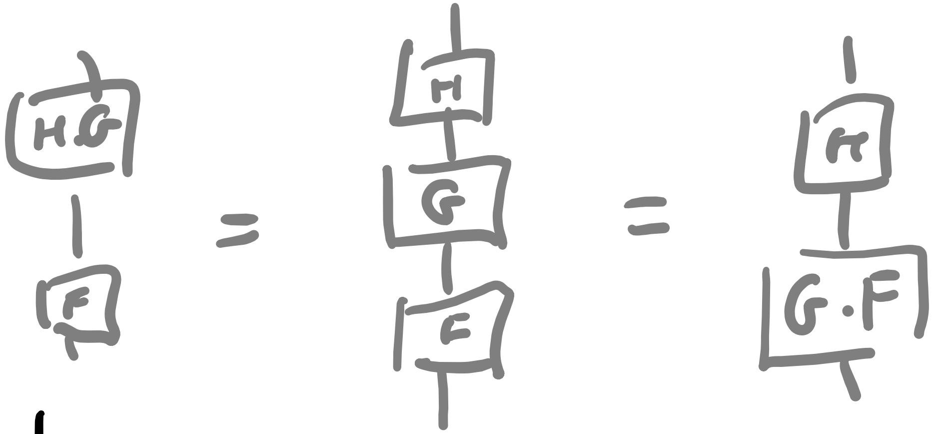
Quantum in Pictures Lecture Series

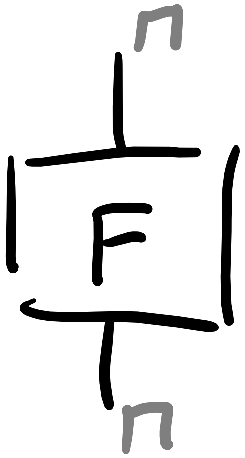
Lecturer: Stefano Gogioso

Mon 26 June 2023 – Afternoon Lecture

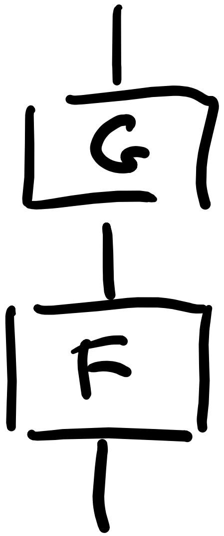
INDIANA UNIVERSITY BLOOMINGTON

$(\Pi, \cdot, 1)$





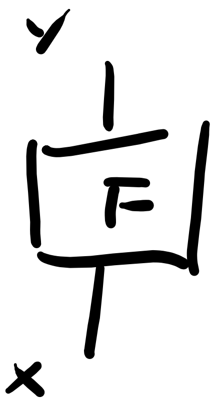
$F \in \Pi$

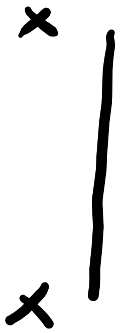


$:= G \cdot F$

$:= 1$

$(\mathrm{Set}, \circ, \mathrm{id})$ Category





$:= id_x$

$= G \cdot F$

$x, y \in \mathrm{Set}$

$F : x \to y$

$\{x \mapsto x\}$

$\{x \mapsto G(F(x))\}$

$$X \times Y \longrightarrow Z \times W$$

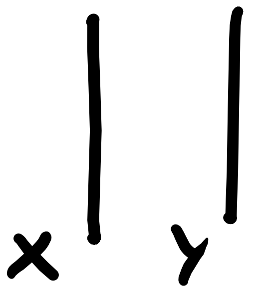
$$\mathrm{swap}: X \times Y \longrightarrow Y \times X$$
$$(x, y) \longmapsto (y, x)$$

$f : X \to Z$

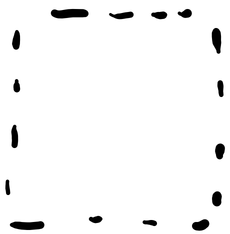
$g : Y \to W$

$$f \times g : X \times Y \longrightarrow Z \times W$$
$$(x, y) \longmapsto (f(x), g(y))$$

$(\mathrm{Set}, \cdot, id, \times, \{*\})$ Monoidal Category

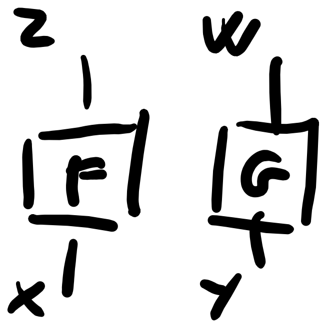


$:= id_{X\times Y}$

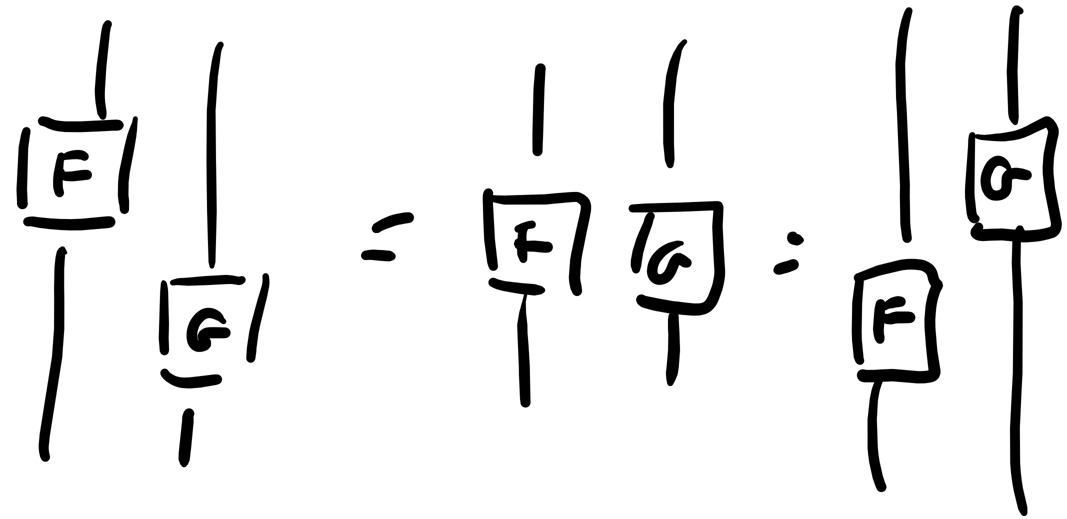


$:= id_{\{*\}}$

$X \times \{*\} \cong X$

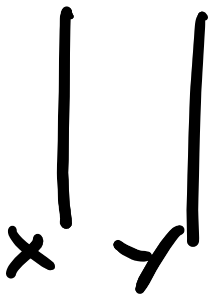


$:= F\times G$

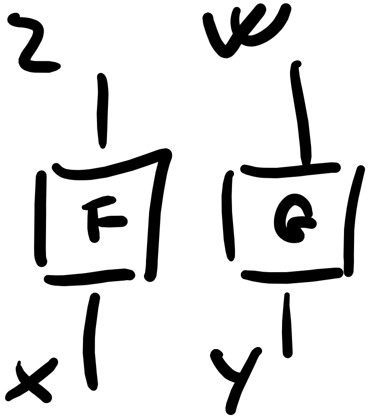


$(F\times id)\cdot(id\times G)$ $F\times G$ $(id\times G)\cdot(F\times id)$

$(\mathbf{Set}, \cdot, \mathrm{id}, \sqcup, \emptyset)$ Monoidal



$:= \mathrm{id}_{X \sqcup Y}$

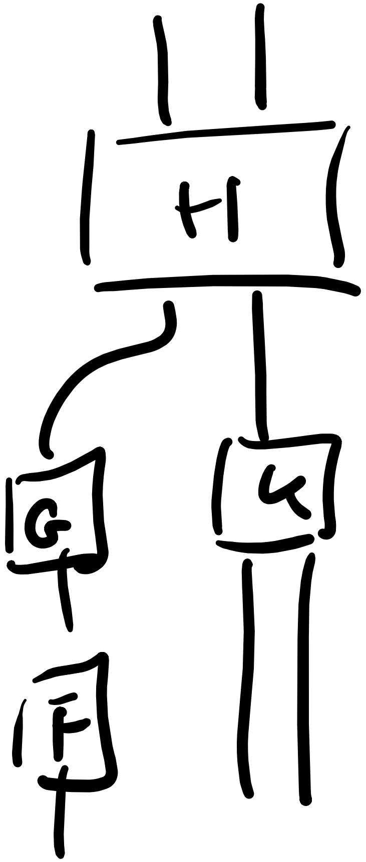


$:= F \sqcup G$

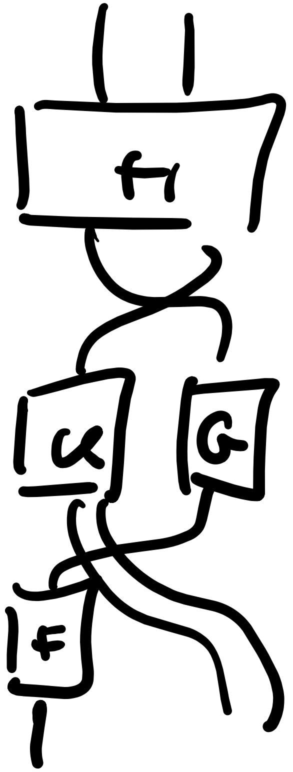
$$F \sqcup G : X \sqcup Y \to Z \sqcup W$$

