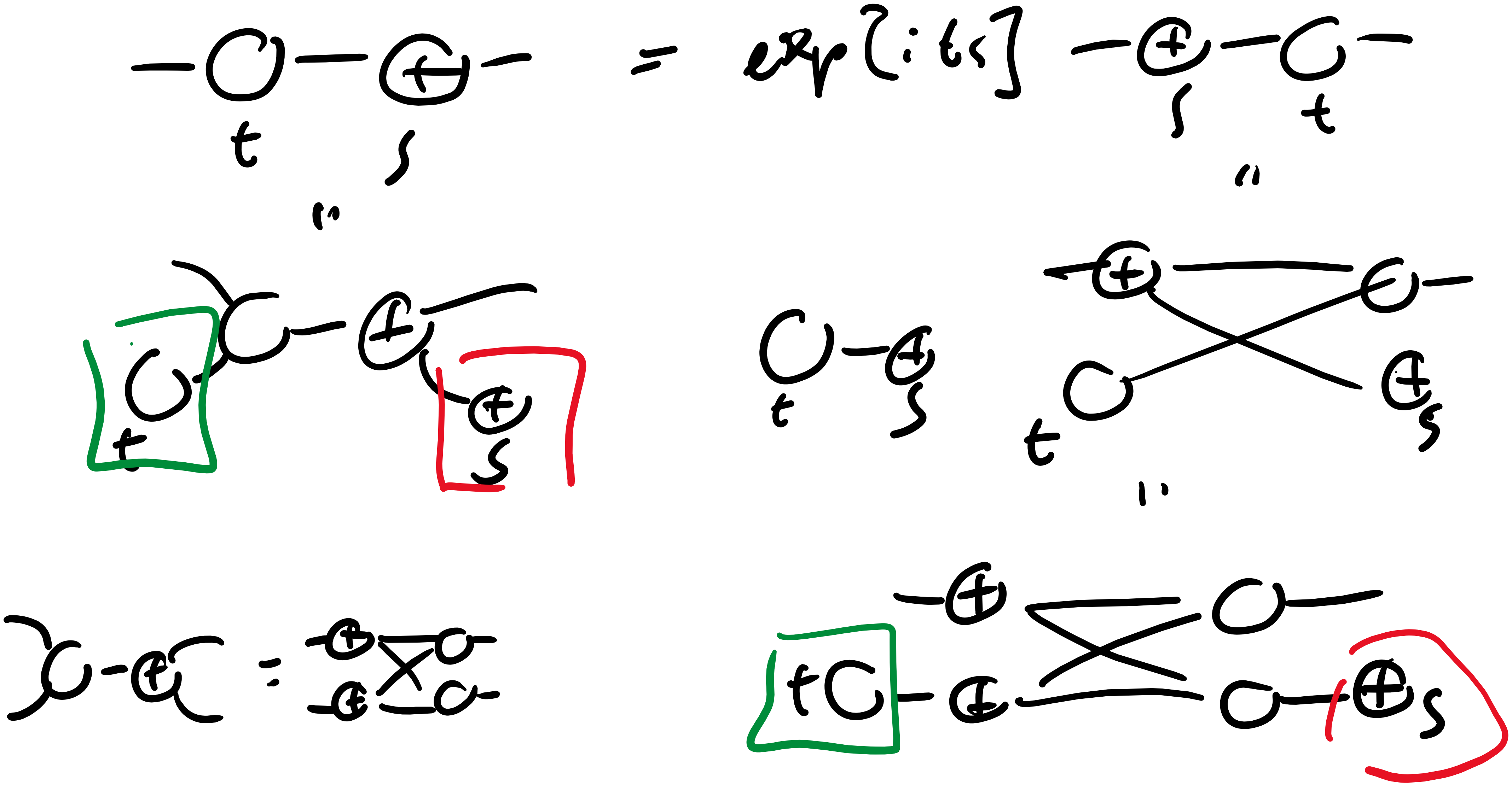
translation by x in position space



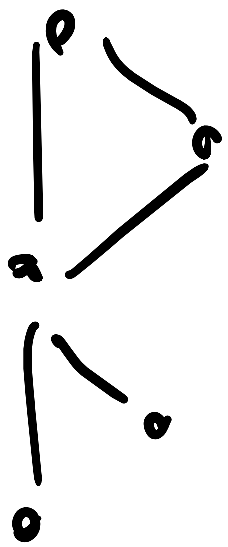
