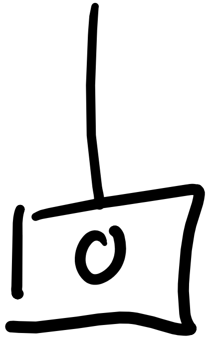
Quantum in Pictures Lecture Series

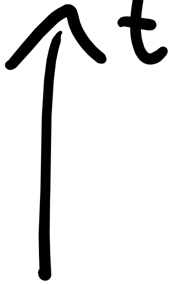
Lecturer: Stefano Gogioso

Mon 26 June 2023 – Morning Lecture

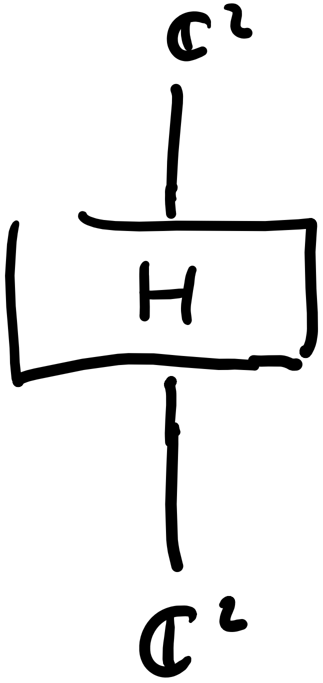
INDIANA UNIVERSITY BLOOMINGTON

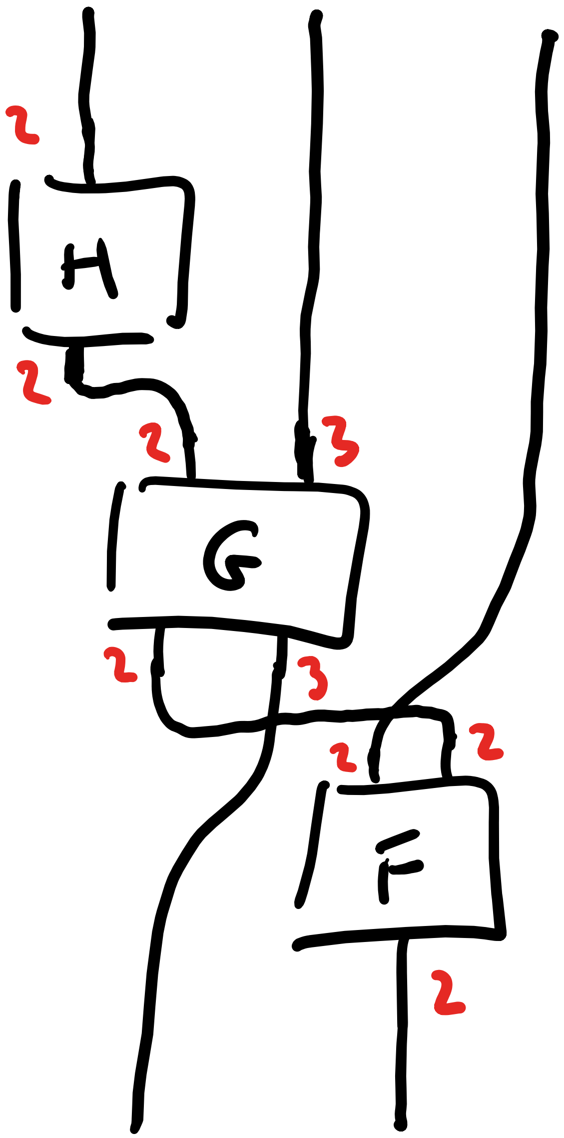
$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

