

# Quantum in Pictures Lecture Series

Lecturer: Stefano Gogioso

Mon 26 June 2023 – Afternoon Lecture

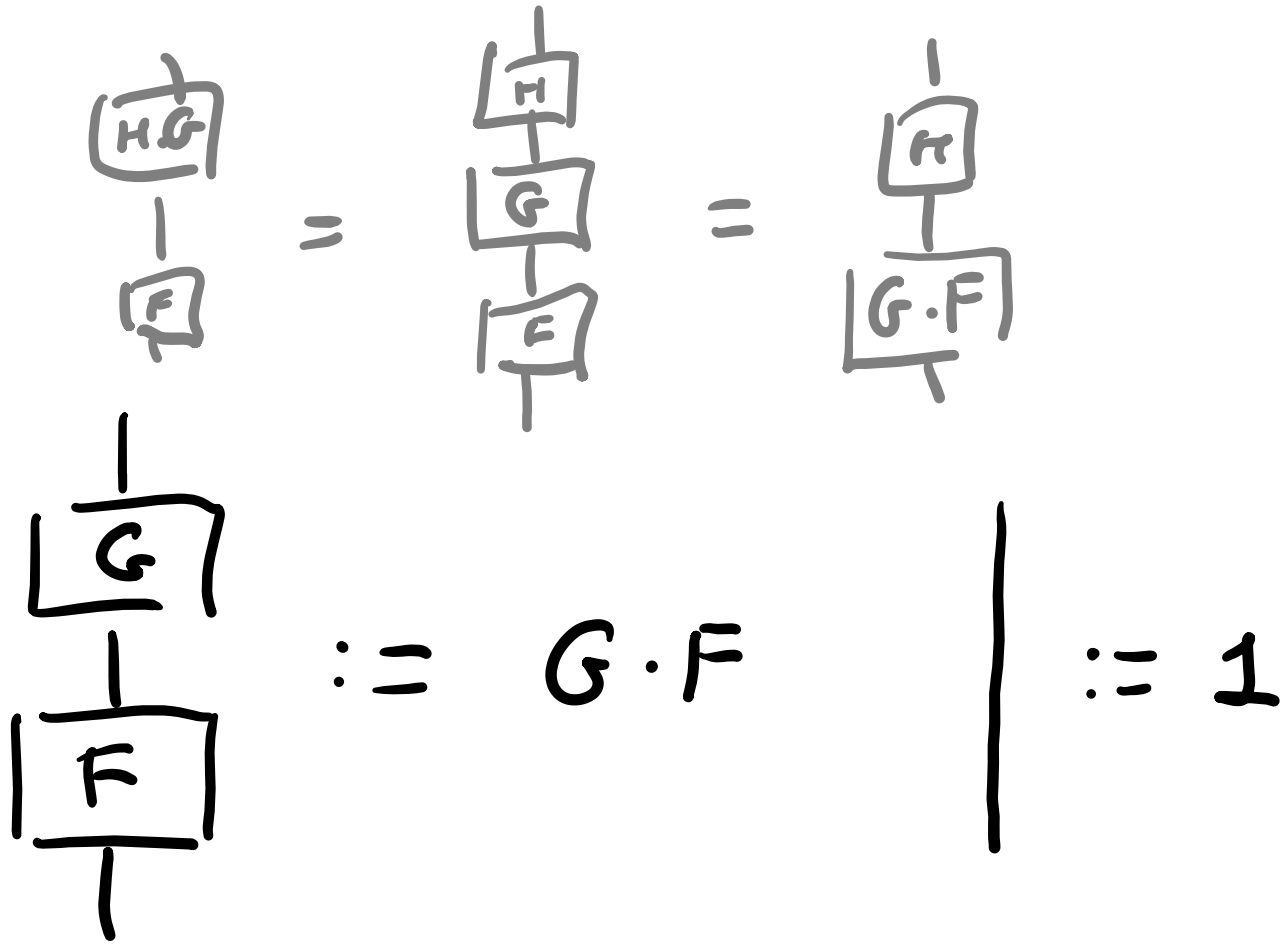


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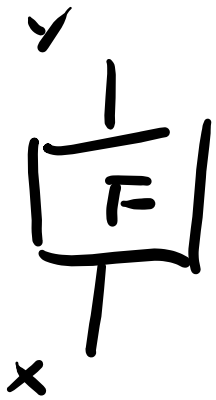
$(\Pi, \cdot, 1)$



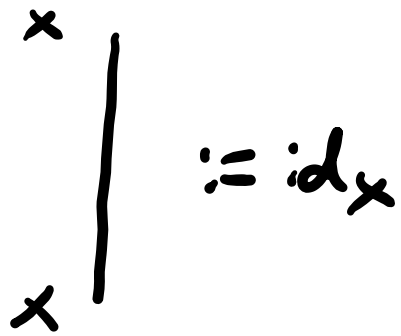
$F \in \Pi$



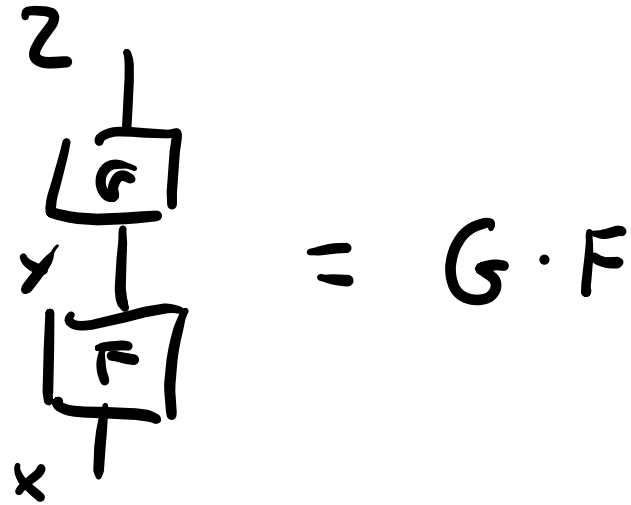
$(\text{Set}, \cdot, \text{id})$  Category



$x, y \in \text{Set}$   
 $F: x \rightarrow y$



$\{x \mapsto x\}$



$\{x \mapsto G(F(x))\}$

$$X \times Y \longrightarrow Z \times W$$

$$\text{Swapp: } X \times Y \longrightarrow Y \times X \\ (x, y) \longmapsto (y, x)$$

$$f: X \rightarrow Z \\ g: Y \rightarrow W$$

$$f \times g: X \times Y \longrightarrow Z \times W \\ (x, y) \longmapsto (f(x), g(y))$$

$(\text{Set}, \cdot, \text{id}, \times, \{*\})$

$$\begin{array}{c} | \\ x \downarrow \\ | \\ y \downarrow \end{array} := \text{id}_{x \times y}$$

$$\boxed{\phantom{x}} := \text{id}_{\{*\}}$$

$$X \times \{*\} \cong X$$

Monoidal Category

$$\begin{array}{c} z \downarrow \\ | \\ \boxed{F} \\ | \\ x \downarrow \end{array} \begin{array}{c} w \downarrow \\ | \\ \boxed{G} \\ | \\ y \downarrow \end{array} := F \times G$$

$$\begin{array}{c} | \\ \boxed{F} \\ | \\ | \\ \boxed{G} \\ | \\ | \end{array} = \begin{array}{c} | \\ \boxed{F} \\ | \\ | \\ \boxed{G} \\ | \\ | \end{array} : \begin{array}{c} | \\ | \\ \boxed{F} \\ | \\ | \\ \boxed{G} \\ | \\ | \end{array}$$

$$(F \times \text{id}) \cdot (\text{id} \times G)$$

$$F \times G$$

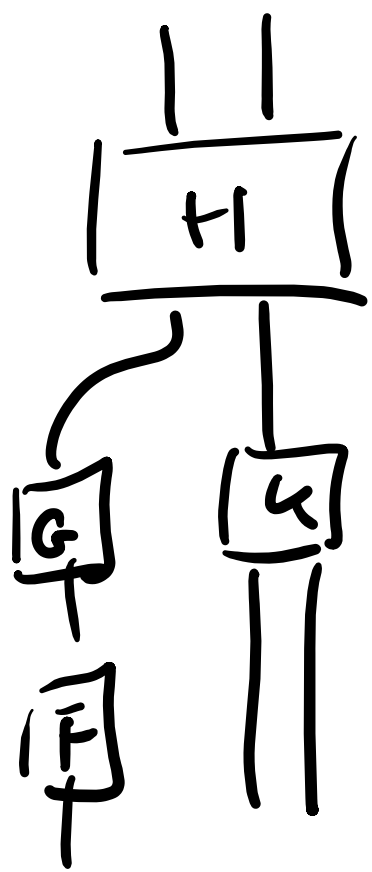
$$(\text{id} \times G) \cdot (F \times \text{id})$$