$= \exp[itP]$

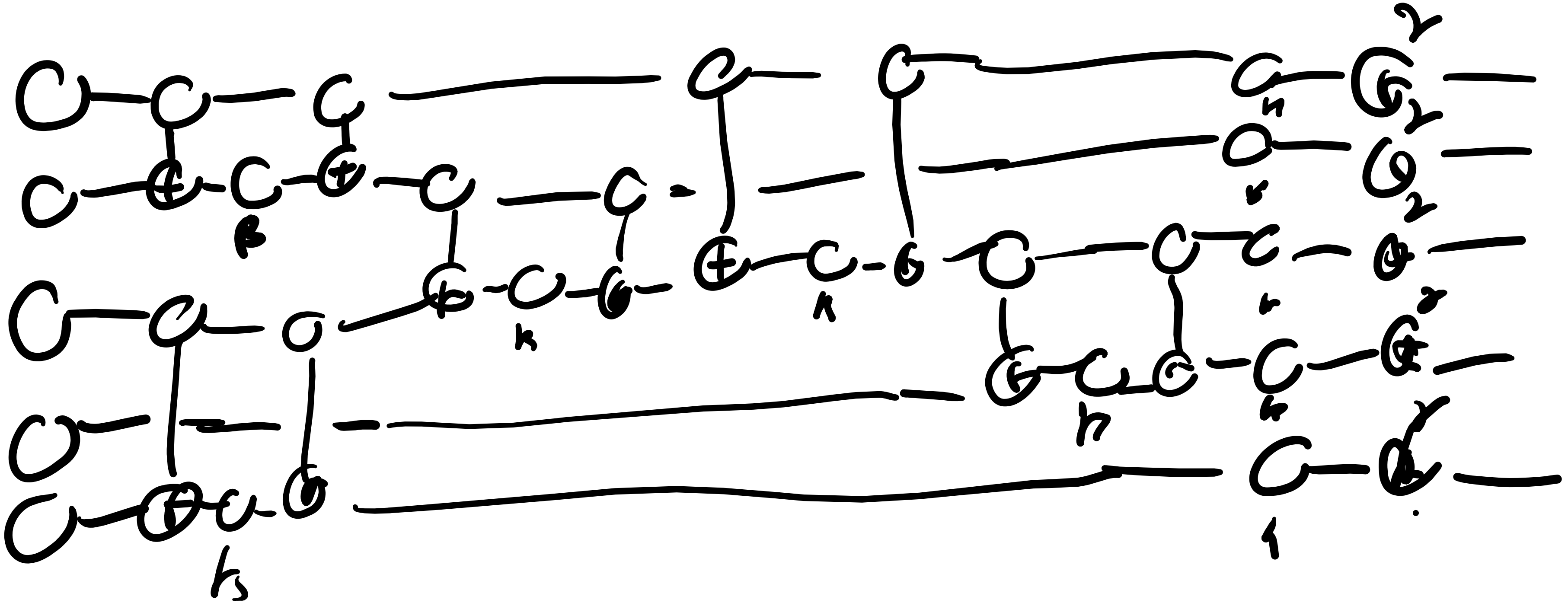


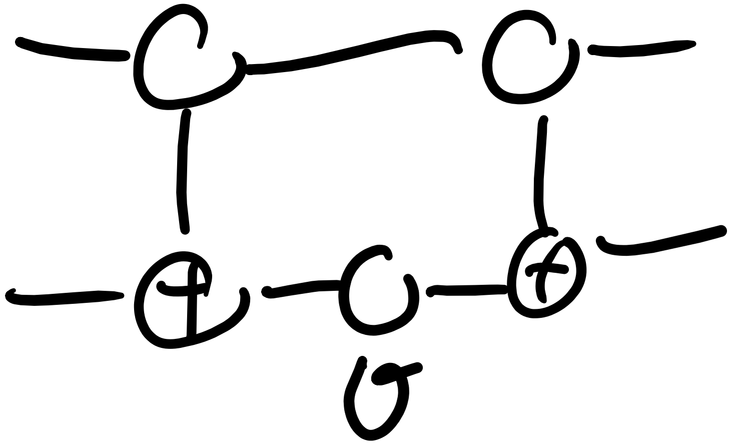
$= \exp[isn]$