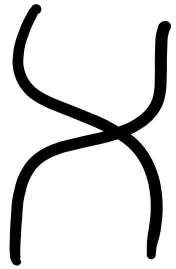
$$x \in X \mapsto F(x) \in Z$$

$$y \in Y \mapsto G(y) \in W$$

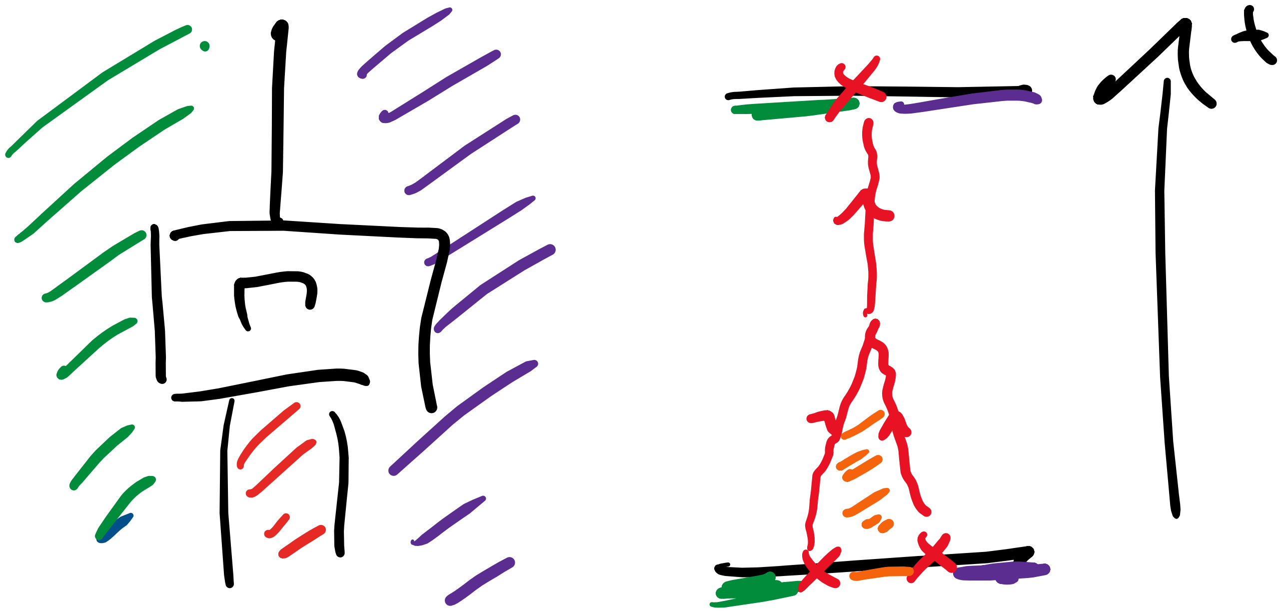


$\overset{?}{=}$

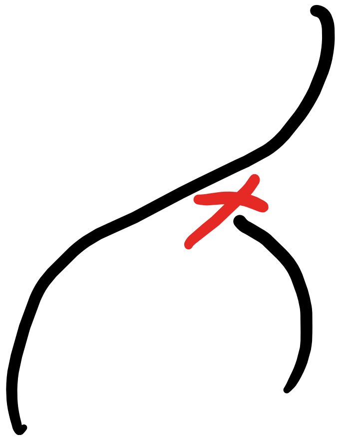




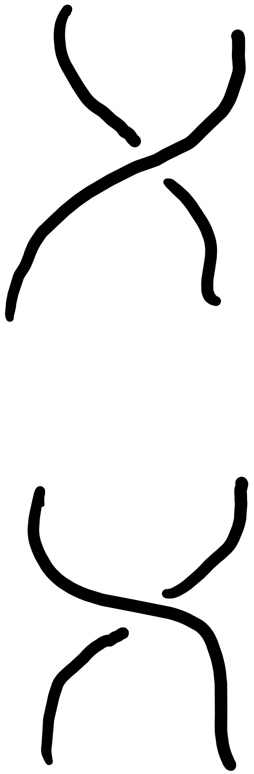
$= ?$


t

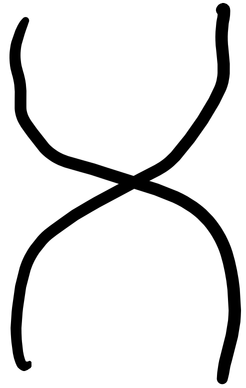
2D

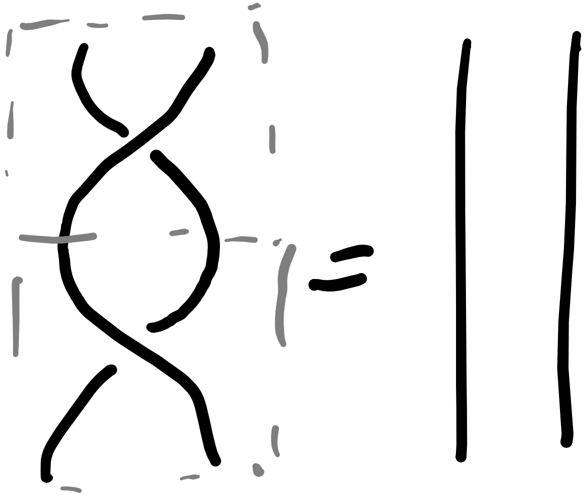


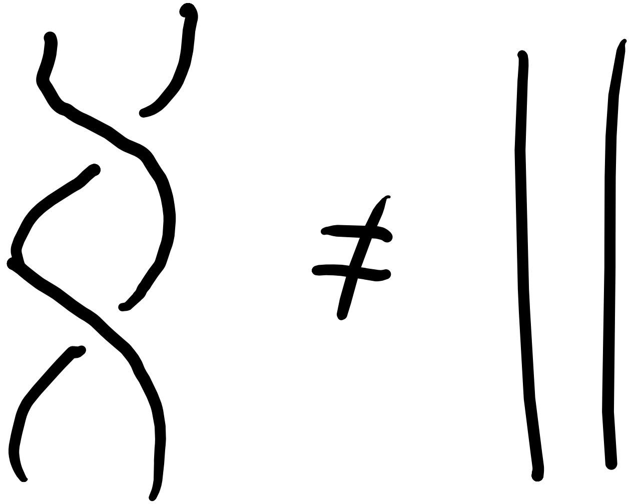
3D



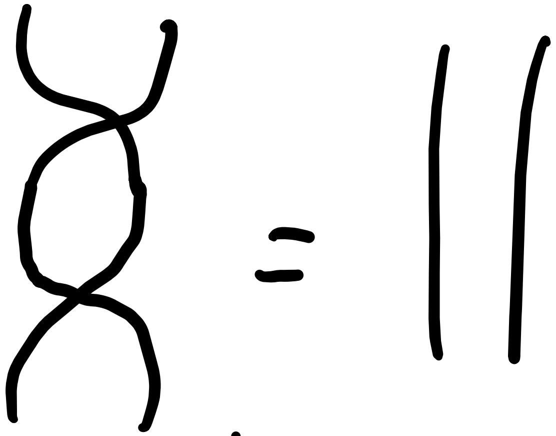
4D





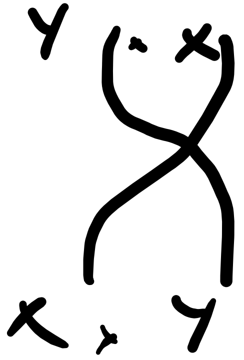


in 3D



in 4D

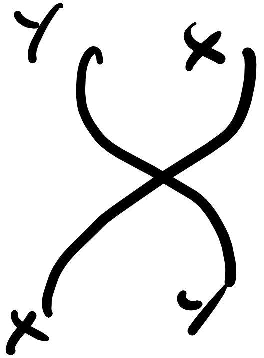
$(\times, \{*\})$



$X \times Y \longrightarrow Y \times X$

$(x, y) \longmapsto (y, x)$

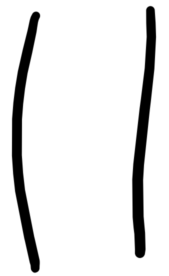
$(\sqcup, \emptyset)$