$(\text{Set}, \cdot, \text{id}, \cup, \emptyset)$   $\Pi$ -coidal

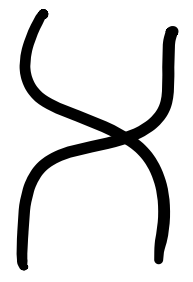
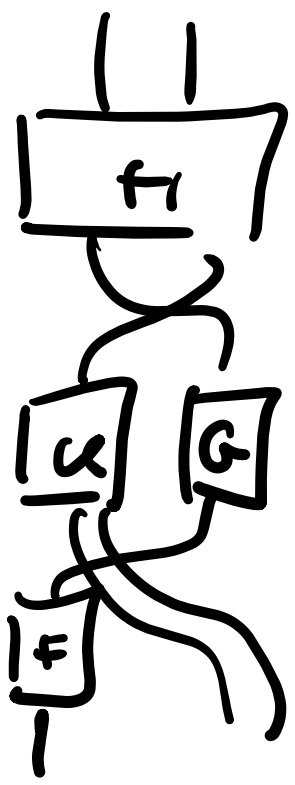
$$\downarrow_x \downarrow_y := \text{id}_{x \cup y}$$

$$\begin{array}{ccc} & z & w \\ & \downarrow & \downarrow \\ \boxed{F} & & \boxed{G} \\ & \downarrow_x & \downarrow_y \end{array} := F \cup G$$

FLIG:  $x \cup y \rightarrow z \cup w$   
 $x \in x \mapsto F(x) \in z$   
 $y \in y \mapsto G(y) \in w$

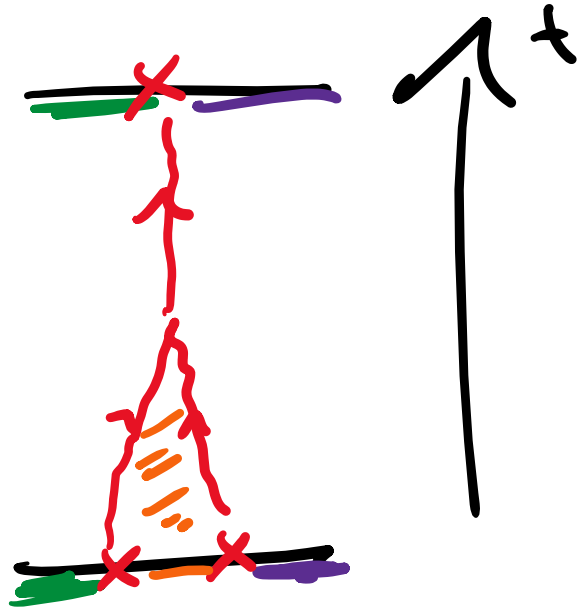
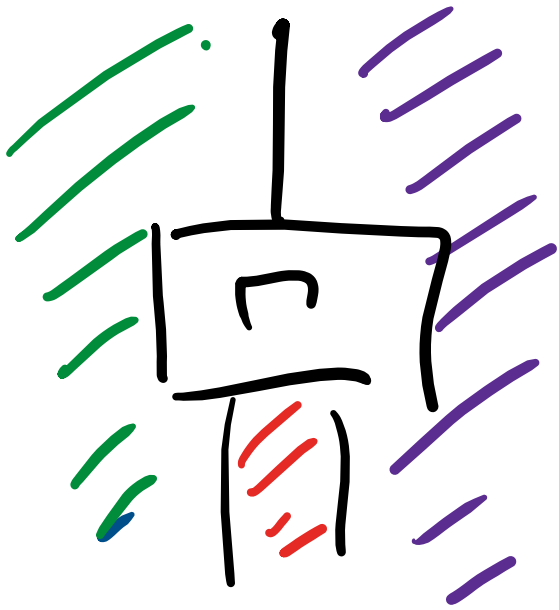


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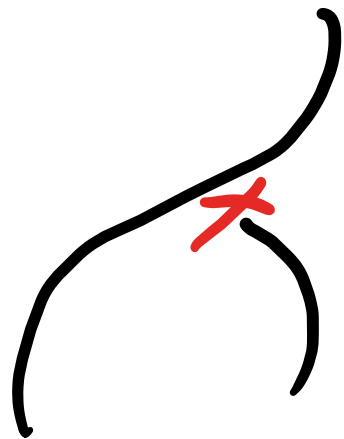
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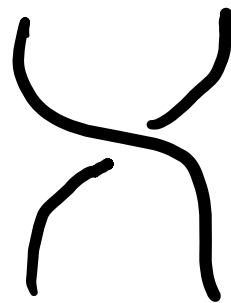
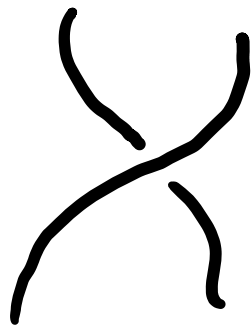




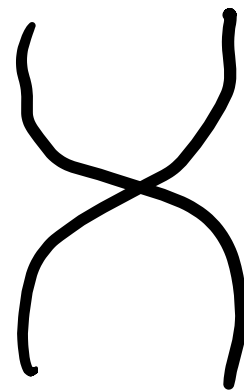
2D

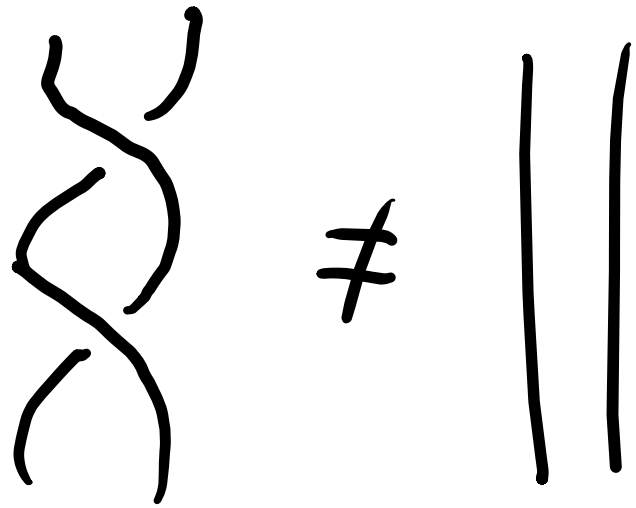
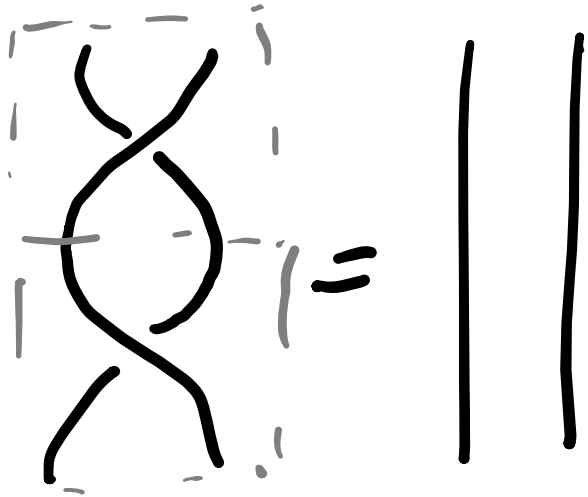