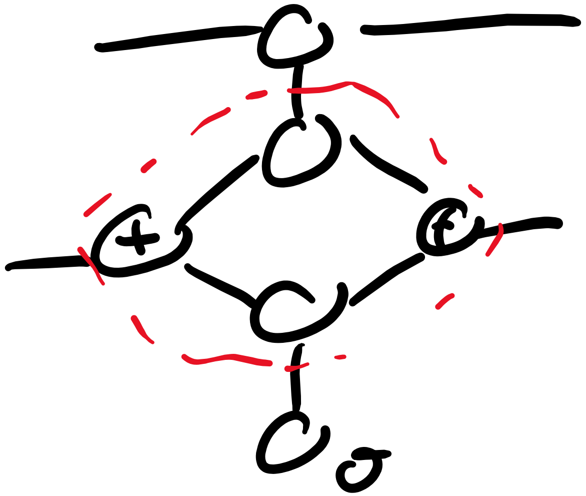
=
exp[its]
t
s
=
=
=
=







Rule =



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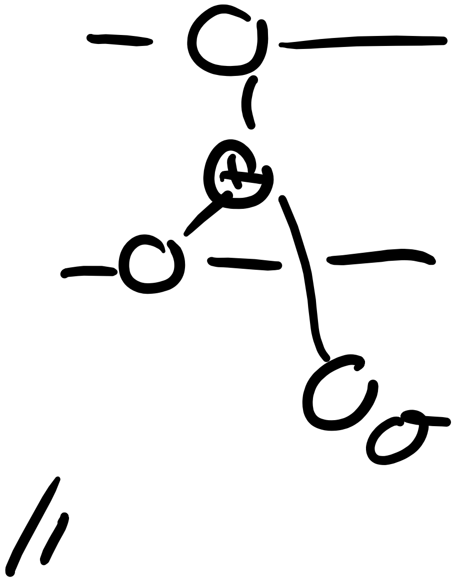
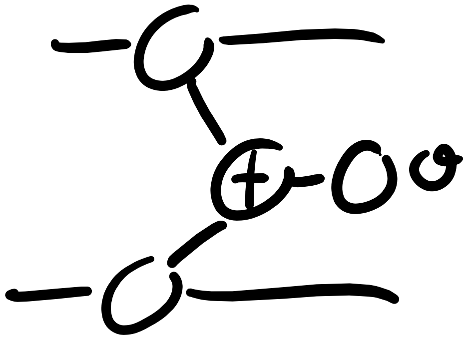