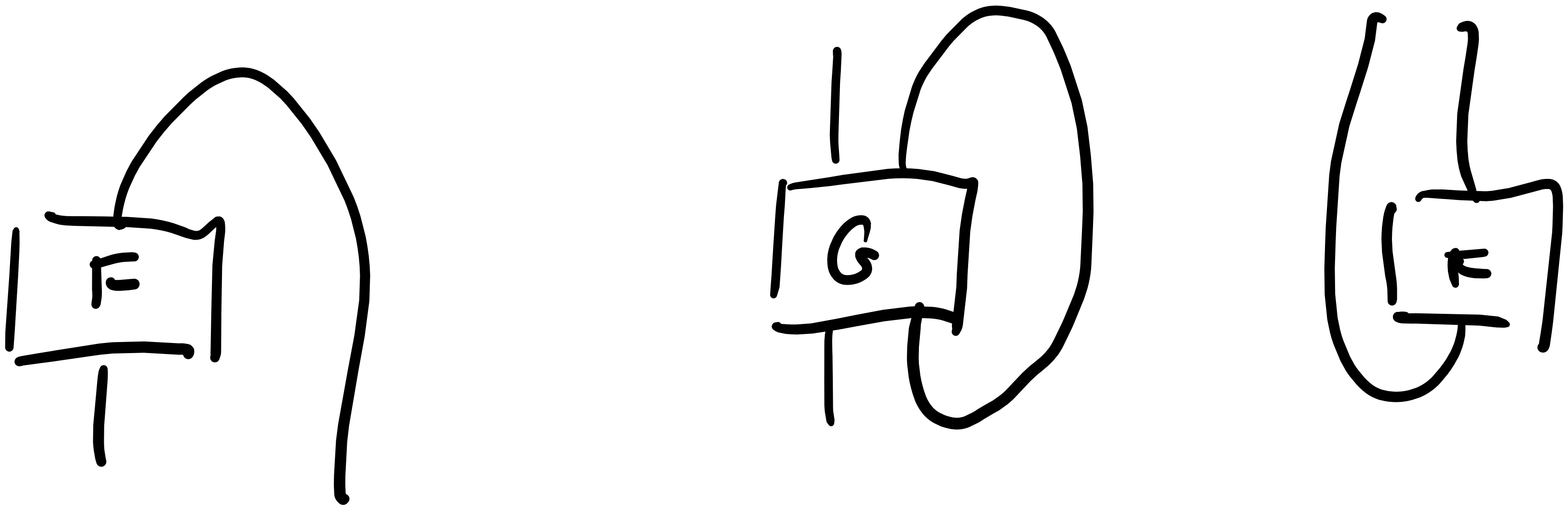


$X \sqcup Y \longrightarrow Y \sqcup X = X \sqcup Y$

$x \in X \longmapsto x$

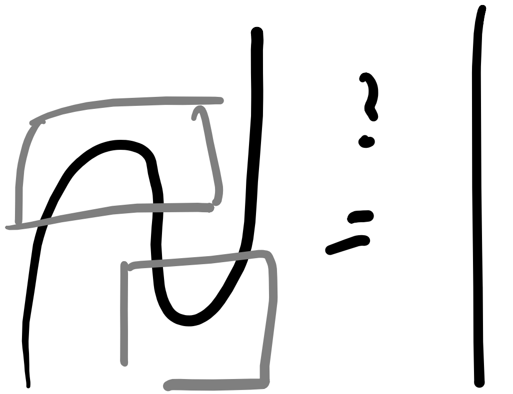
$y \in Y \longmapsto y$

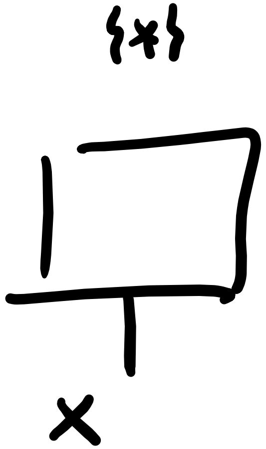
$=$ 


F
G
F

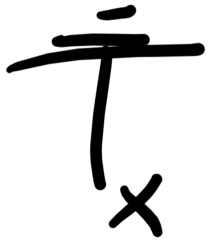

=
=

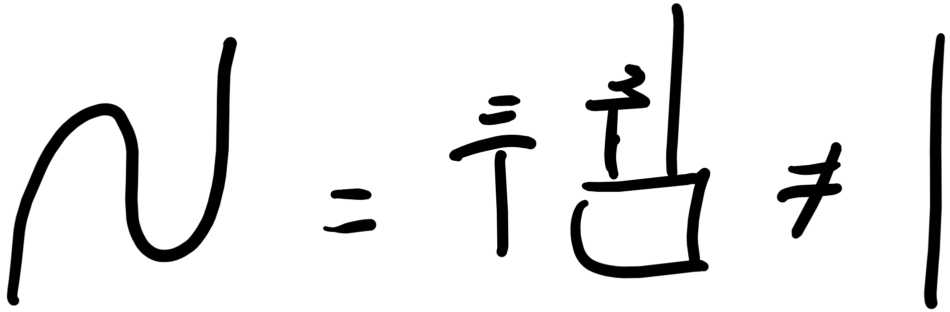
Set_X



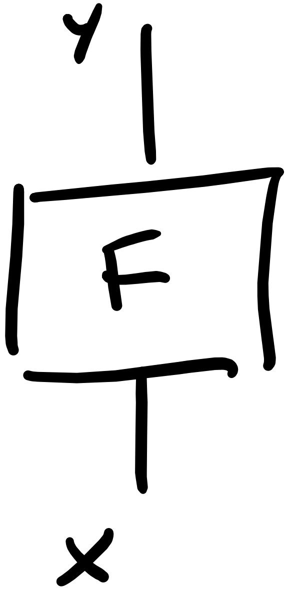


$= x \mapsto *$





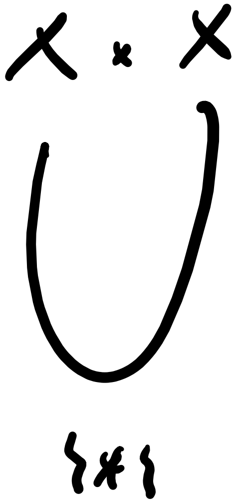
Set is not compact closed



$F \subseteq X \times Y$ $\qquad$ $F \in \{0,1\}^{Y \times X}$

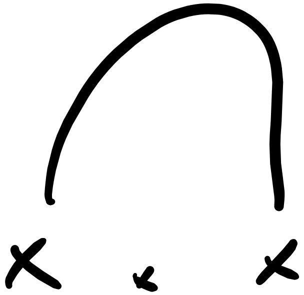
$$(\mathrm{Rel}, \circ, 1, \times, \{*\})$$

Relation on $\{*\}$: $\begin{cases} \{*\} \leftrightarrow 1 \\ \emptyset \leftrightarrow 0 \end{cases}$