3D

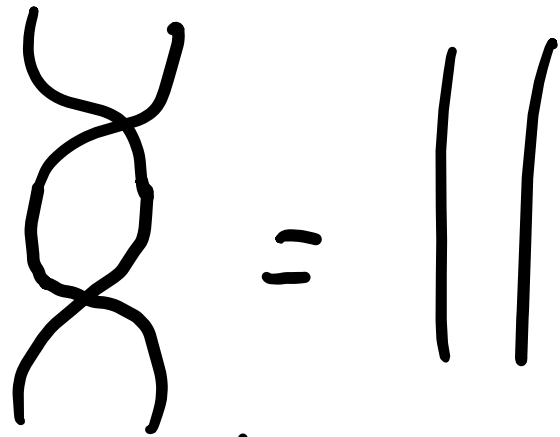


4D



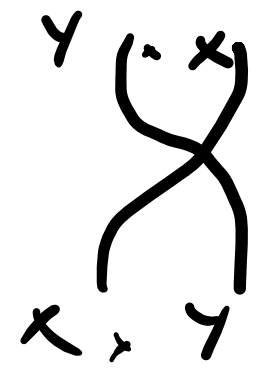


in 3D



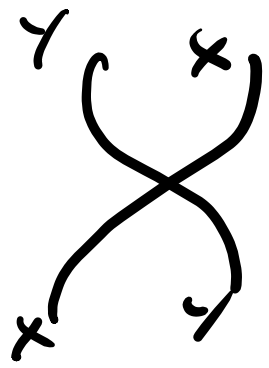
in 4D

$(X, \{*\})$



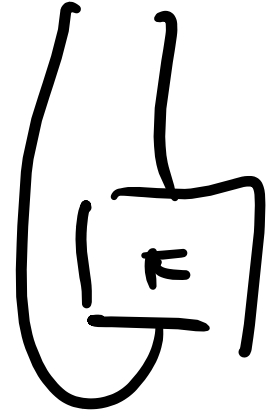
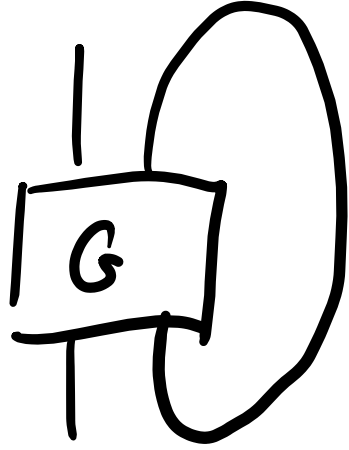
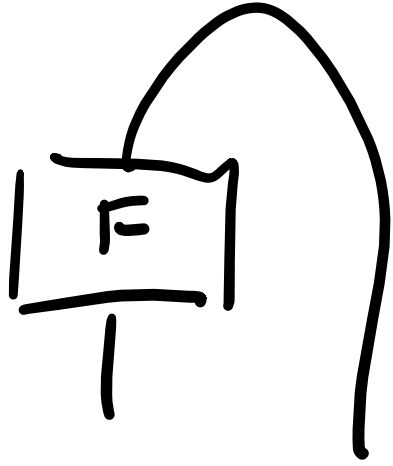
$$\begin{array}{l}
 X * Y \longrightarrow Y * X \\
 (x, y) \longmapsto (y, x)
 \end{array}$$

$(\sqcup, \emptyset)$

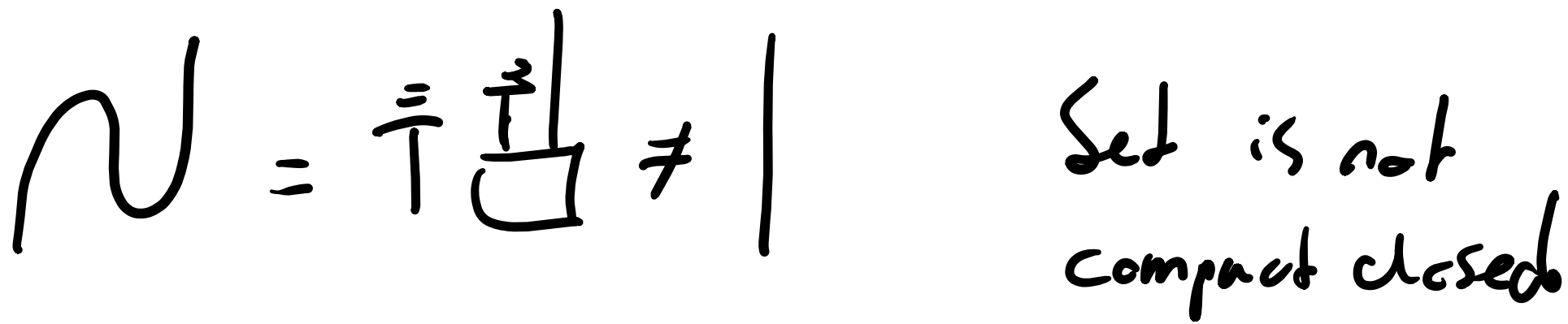
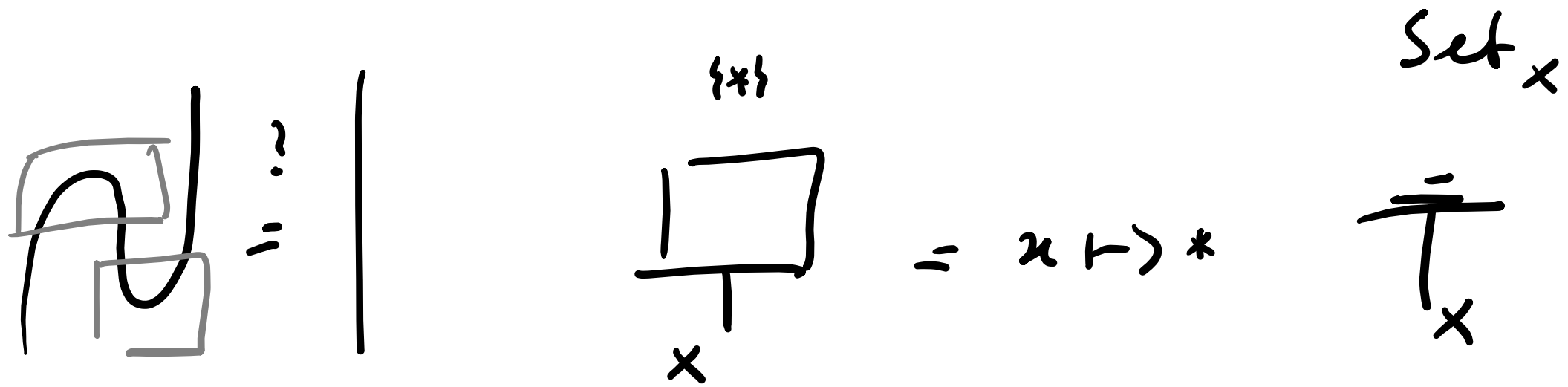


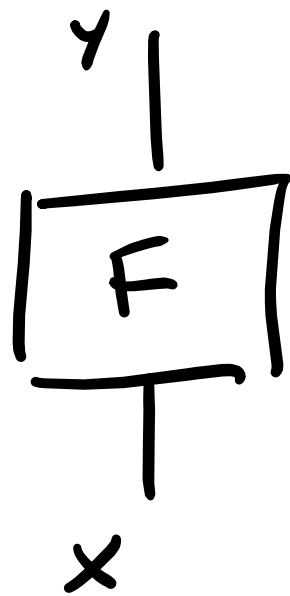
$$\begin{array}{l}
 X \sqcup Y \longrightarrow Y \sqcup X = X \sqcup Y \\
 x \in X \longmapsto x \\
 y \in Y \longmapsto y
 \end{array}$$