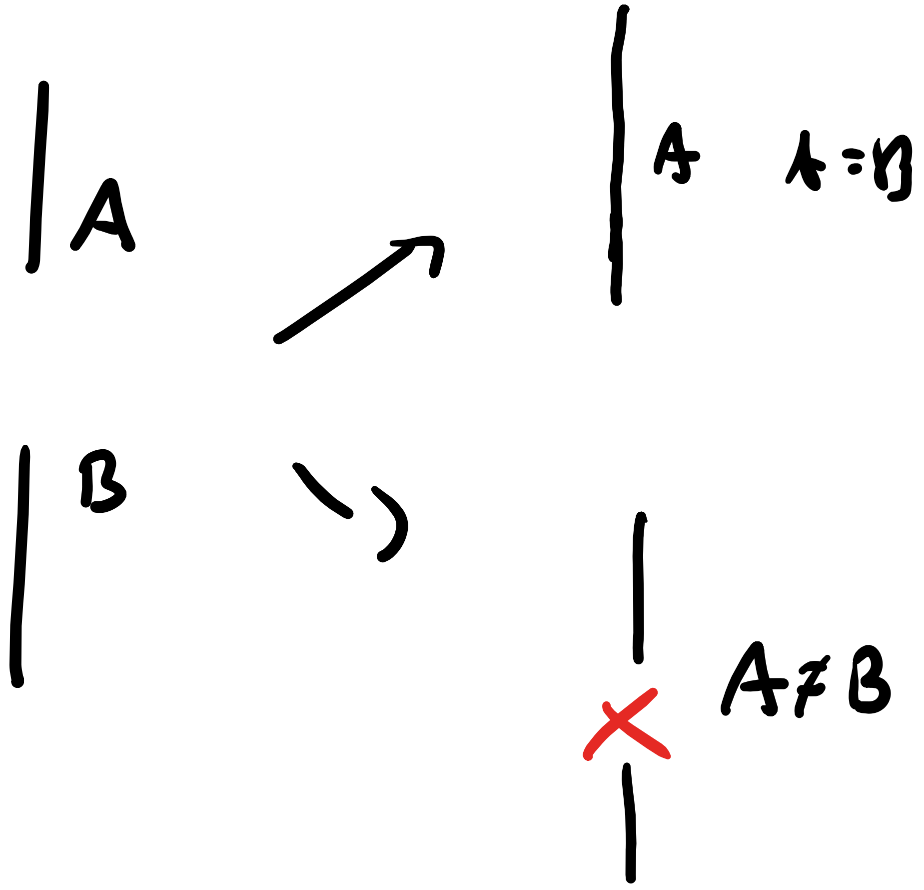


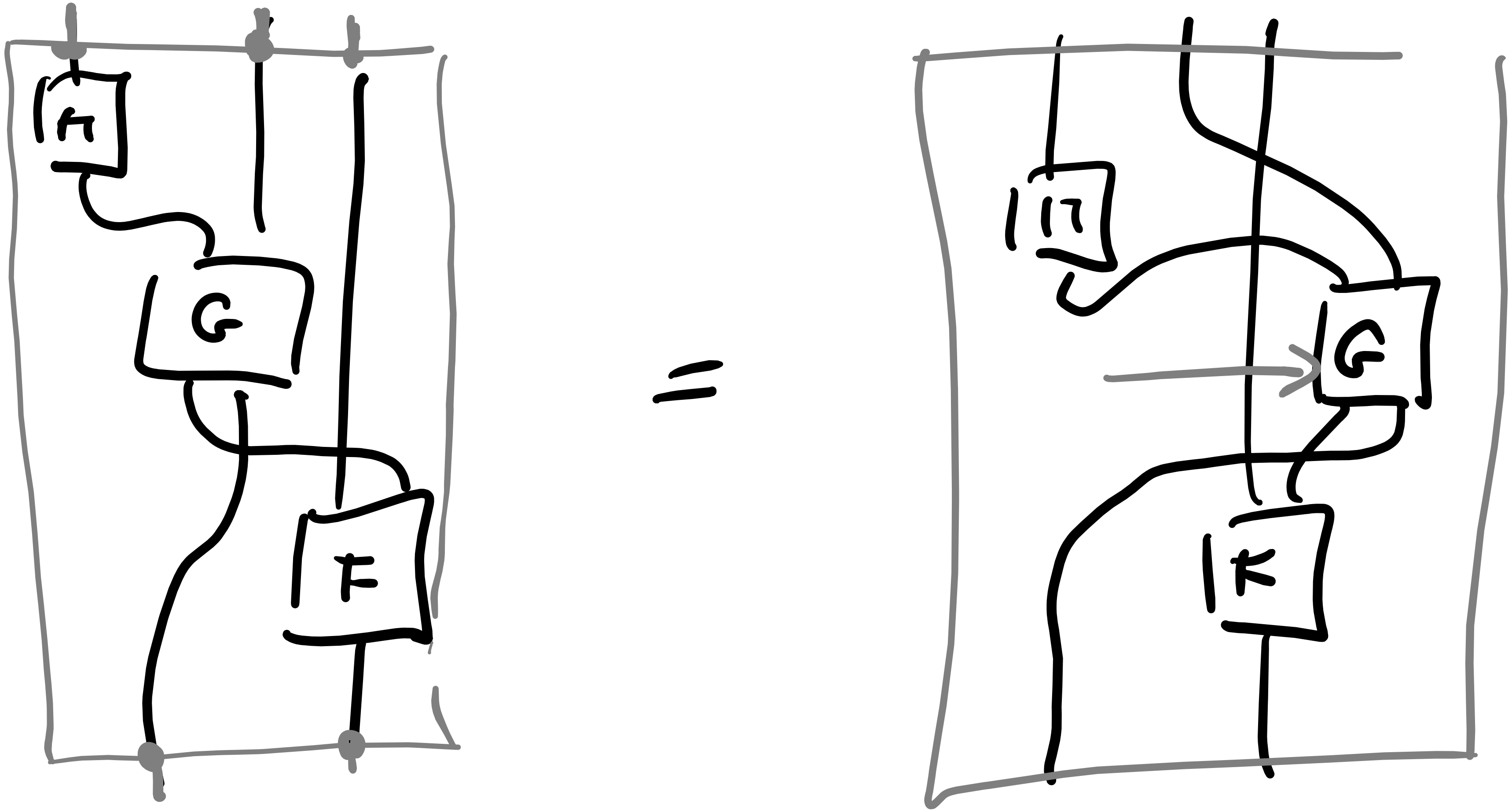