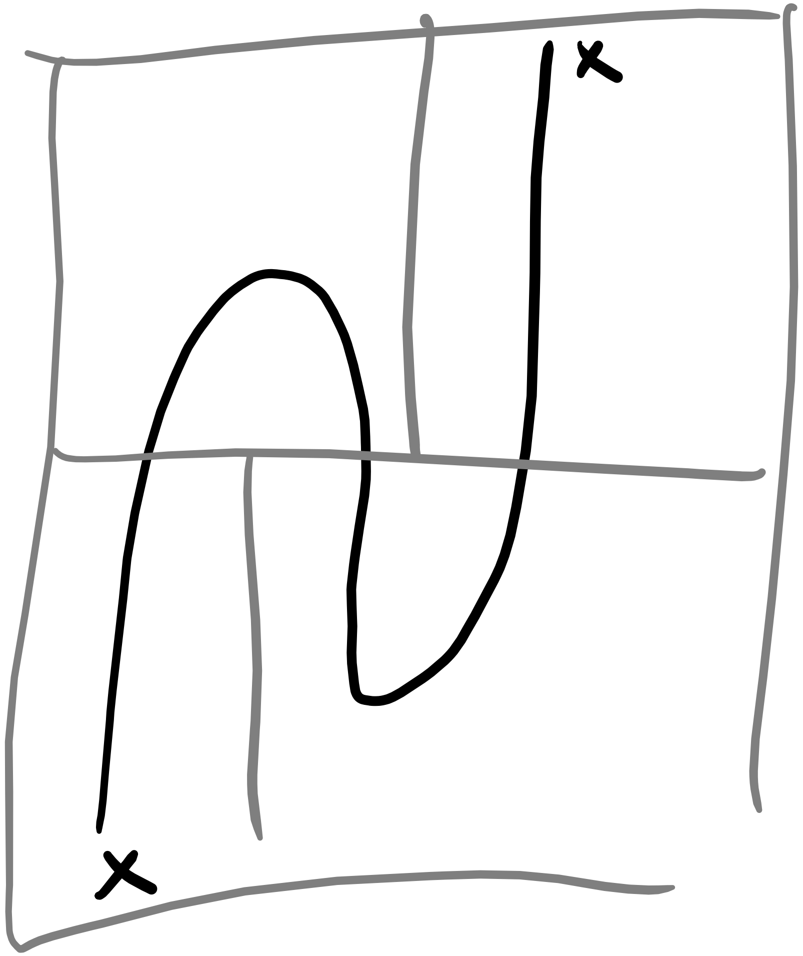
$$\{ * \longmapsto (x,x) \mid x \in X \}$$

Subset of $X \times X$



$$\{ (x,x) \mapsto * \mid x \in X \}$$

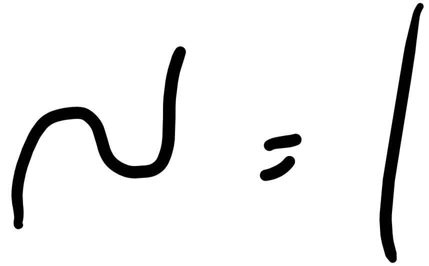
Partial function $X \times X \to \{*\}$

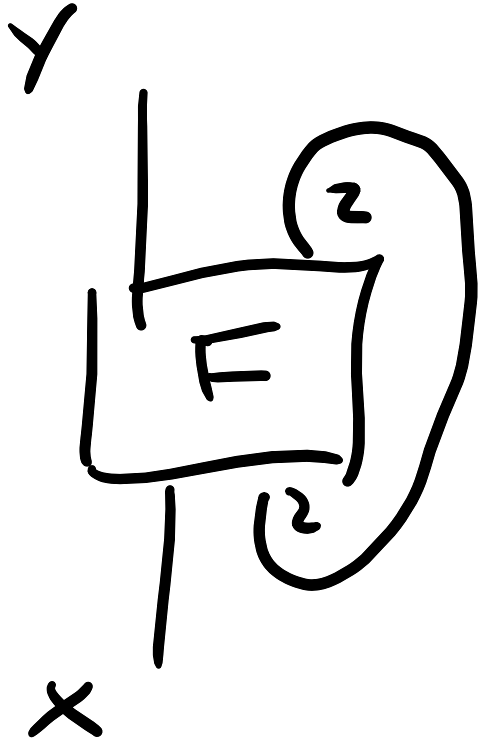


$$\{(x,x,y) \mapsto y \mid x,y \in X\}$$

$$\{x \mapsto (x,y,y) \mid y \in X\}$$

$$= \{x \mapsto x \mid x \in X\} = \mathrm{id}_X$$





$x \mapsto F(x,z)_Y$ for some $z \in Z$

s.t. $F(x,z)_Z = z$

Partial function

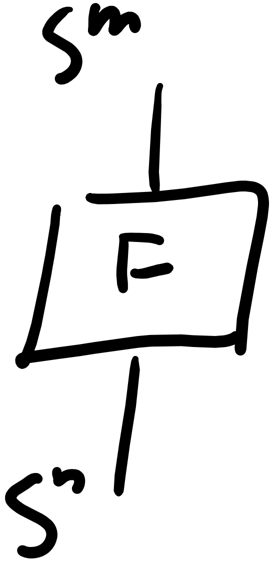
$$\begin{pmatrix} .75 & 1 \\ .25 & 0 \end{pmatrix} \xrightarrow[\;x>0 \,\mapsto\, 1\;]{\;0 \,\mapsto\, 0\;} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$(\mathbb{R}^+, +, 0, \cdot, 1) \qquad\qquad (\{0,1\}, \vee, 0, \wedge, 1)$$

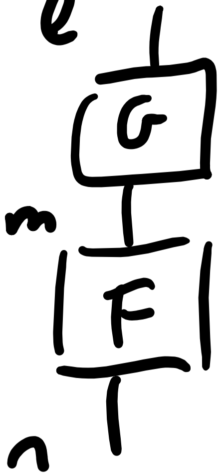
$(S, +, 0, \cdot, 1)$

$S = \mathbb{R}^+, \mathbb{R}, \mathbb{C}, \mathbb{B}, \ldots$

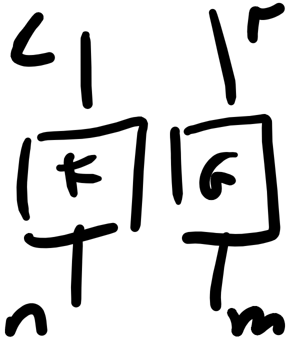
$n, m \in \mathbb{N}^+$



$F \in S^{m \times n}$



$:= G \cdot F$



$:= F \otimes G$

$$(\mathbb{C}\text{-Mat}, \cdot, 1, \otimes, \mathbb{C})$$

Multiplicative

$$\begin{array}{ccc} |0\rangle & \otimes & |+\rangle \\ \begin{pmatrix} 1 \\ 0 \end{pmatrix} & \otimes & \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \cdot \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \\ 0 \cdot \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \end{pmatrix}$$

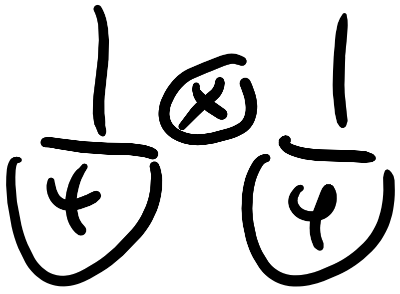
$$(\mathbb{C}\text{-Mat}, \cdot, 1, \oplus, \{1\})$$

Additive

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \oplus \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \\ \hline 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \end{pmatrix}$$

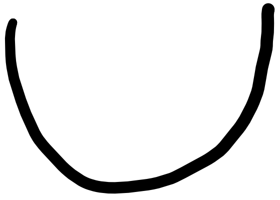
$$\mathbb{C}^n \otimes \mathbb{C}^m = \mathbb{C}^{n \times m}$$