" ( (



$N = 1 - h$



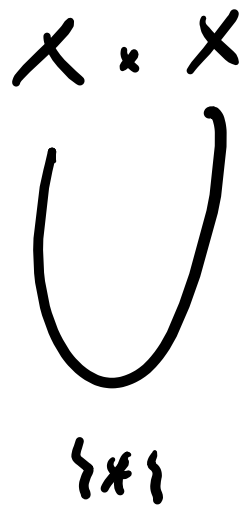


$$F \subseteq X \times Y$$

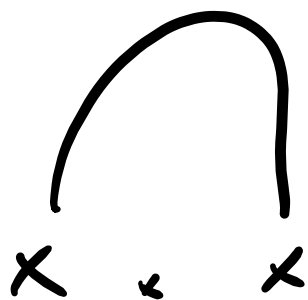
$$\bar{F} \in \{0,1\}^{Y \times X}$$

$$(\text{Rel}, \cdot, 1, \times, \{*\})$$

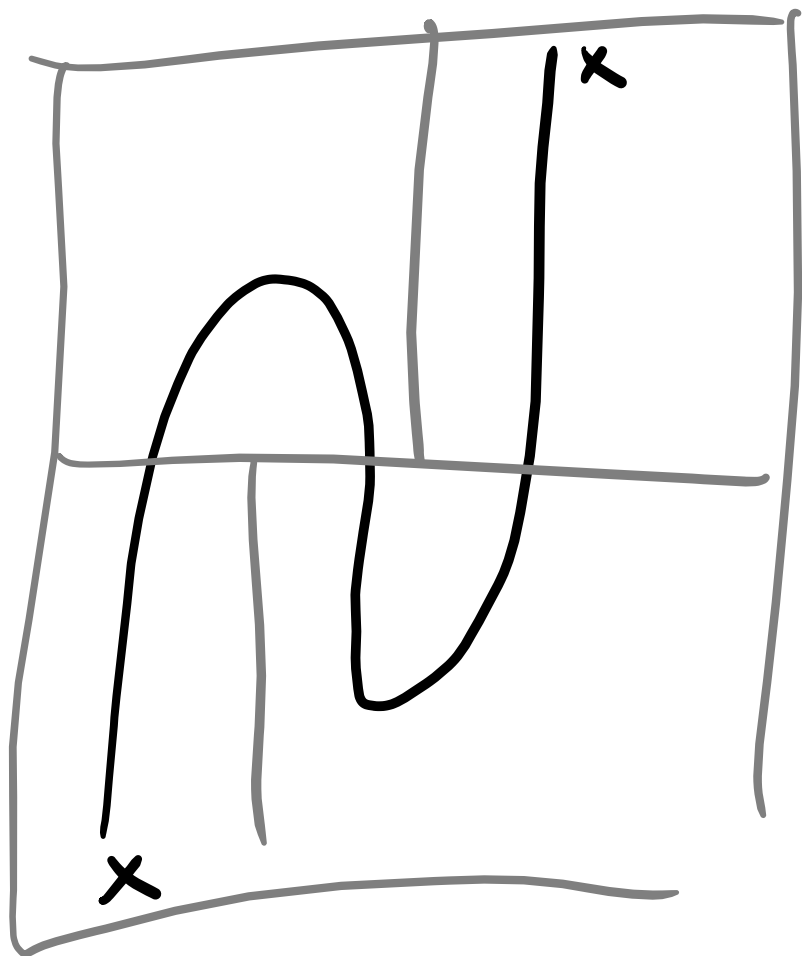
$$\text{Relation on } \{*\} : \begin{cases} \{*\} \mapsto 1 \\ \emptyset \mapsto 0 \end{cases}$$



$$\left\{ * \mapsto (x, x) \mid x \in X \right\} \quad \text{Subst. of } X \times X$$



$$\left\{ (x, x) \mapsto * \mid x \in X \right\} \quad \text{Partial function } X \times X \rightarrow \{*\}$$



$$\{ (x, y, z) \mapsto y \mid x, y \in X \}$$

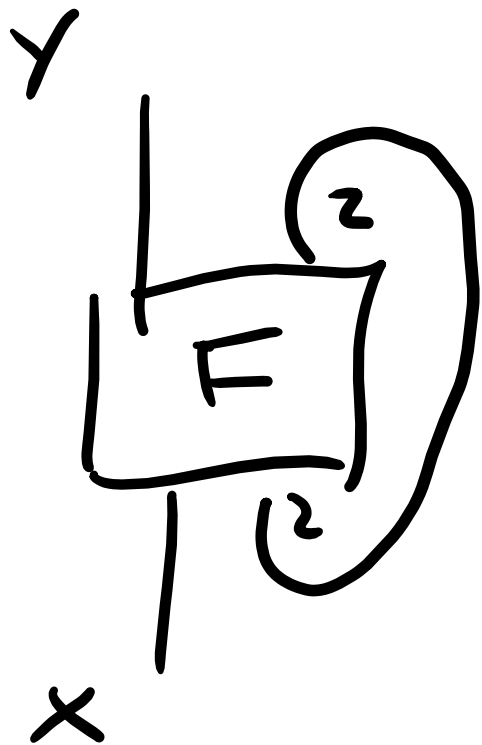
$$\{ x \mapsto (x, y, y) \mid y \in X \}$$

$$= \{ x \mapsto x \mid x \in X \}$$

" id<sub>x</sub>

$$\sim = |$$





$$x \mapsto F(z, z)_{\gamma} \text{ for some } z \in \mathbb{Z}$$

$$\text{st. } F(z, z)_{\mathbb{Z}} = z$$

Partial fraction

$$\begin{pmatrix} .75 & 1 \\ .25 & 0 \end{pmatrix}$$

$$(\mathbb{R}^+, +, 0, \cdot, 1)$$

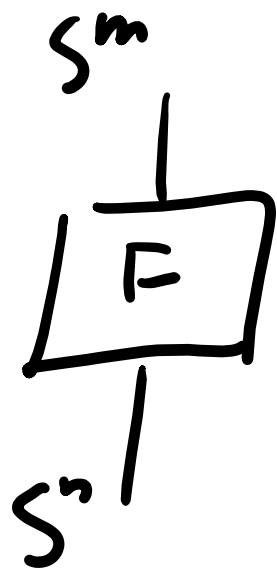
$$\begin{array}{l} 0 \mapsto 0 \\ x > 0 \mapsto 1 \end{array}$$
$$\longrightarrow$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$(\{0, 1\}, \vee, \wedge, 0, 1, 1)$$

$$(S, +, 0, \cdot, 1)$$

$$S = \mathbb{R}^+, \mathbb{R}, \mathbb{C}, \mathbb{B}, \dots$$



$$F \in S^{m \times n}$$

$$n, m \in \mathbb{N}^+$$

A diagram showing two boxes stacked vertically. The bottom box is labeled 'F' and has a vertical line below it labeled  $n$ . The top box is labeled 'G' and has a vertical line above it labeled  $e$ . A vertical line connects the top of the 'F' box to the bottom of the 'G' box. To the right of this diagram is the equation  $= G \cdot F$ .

A diagram showing two boxes side-by-side. The left box is labeled 'F' and has a vertical line below it labeled  $n$ . The right box is labeled 'G' and has a vertical line below it labeled  $m$ . A vertical line above the 'F' box is labeled  $l$ . A vertical line above the 'G' box is labeled  $r$ . To the right of this diagram is the equation  $= F \otimes G$ .

$$(\mathbb{C}\text{-Mat}, \cdot, 1, \otimes, \mathbb{C})$$

Multiplicative

$$\begin{matrix} 10) & \otimes & 14) \\ \begin{pmatrix} 1 \\ 0 \end{pmatrix} & \otimes & \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \end{matrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \\ 0 \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \end{pmatrix}$$

$$(\mathbb{C}\text{-Mat}, \cdot, 1, \oplus, \{1\})$$

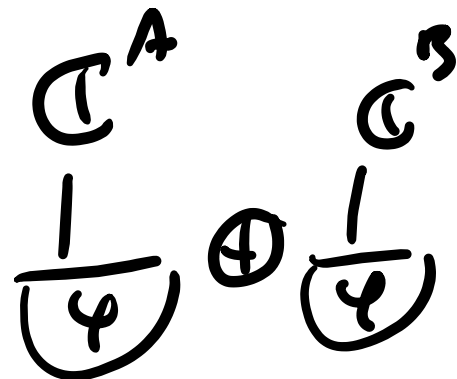
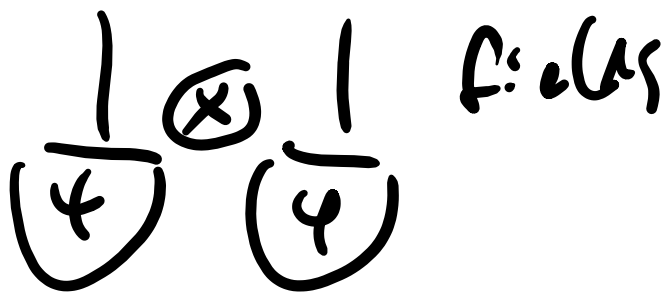
Additive

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \oplus \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