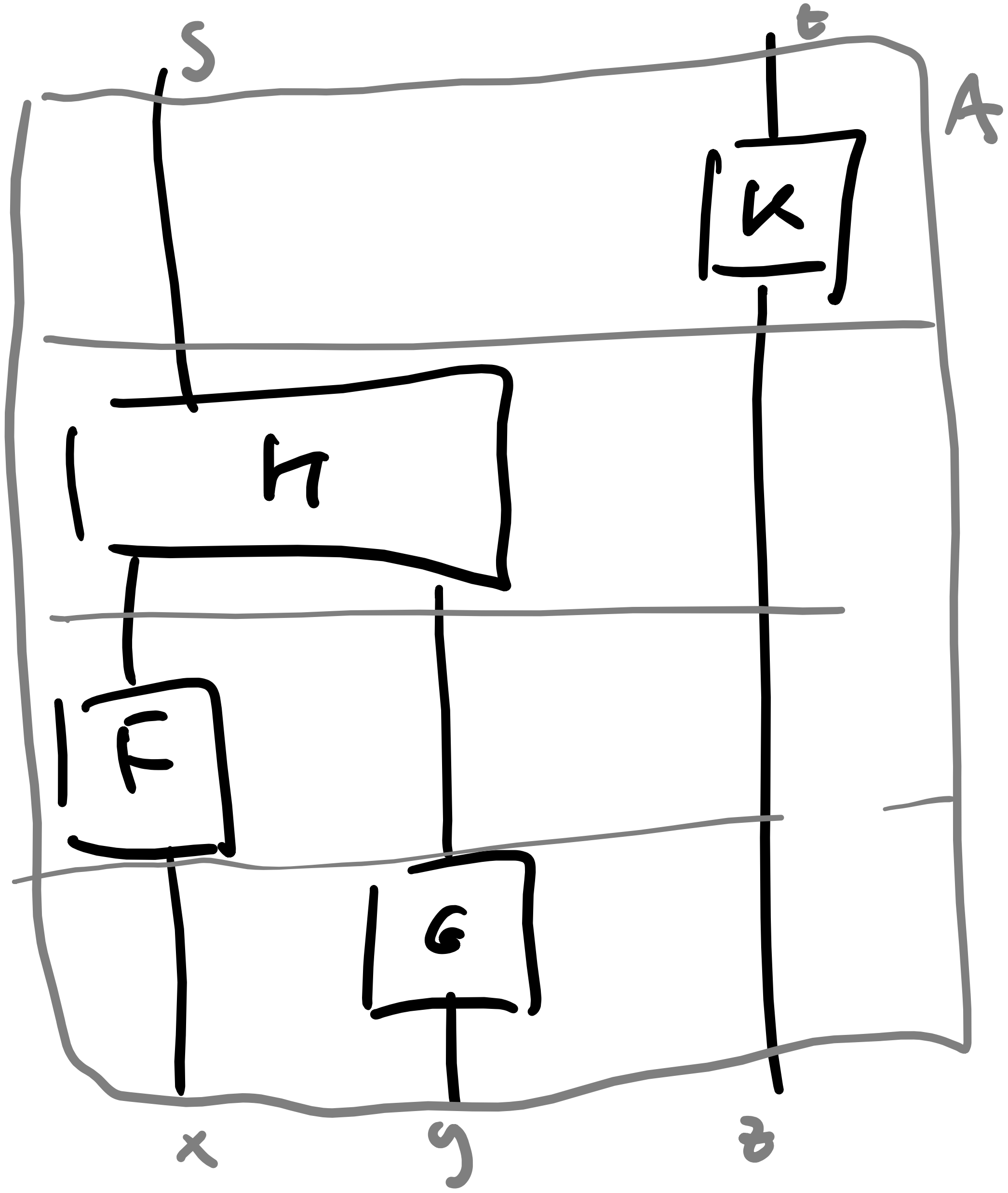
$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