fields

Allows Entanglement

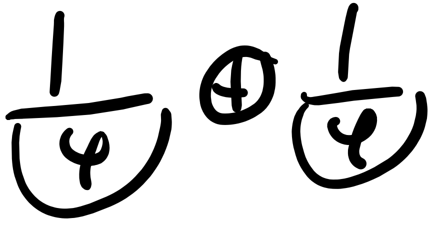
$\frac{1}{\sqrt{2}}$

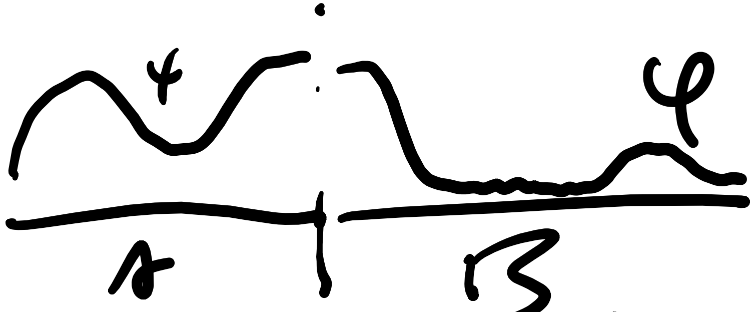


$$\mathbb{C}^n \oplus \mathbb{C}^m$$

$$\mathbb{C}^{n+m}$$

$\mathbb{C}^A$ $\mathbb{C}^B$





gluing of wavefunctions

$$\bigotimes_{x \in X} \mathcal{H}_x$$

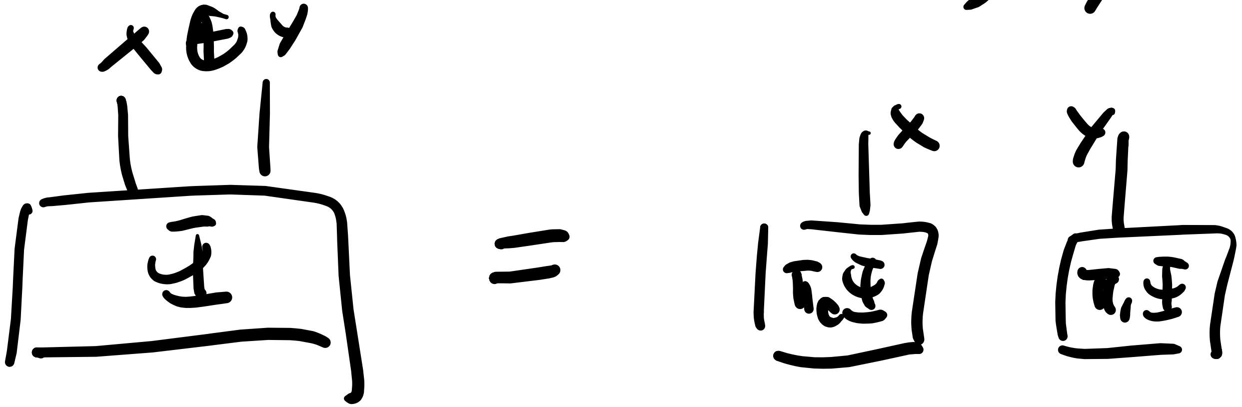
$$\bigoplus_{x \in X} \mathbb{C} \cong L^2[X]$$

$$\bigoplus_{x \in X} H_x \oplus \bigoplus_{y \in X} H_y = \bigoplus_{x \in X \sqcup Y} H_x$$

$$(\mathbb{C}\text{-Mat}, \cdot, 1, \oplus, 0)$$

$$\left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right)$$

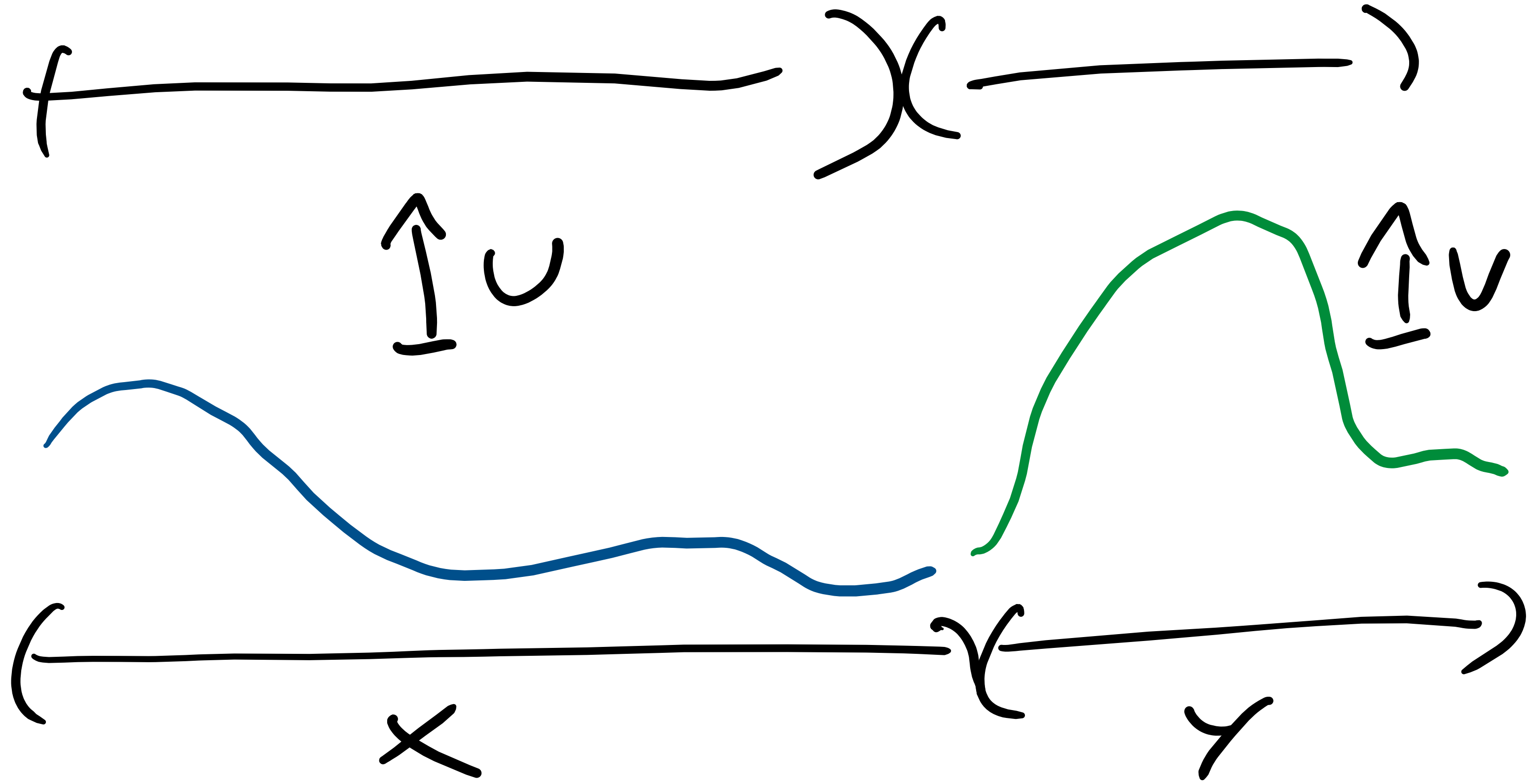
$$
\begin{tikzcd}
X \arrow[dr, "i_0"] & & X \\
& X \oplus Y \arrow[ur, "\pi_0"] \arrow[dr, "\pi_1"] & \\
Y \arrow[ur, "i_1"] & & Y
\end{tikzcd}
$$



No Entanglement!