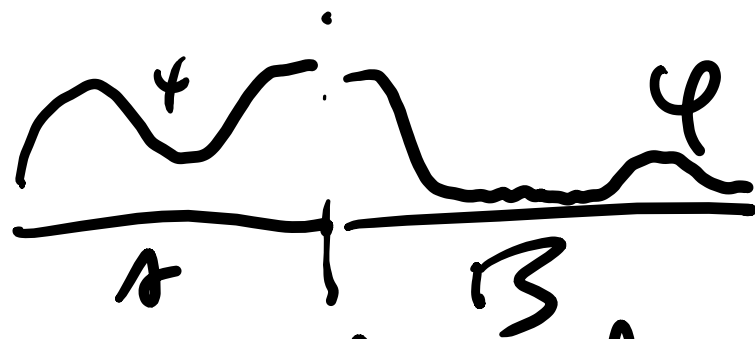
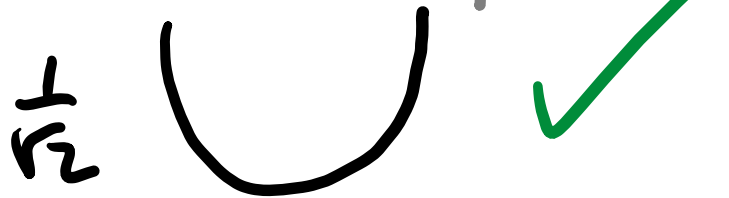
$$= \begin{pmatrix} 1 \\ 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \end{pmatrix}$$

$$\mathbb{C}^n \otimes \mathbb{C}^m = \mathbb{C}^{n \times m}$$

$$\mathbb{C}^n \oplus \mathbb{C}^m = \mathbb{C}^{n+m}$$



Alles ist tangential



gluing of wavefunctions

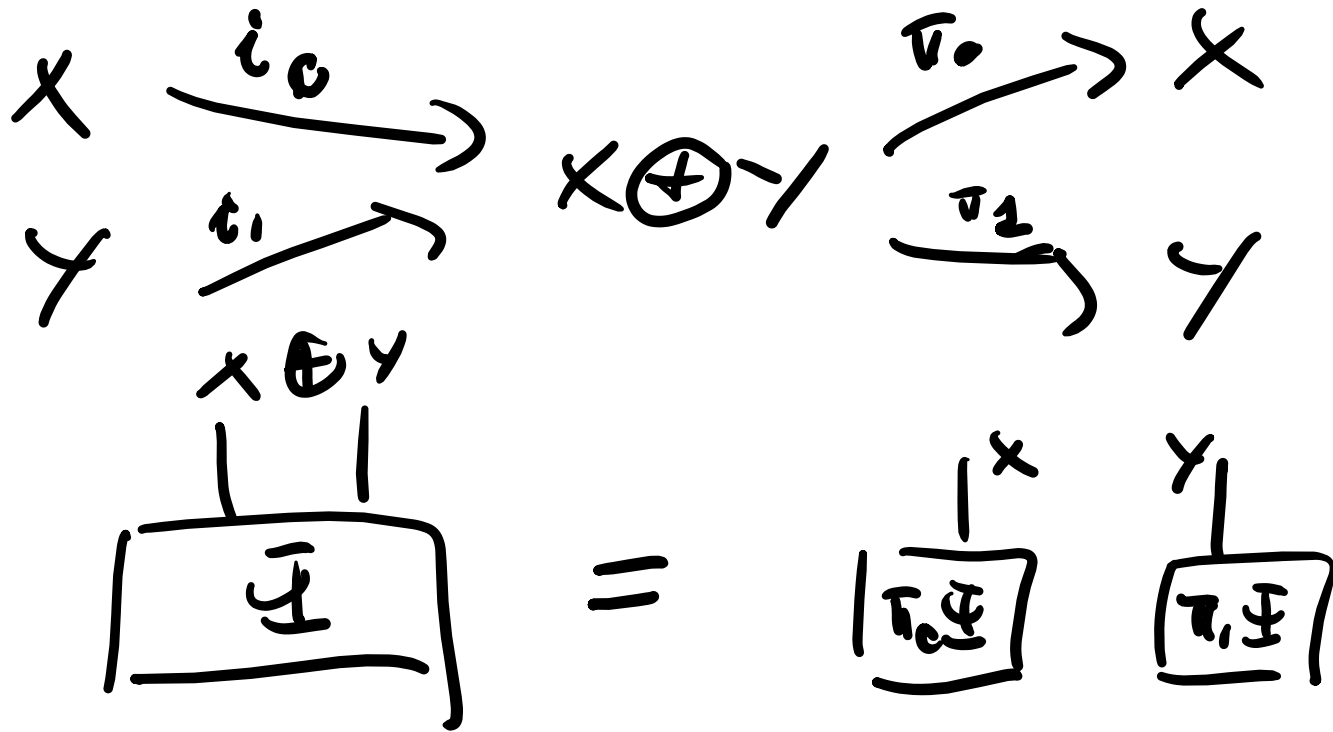
$$\bigoplus_{x \in X} \mathbb{C} \cong \mathbb{C}^X$$

$$\bigoplus_{x \in X} \mathbb{C} \cong L^2[X]$$

$$\bigoplus_{x \in X} H_x \oplus \bigoplus_{y \in X} H_y = \bigoplus_{x \in X \cup Y} H_x$$

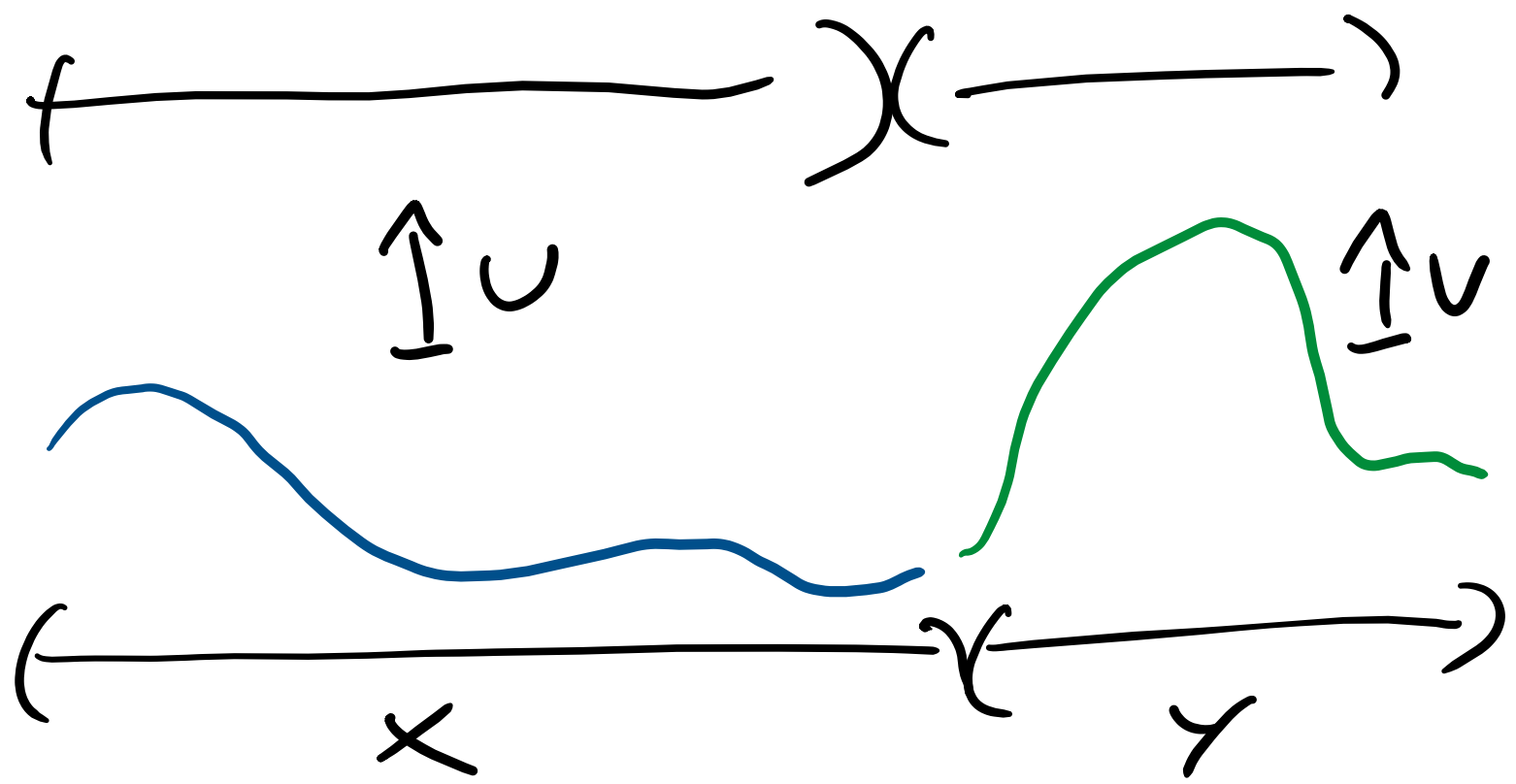
$(\mathbb{C}\text{-Mod}, \cdot, 1, \oplus, \mathbb{0}^{\oplus})$

$\left( \begin{array}{c|c} A & B \\ \hline C & D \end{array} \right)$



No  
 Entanglement!