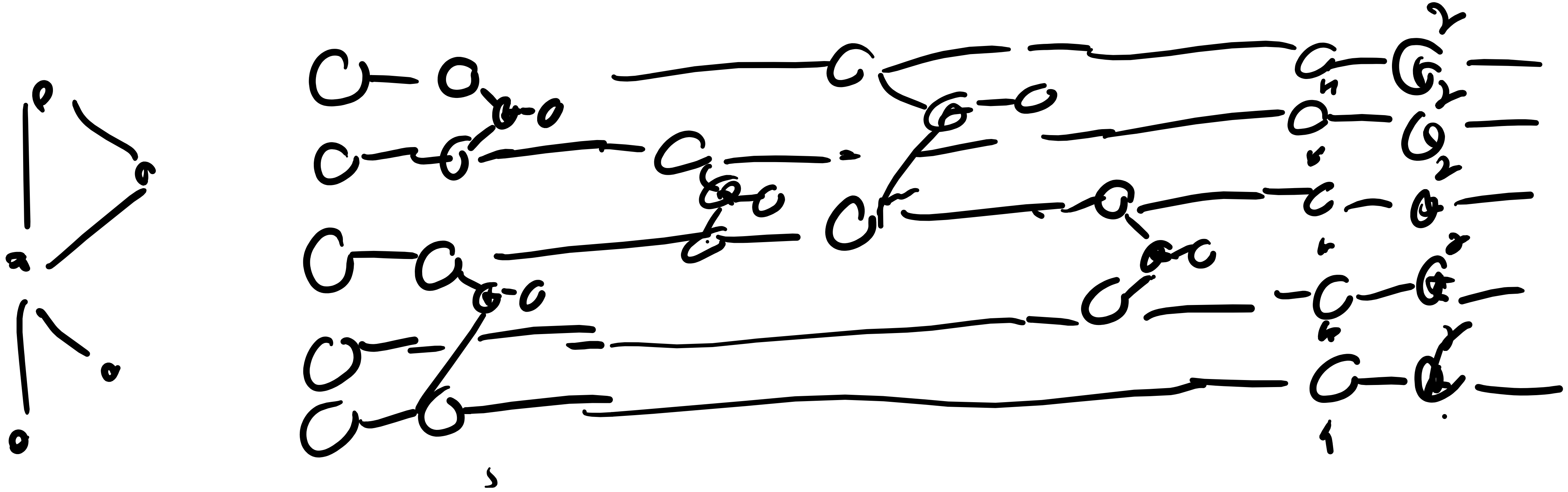
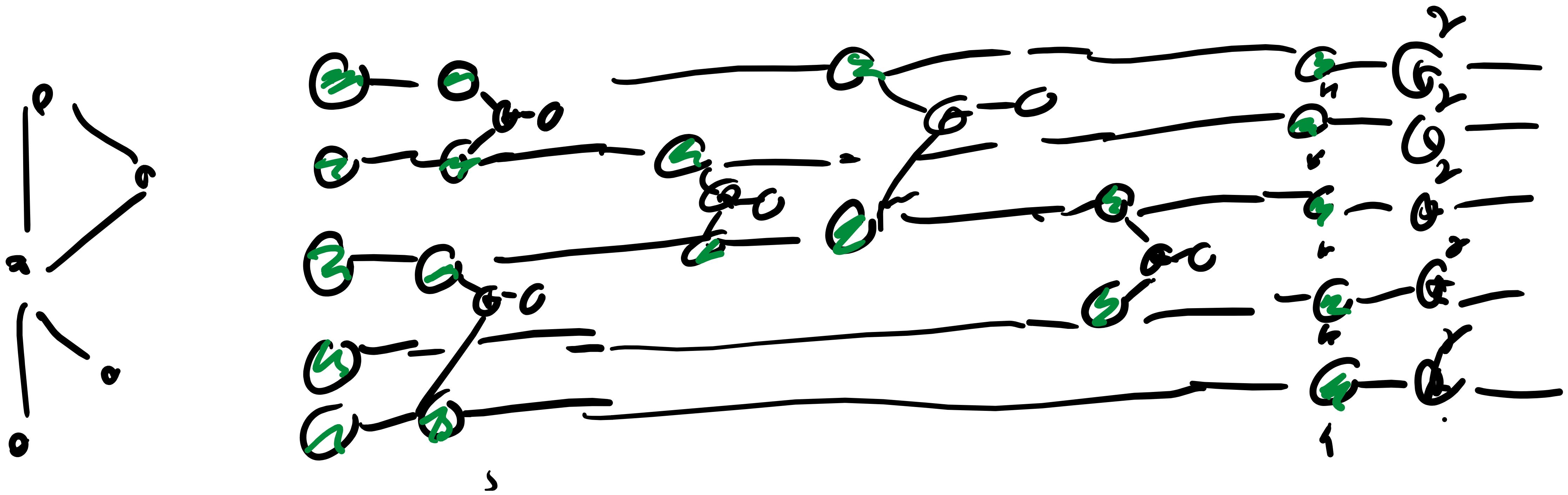
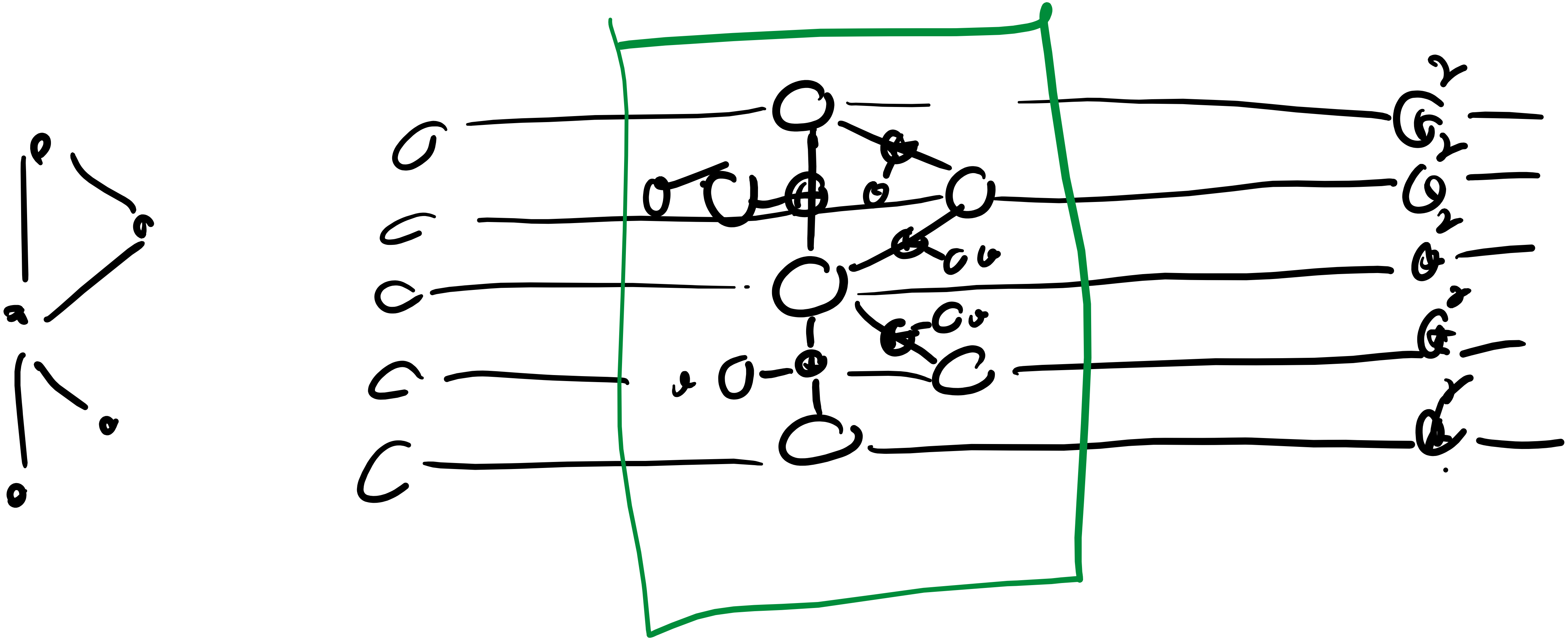