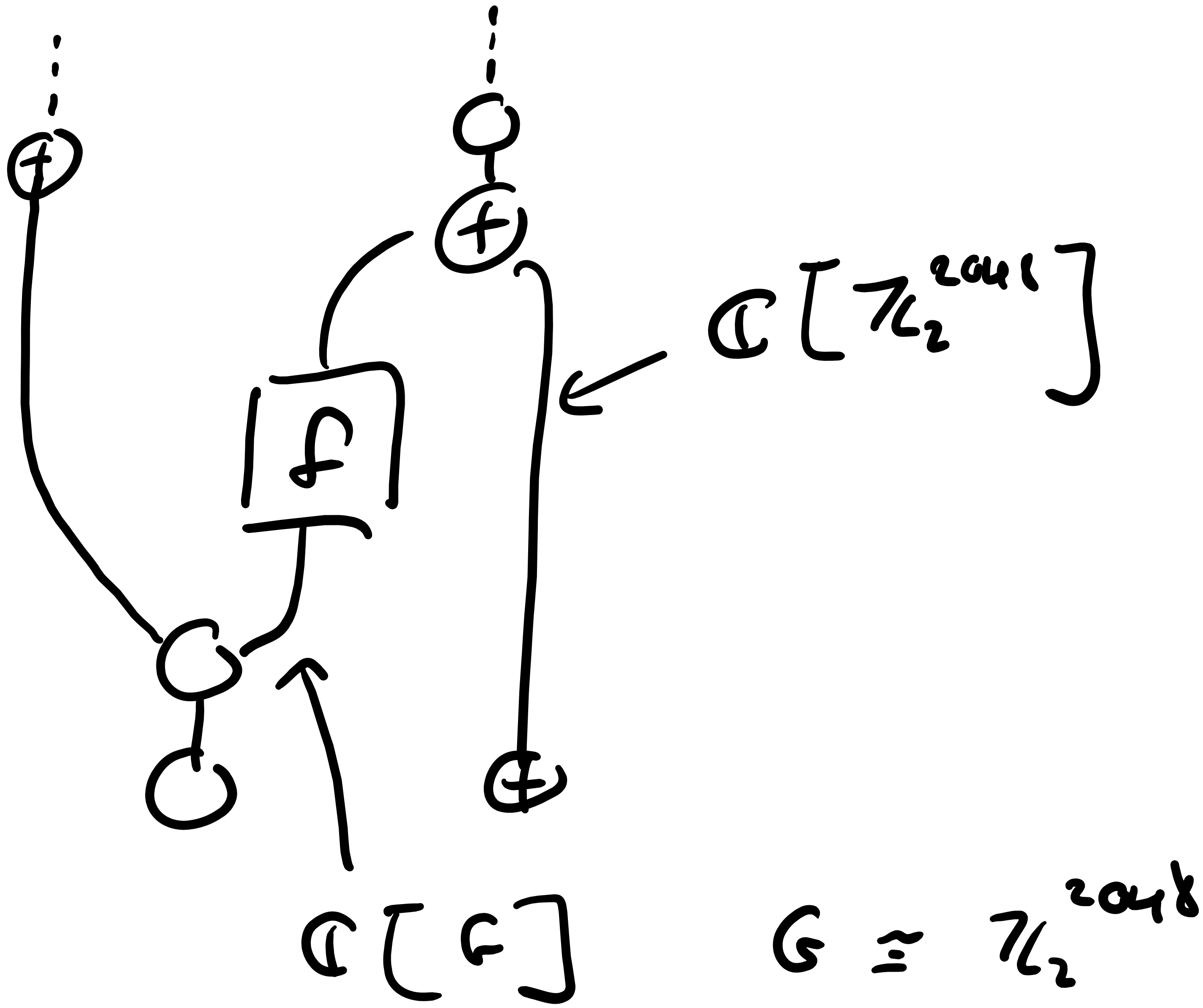
$$\mathbb{C}^X \oplus \mathbb{C}^Y \hat{=} \mathbb{C}^{X \cup Y}$$

$$F \oplus G = \left(\begin{array}{c|c} F & \\ \hline & G \end{array}\right)$$

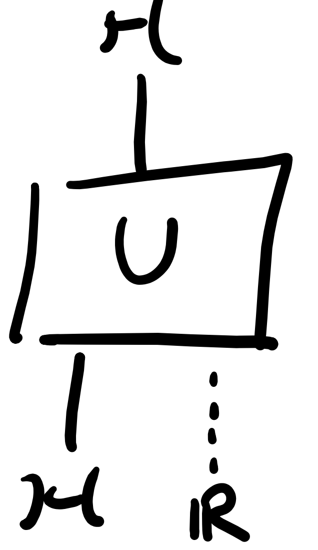


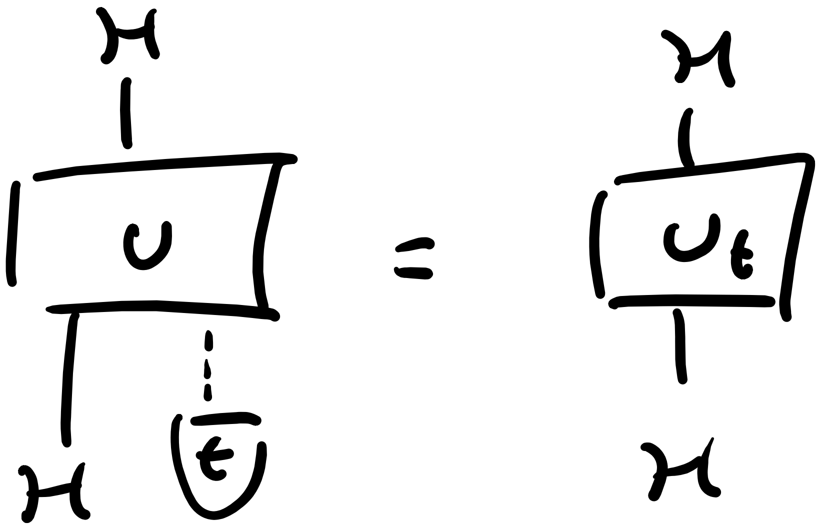
$$U \otimes V \qquad \begin{pmatrix} u_{00}\cdot V & u_{01}\cdot V \\ u_{10}\cdot V & u_{11}\cdot V \end{pmatrix}$$

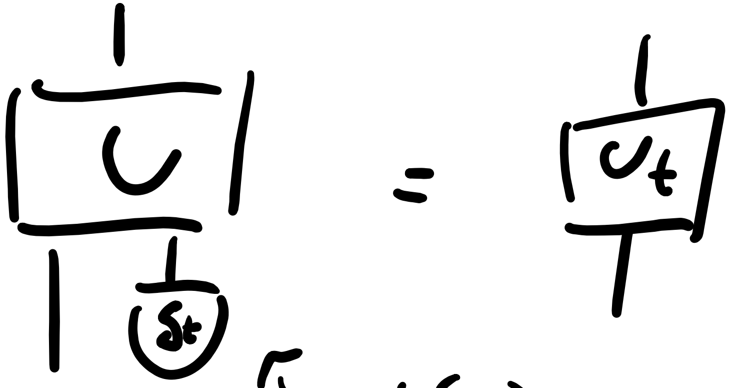
$$U \oplus V \qquad \left(\begin{array}{c|c} U & \\ \hline & V \end{array}\right)$$


$\mathbb{C}[\mathbb{Z}_2^{2041}]$
$\mathbb{C}[G]$
$G \cong \mathbb{Z}_2^{2048}$

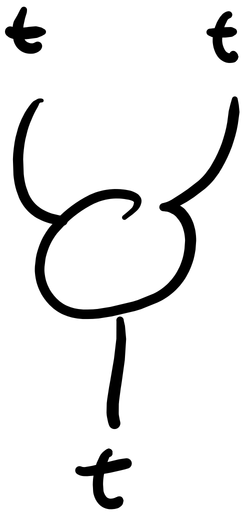
$\mathcal{H}$ Hilbert space $\qquad U_t := e^{itH}$