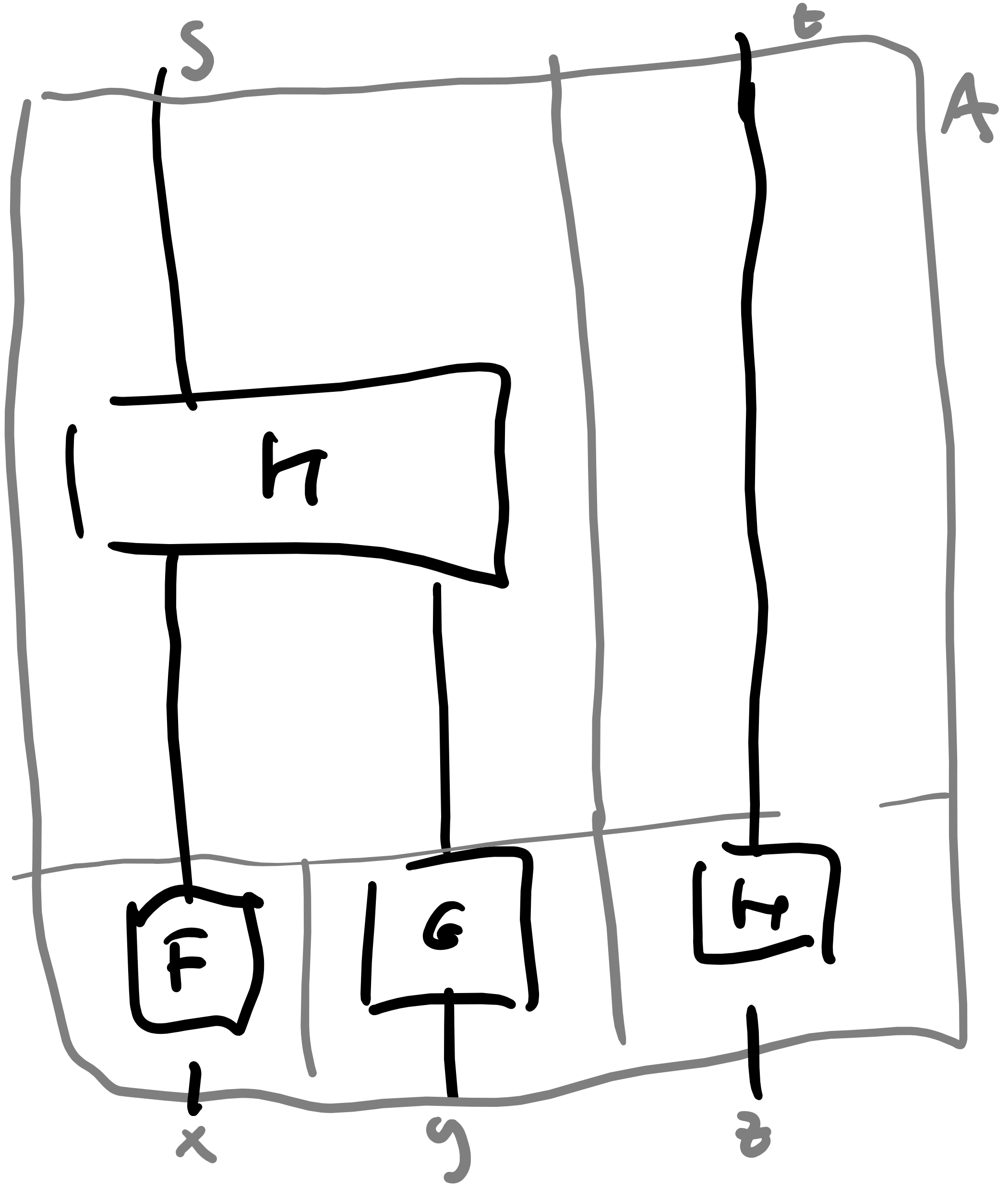

H
G
F
2
2
2
3
2
3
2
2
2


A
B
A
A=B
A≠B