$$\mathbb{C}^x \oplus \mathbb{C}^y \cong \mathbb{C}^{x \cup y} \quad F \oplus G = \left( \begin{array}{c|c} F & \\ \hline & G \end{array} \right)$$



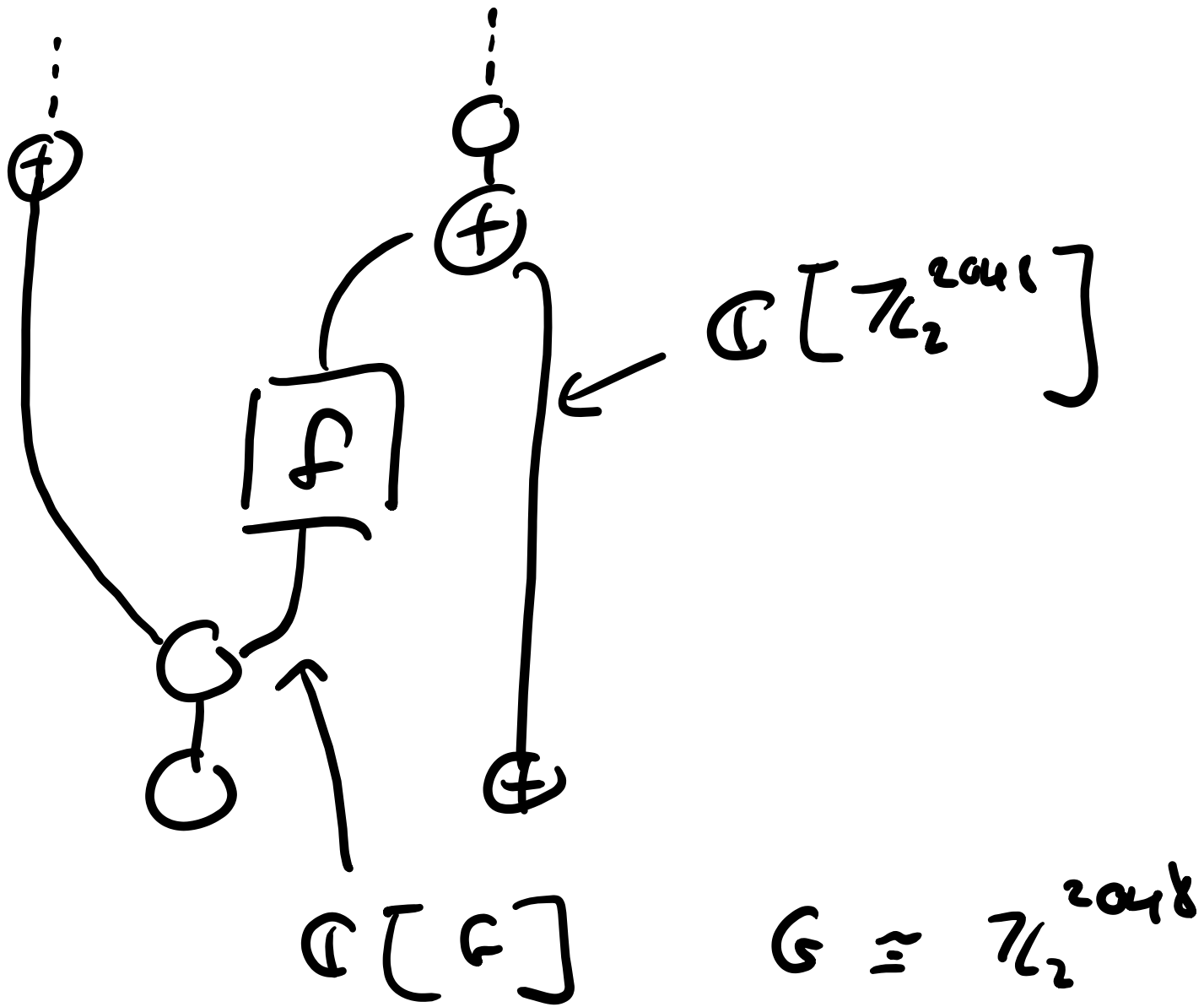


$$U \otimes V$$

$$\begin{pmatrix} U_{00} \cdot V & U_{01} \cdot V \\ U_{10} \cdot V & U_{11} \cdot V \end{pmatrix}$$

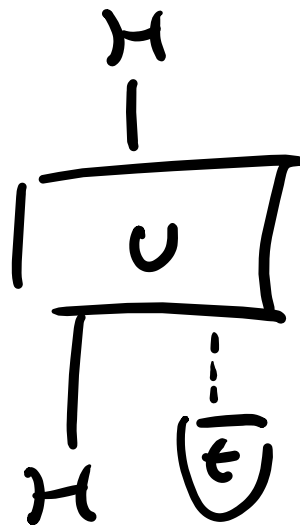
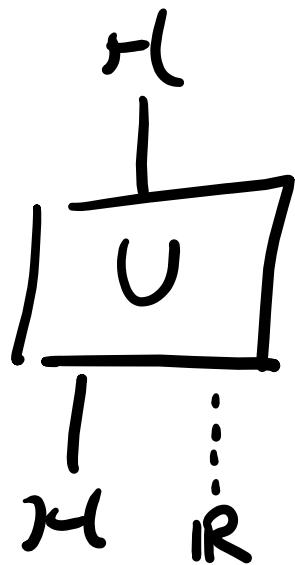
$$U \oplus V$$

$$\begin{pmatrix} U & \\ \hline & V \end{pmatrix}$$

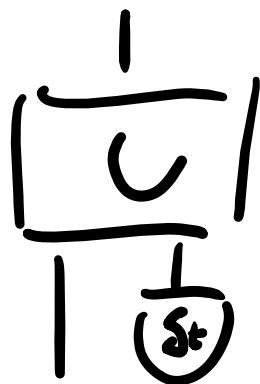
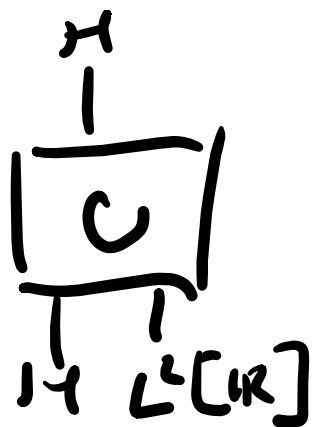
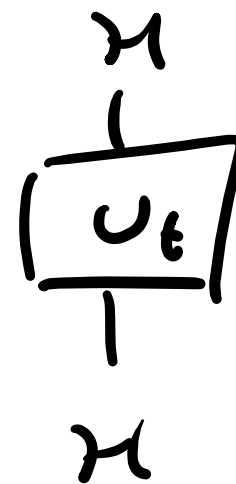