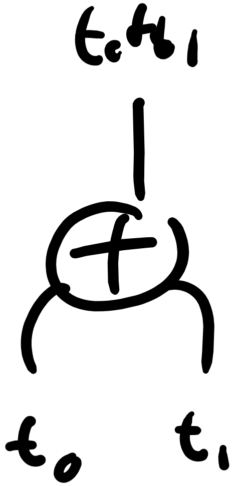


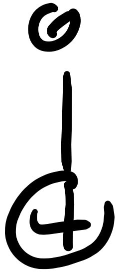




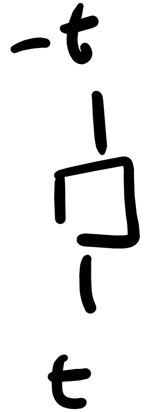
$|\delta_t\rangle$

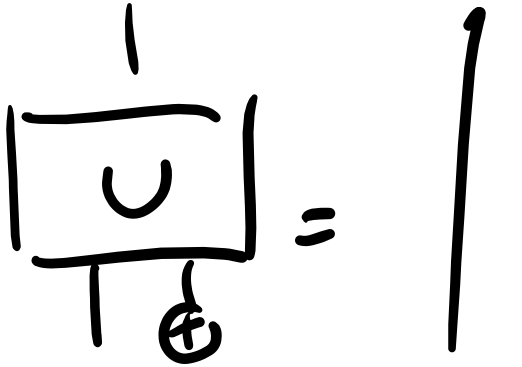




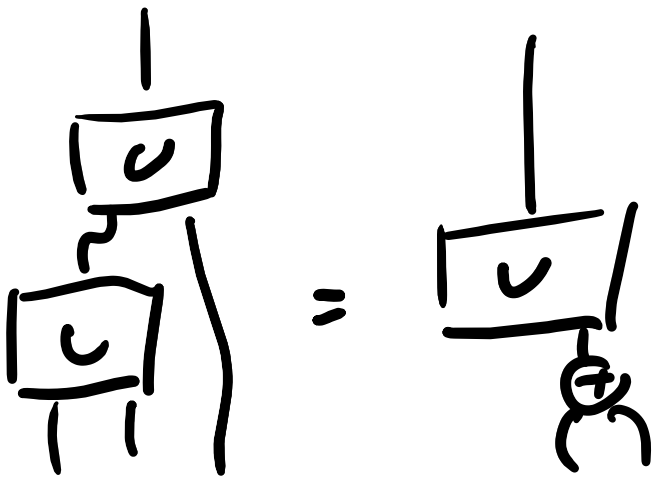


$\mathbb{C}[\mathbb{R}]$

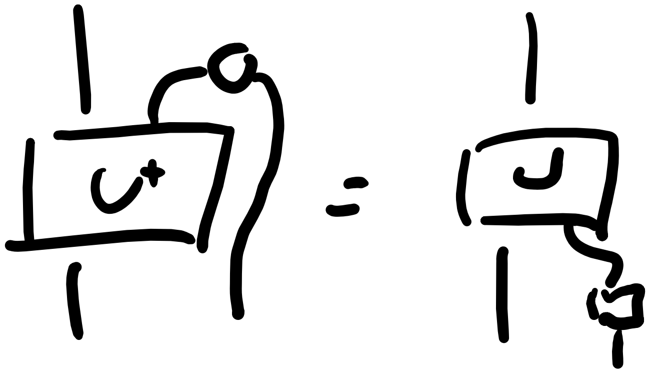




$U = id_0$

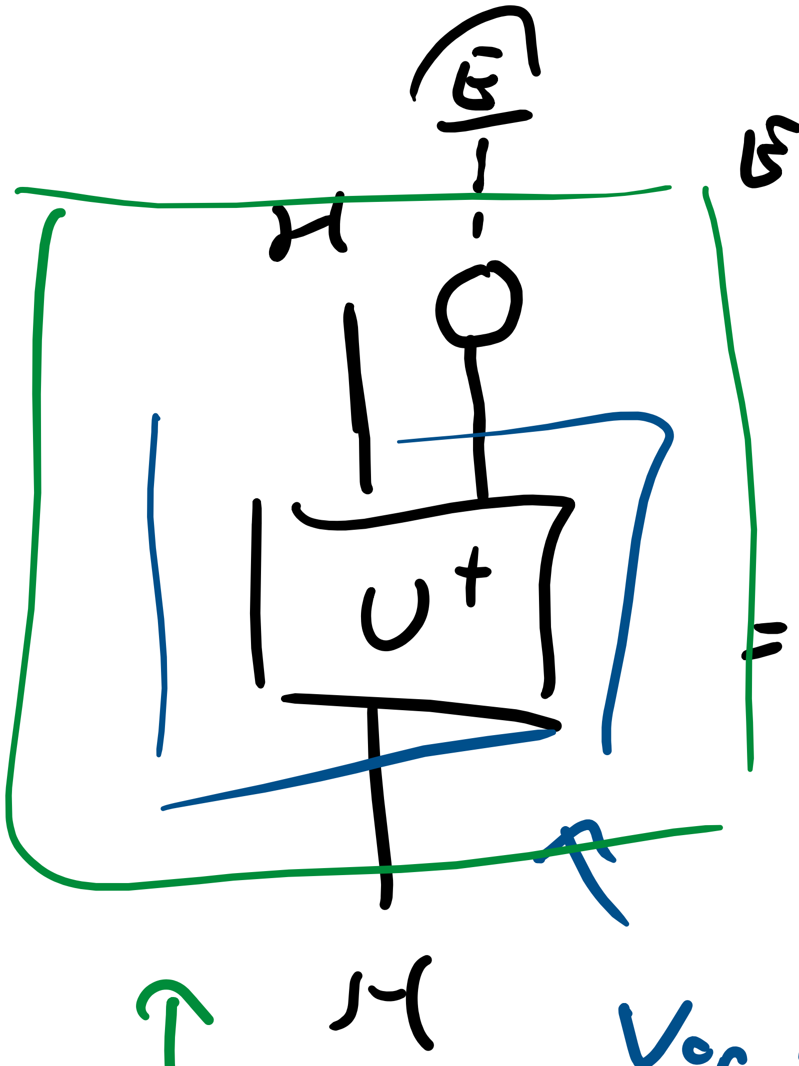


$U_t \cdot U_s = U_{t+s}$

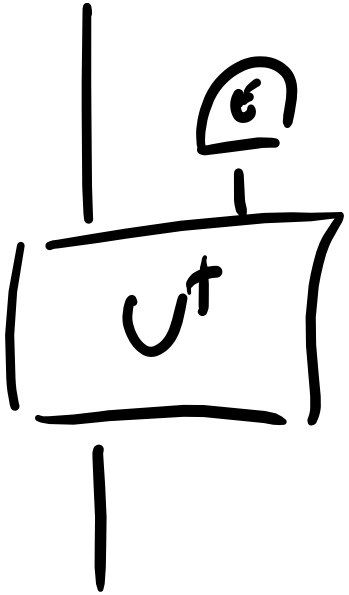


$U_t^{\dagger} = U_{-t}$

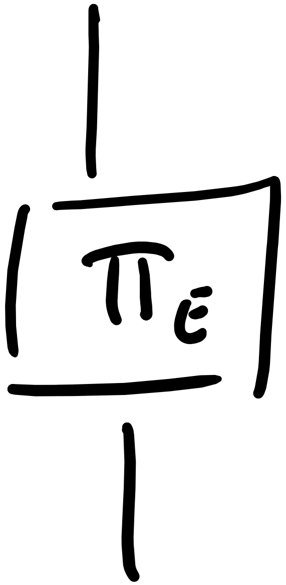
$\varepsilon \in \hat{\mathbb{R}} \cong \mathbb{R}$