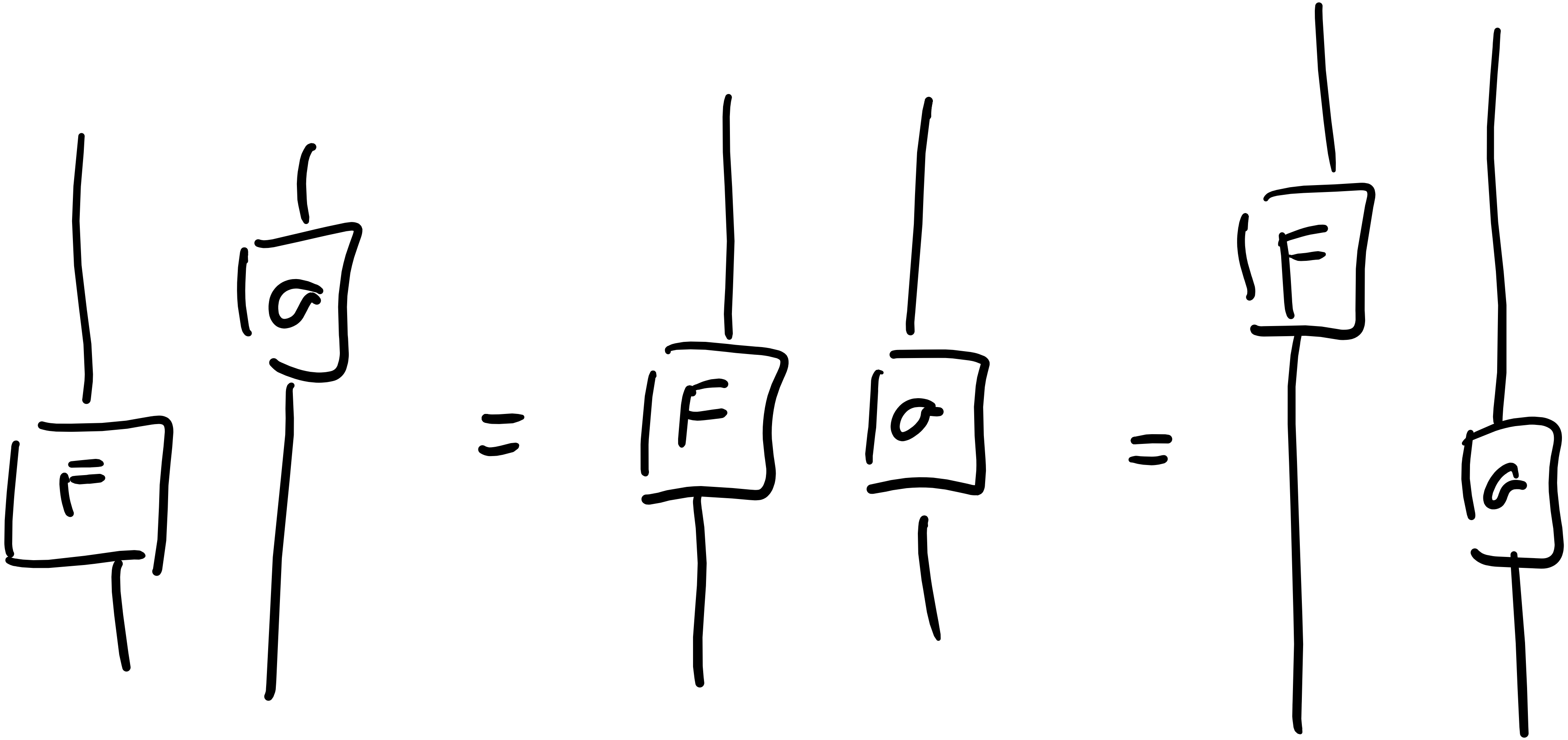

G
F
=
G
K



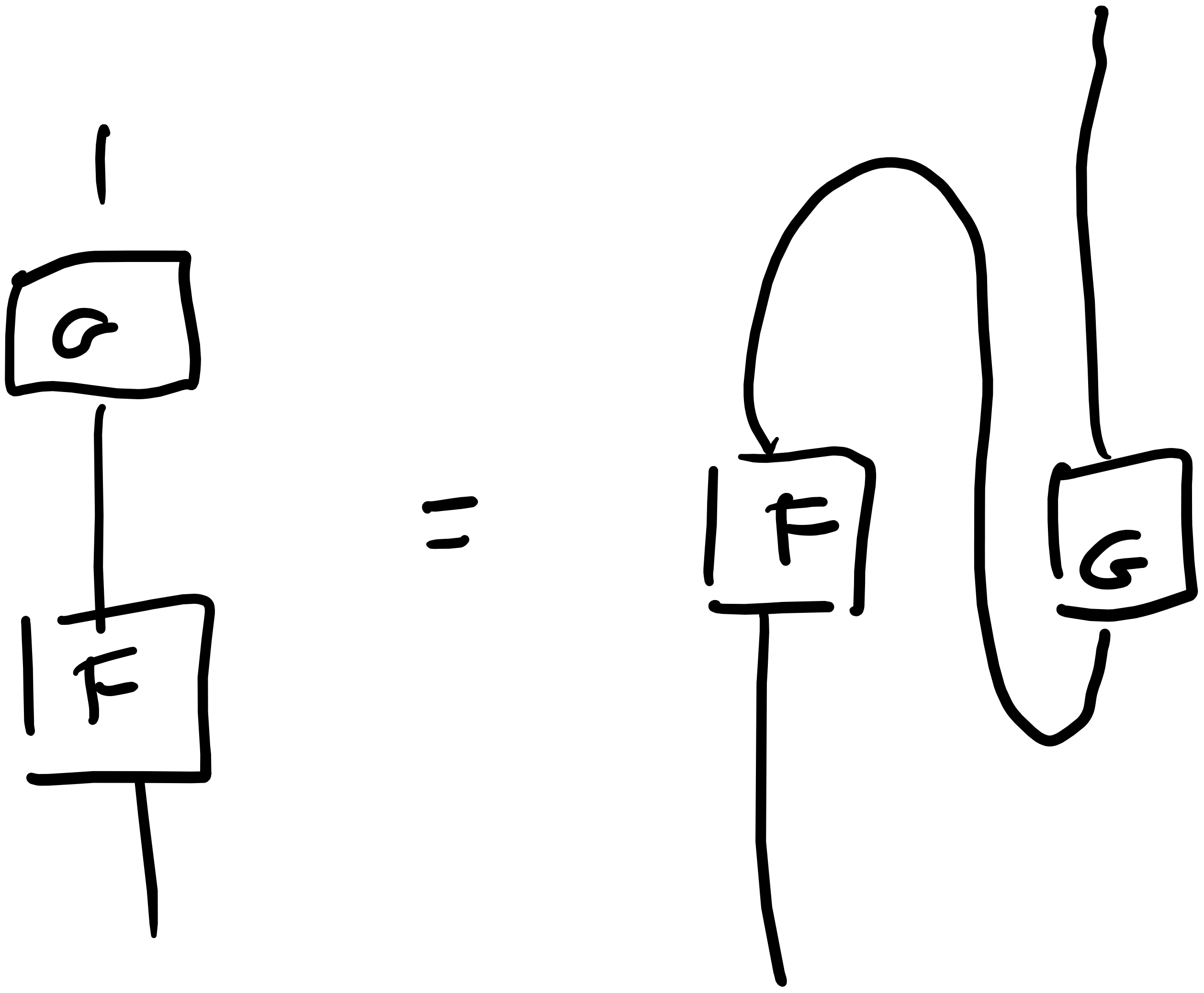

```
def A(x, y, z):
    _y = G(y)
    _x = F(x)
    s = H(x, y)
    t = K(z)
    return s, t
```