4) Hilbert space

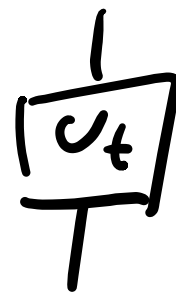
$$U_t := e^{itH}$$



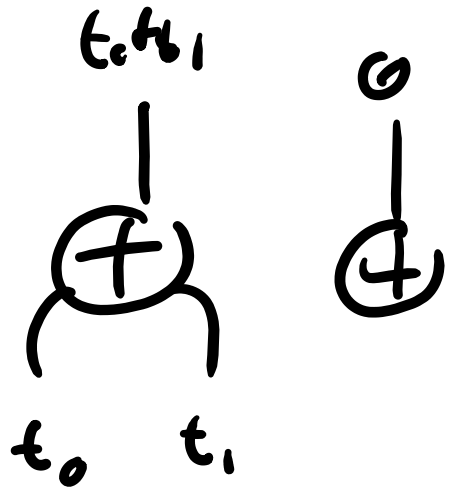
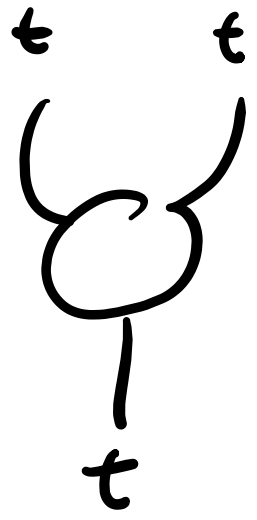
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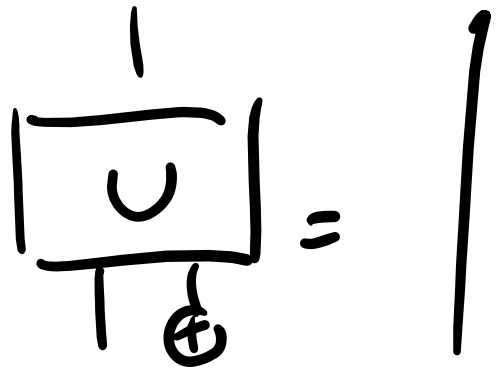
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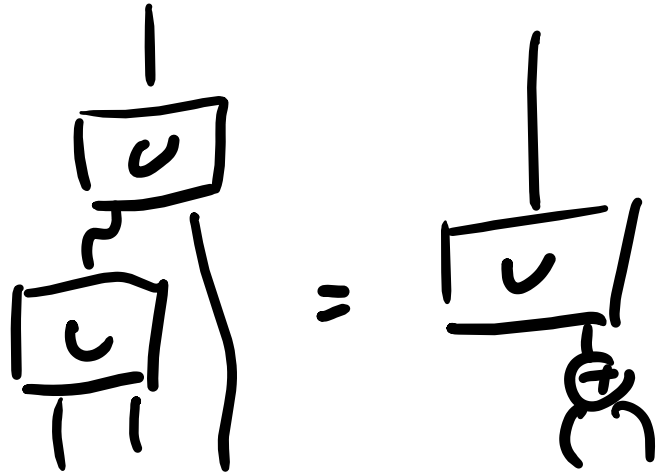
$\sim (\delta_t)$



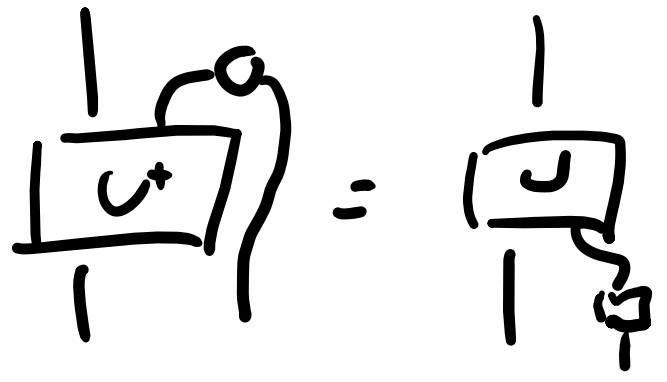
$\mathbb{C}[\mathbb{R}]$



$U = id_0$

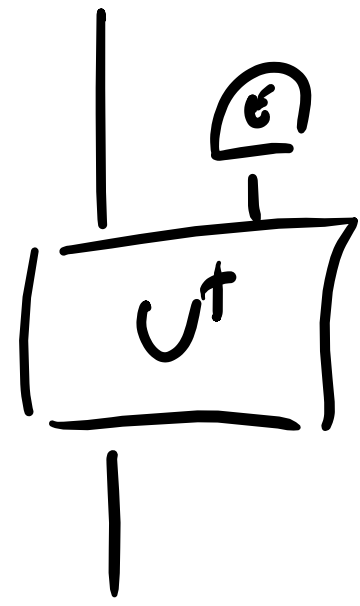
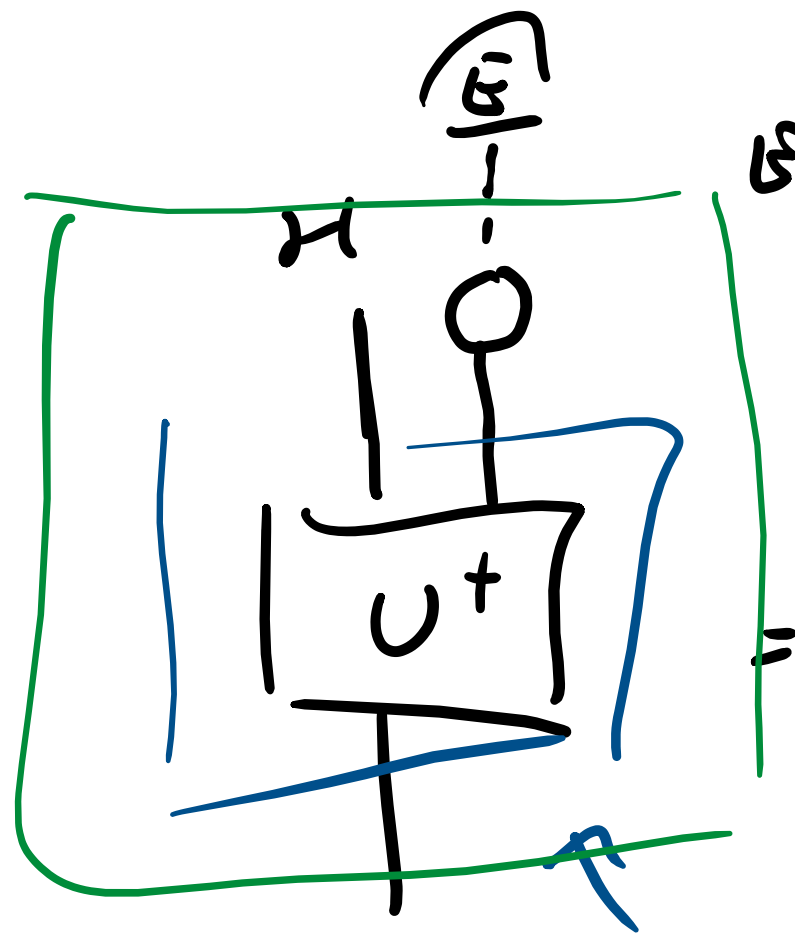


$U_t \cdot U_s = U_{t+s}$

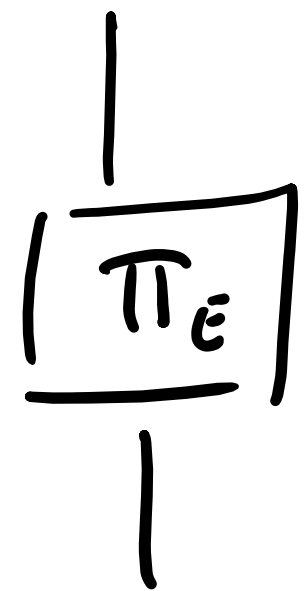


$U_t^\dagger = U_{-t}$

$$\mathcal{E} \in \mathbb{R}^1 \cong \mathbb{R}$$



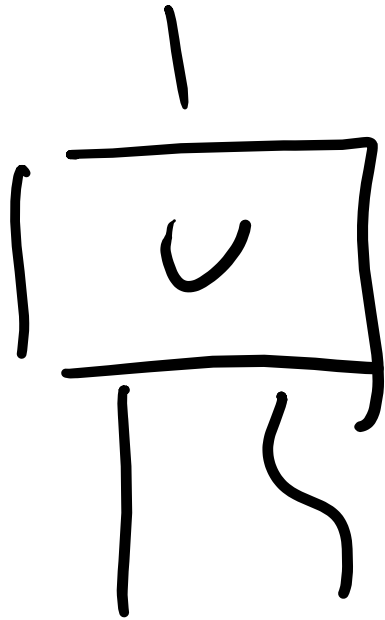
(proof)



Von Messern pro-messungen!