$=$



$\overset{\text{(proof)}}{=}$

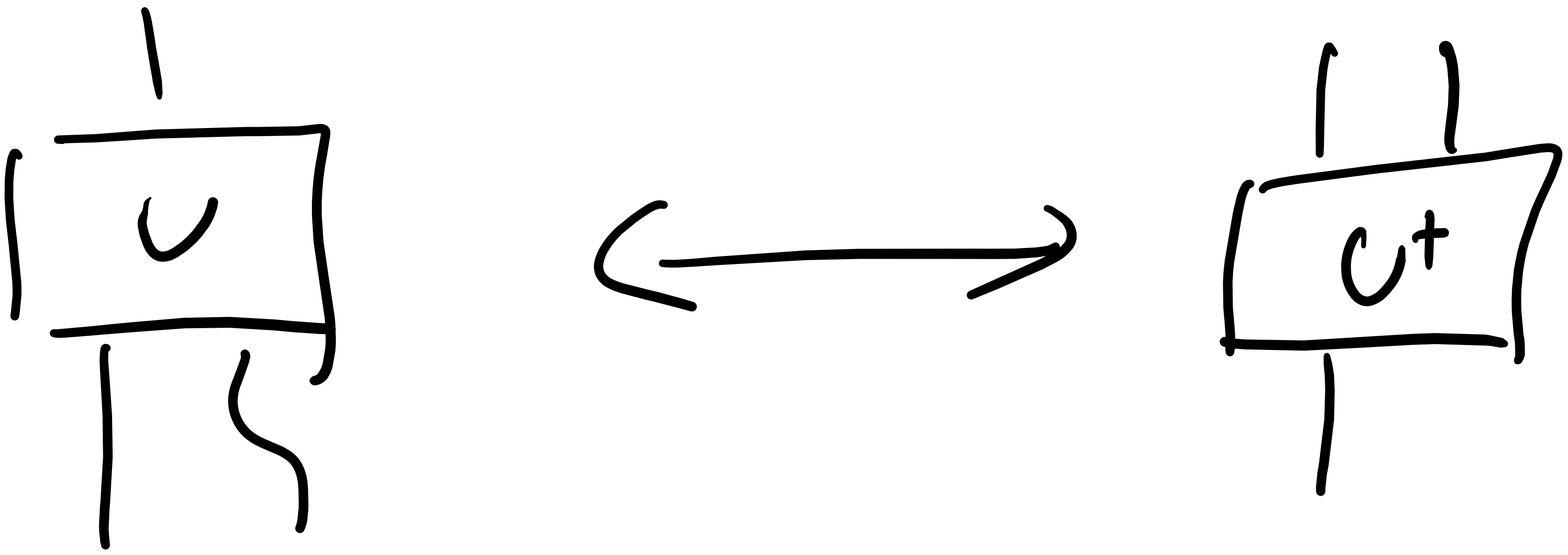


Von Neumann pre-measurement

Energy non-dem meas

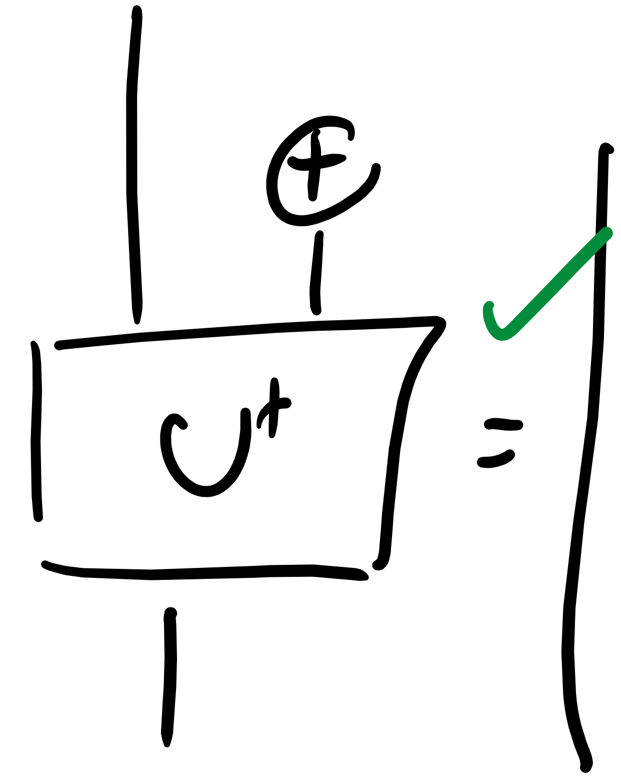
Space translation

Momentum measurement

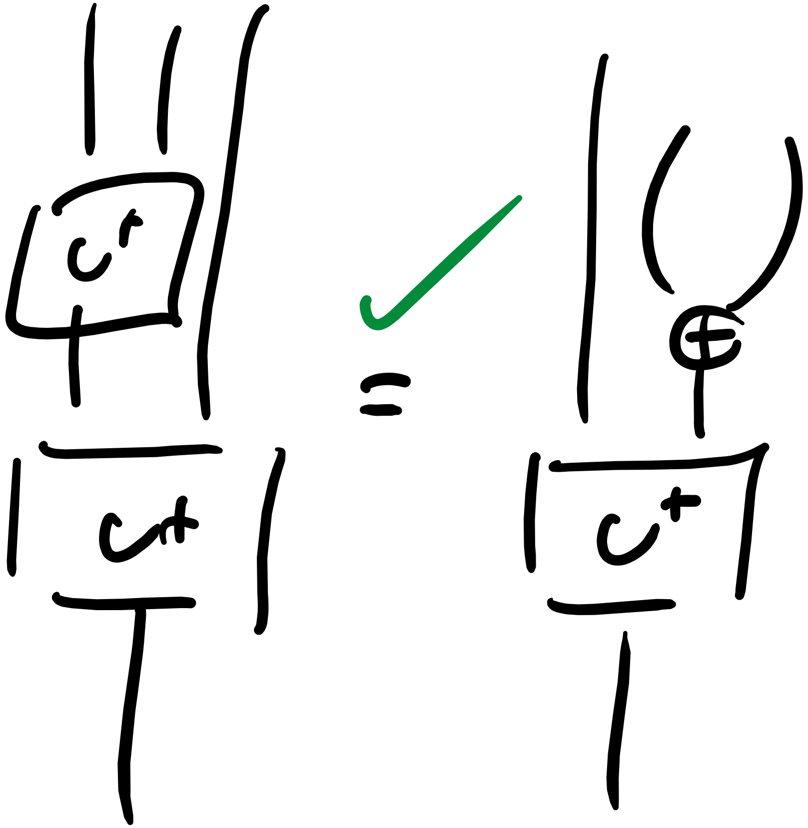


time-translation

Energy meas

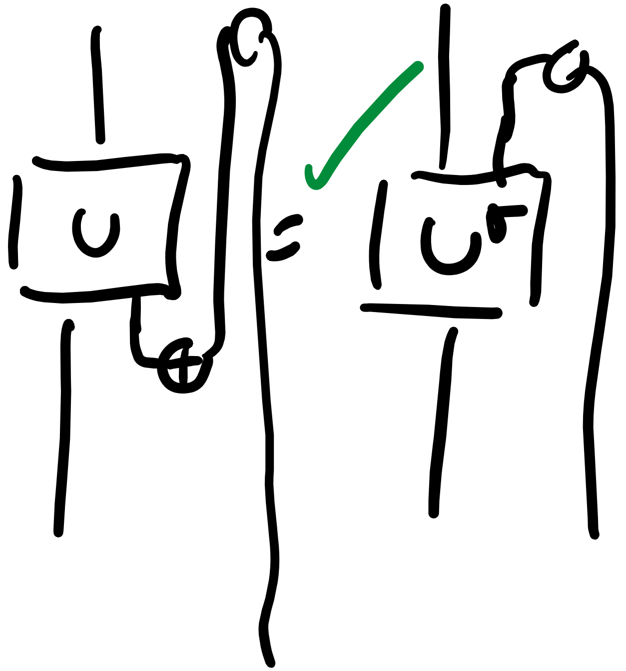


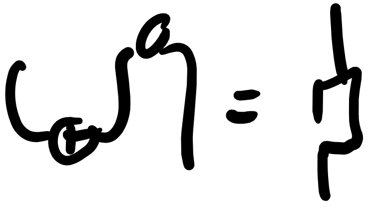
$$\sum_{\epsilon} \pi_\epsilon = id$$

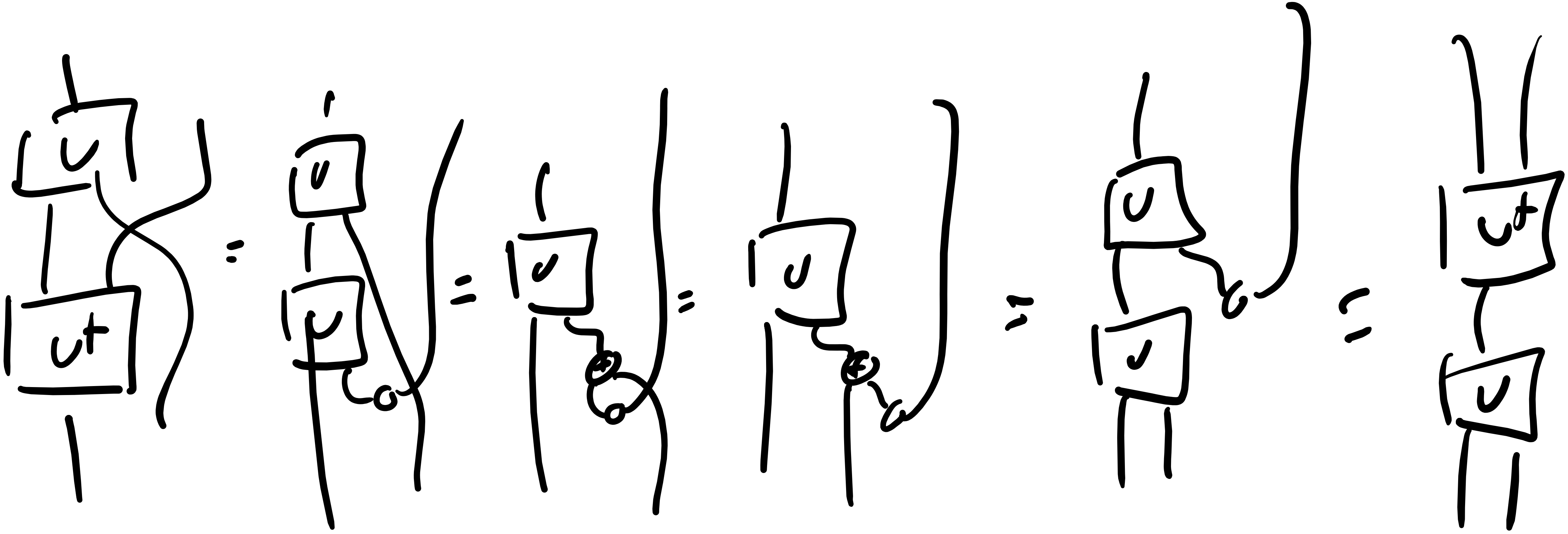


$$\pi_\epsilon \pi_{\epsilon'} = \pi_\epsilon \delta_{\epsilon\epsilon'}$$

$\forall U$






U
U†
=
U
U
=
U
=
U
=
U
U
=
U†
U