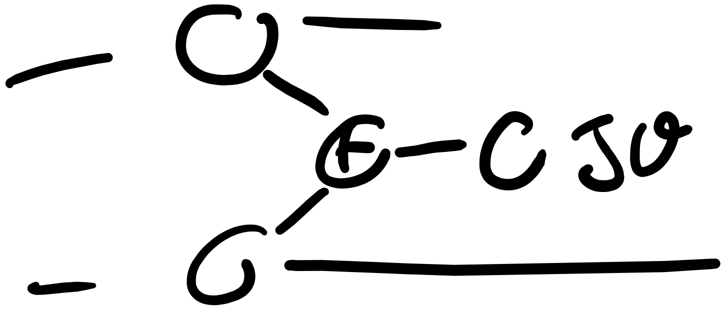


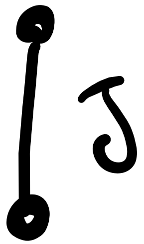
$\exp[i \frac{\theta}{2} ZZ]$

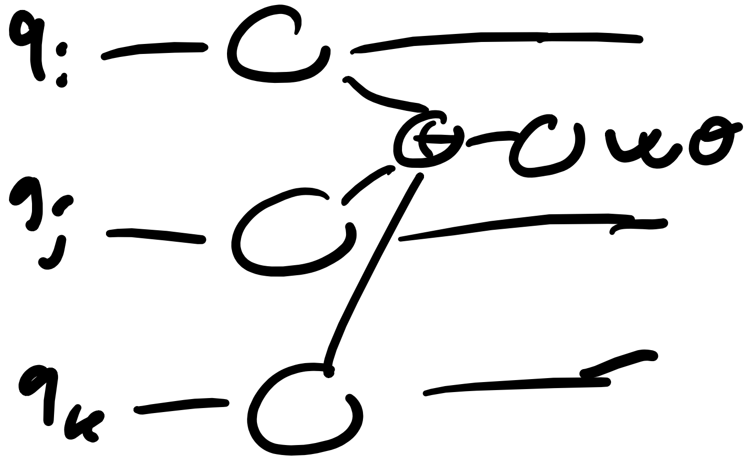


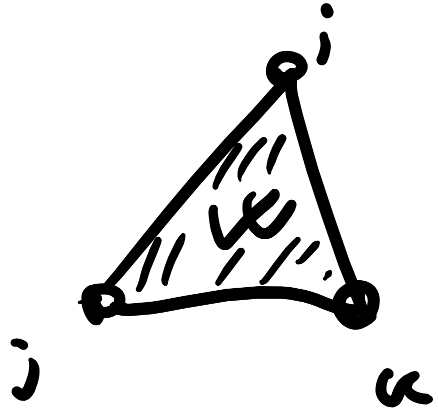


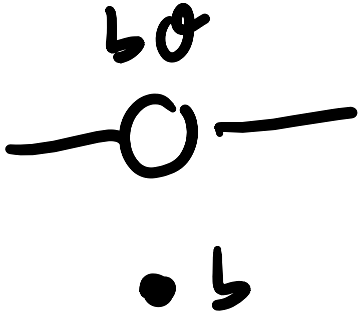


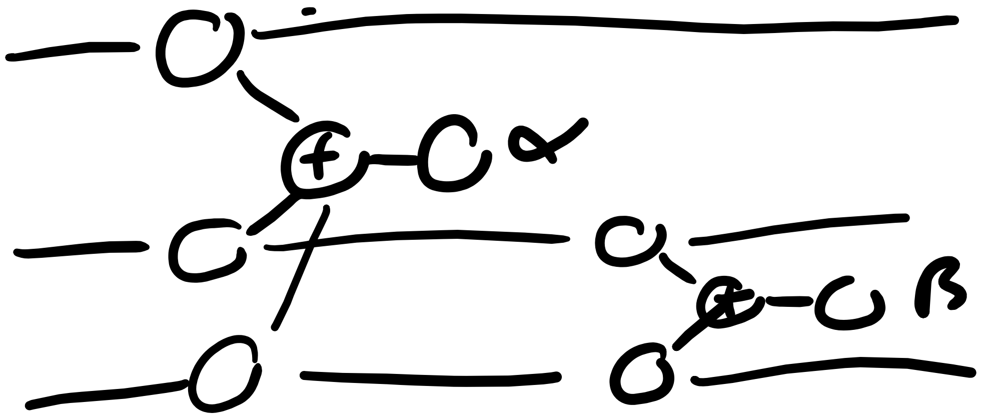