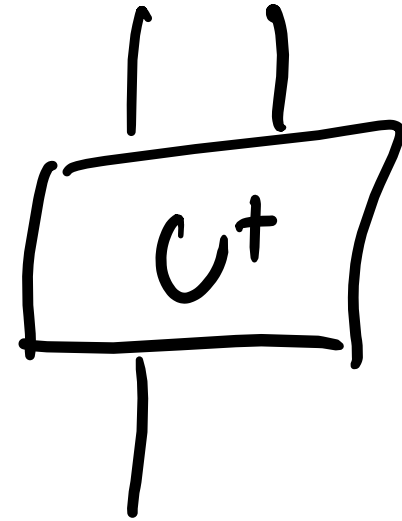
Energy near the meas

Space translation



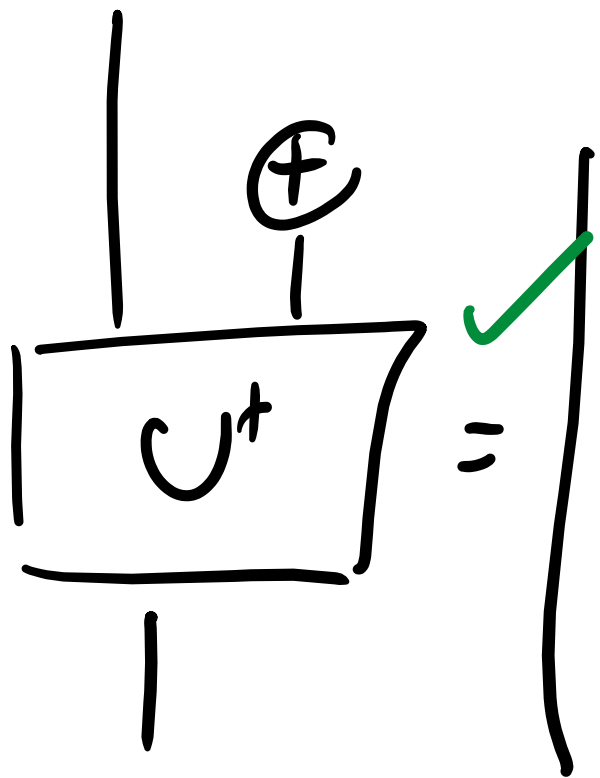
Time-translation

Momentum measurement

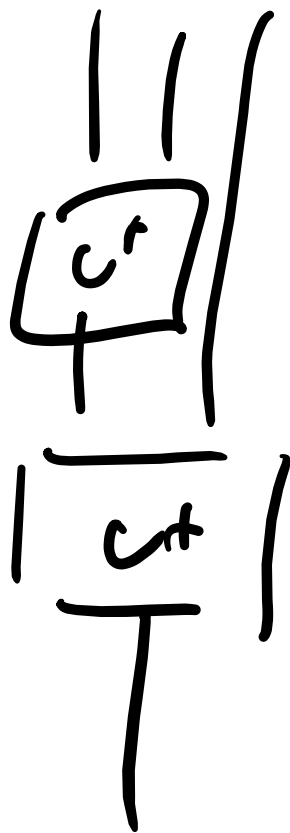


Energy meas

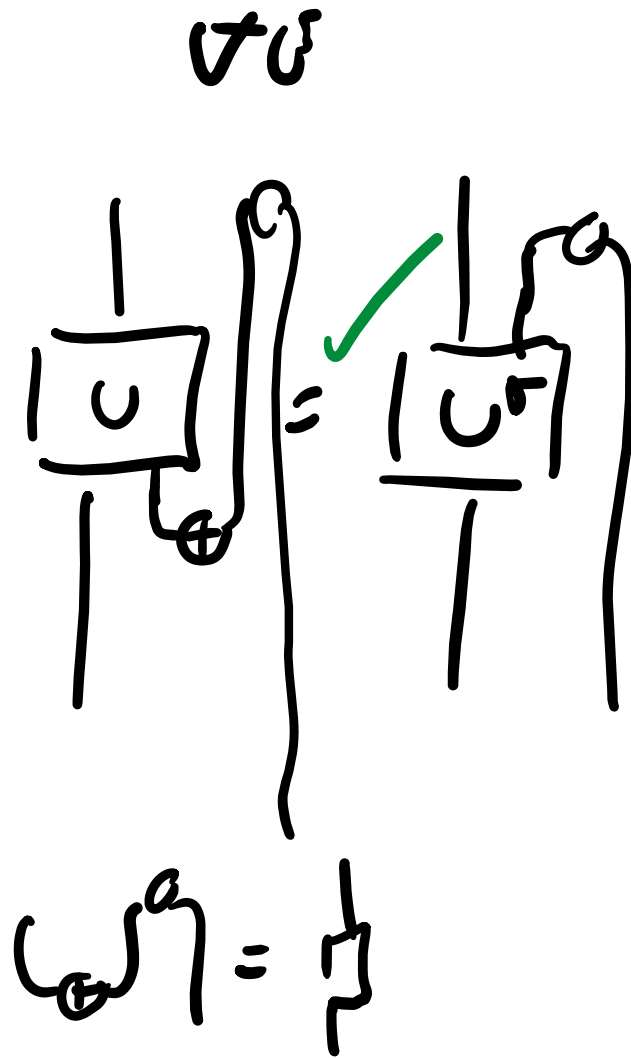
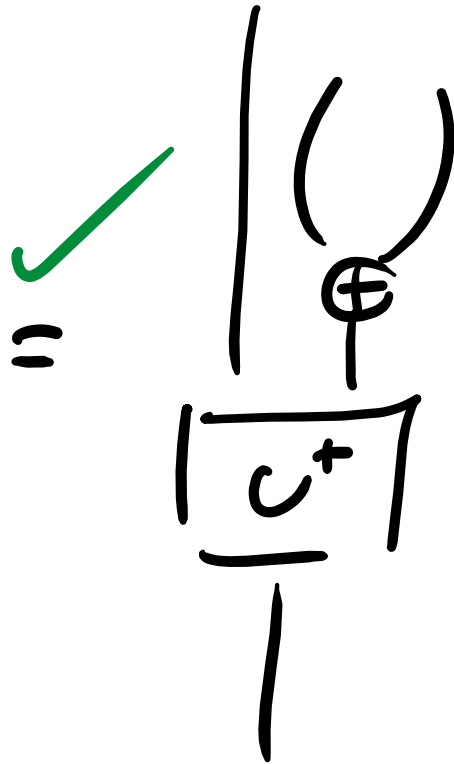


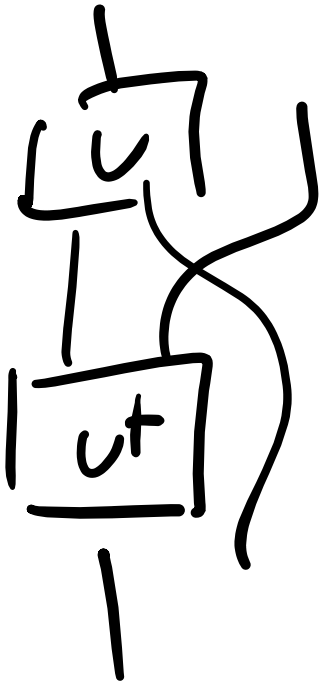


$$\sum_E \pi_E = id$$

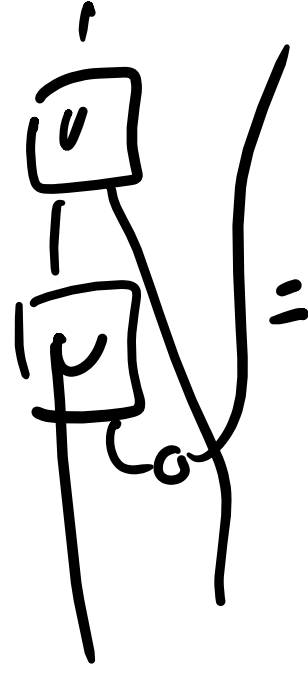


$$\pi_B \pi_{B'} = \pi_B \delta_{B'}$$





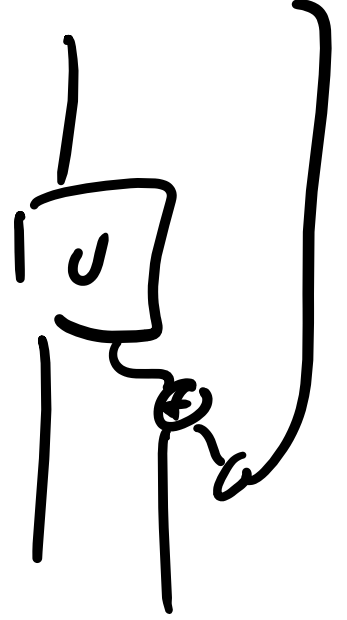
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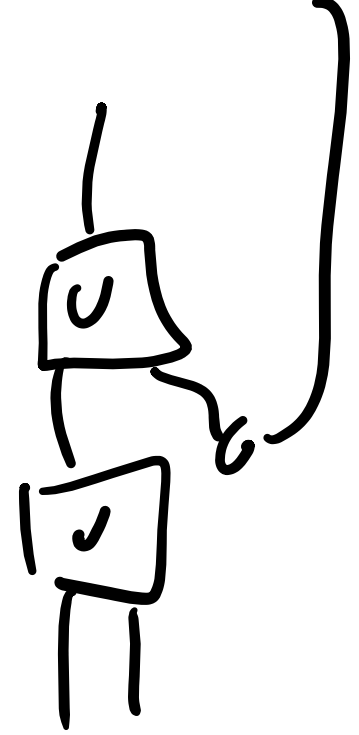
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