CJθ


J

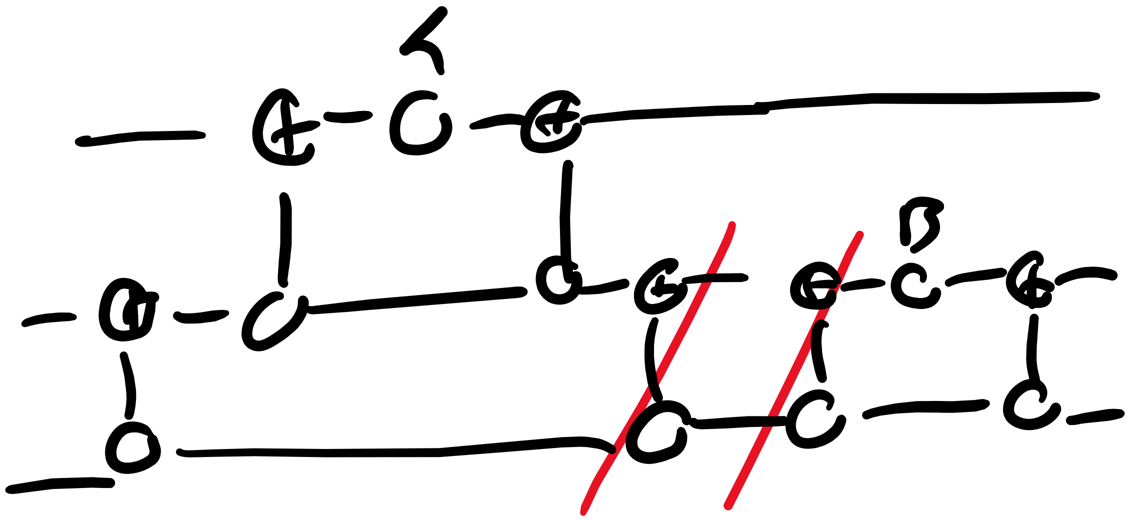

qi
qj
qk
CKθ


i
K
j
k

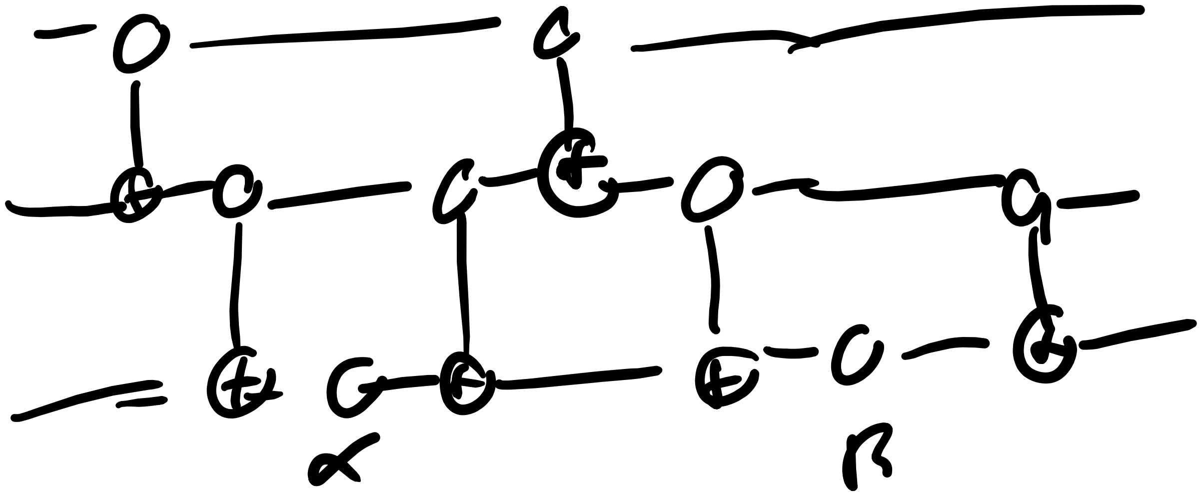

bθ
b

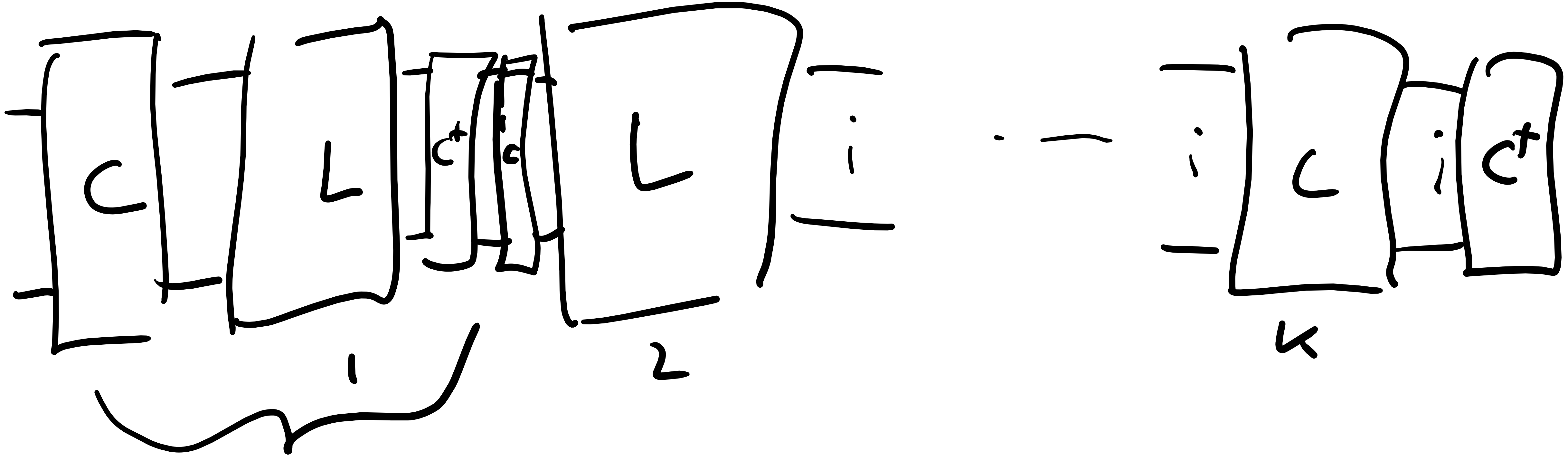

α
β

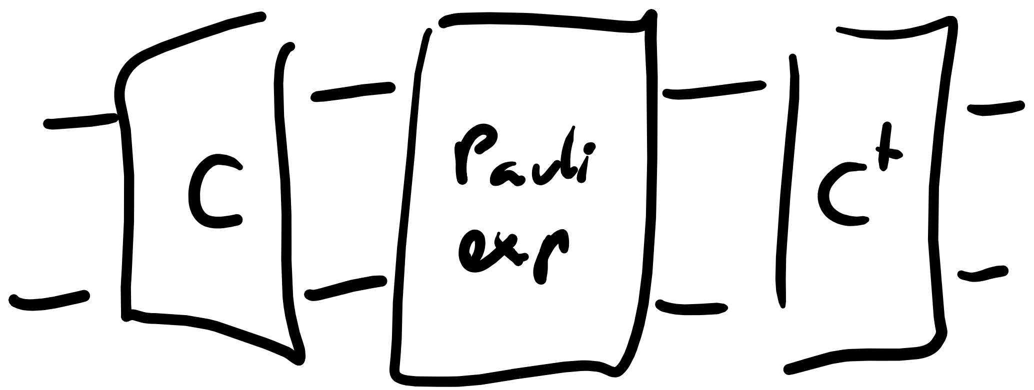
=


α
β

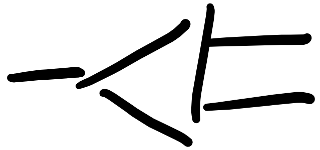
=


α
β

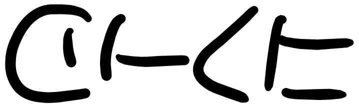

C
L
C†
C
L
1
2
C
C†
K

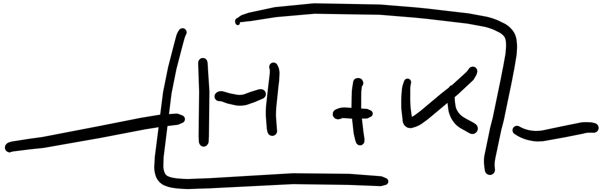

C
Pauli exp
C†

W spider:



$$|00\rangle\langle 0| + \sum_{j>0} \left(|j0\rangle + |0j\rangle \right) \langle j|$$

 $= |01\rangle + |10\rangle$



$=$