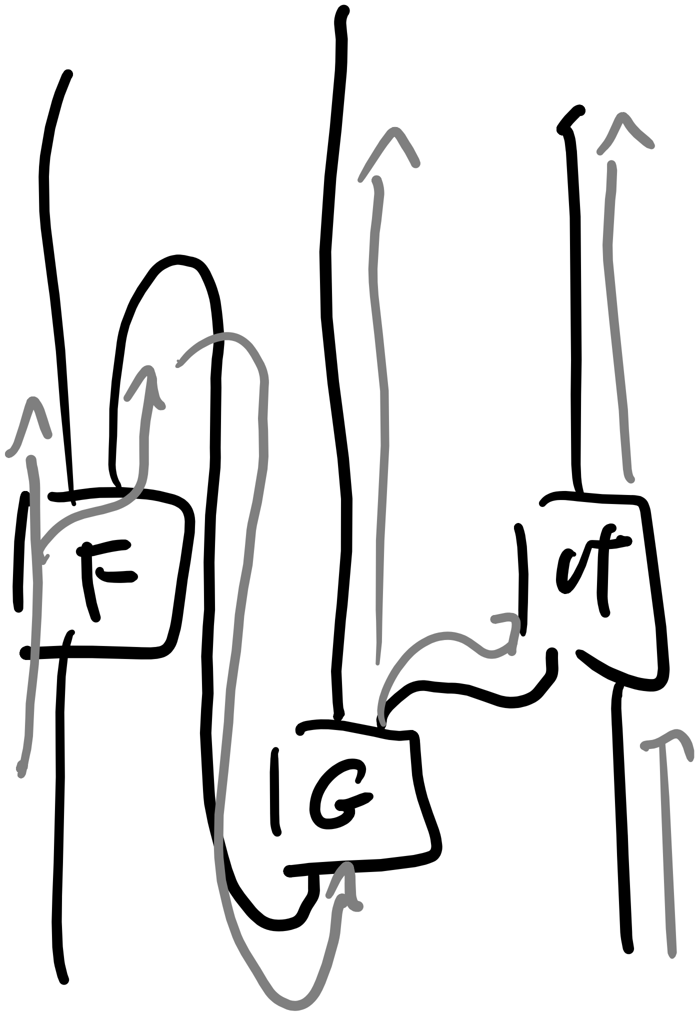


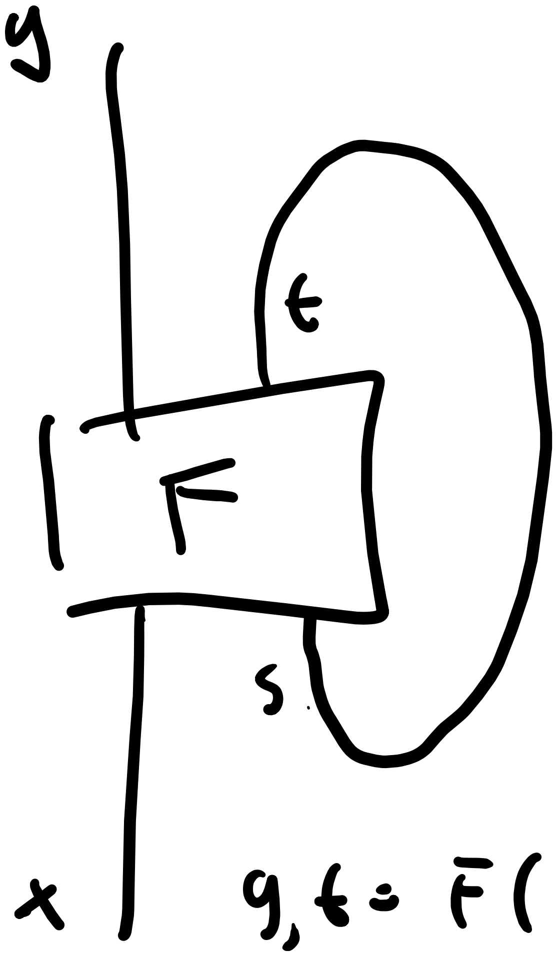

```
def A(x, y, z):
_x, _y, t = F(x), G(y), H(z)
    _
    s = H(x, y)
    t = K(z)
    return s, t
```



F
σ
=
F
σ
=
F
σ

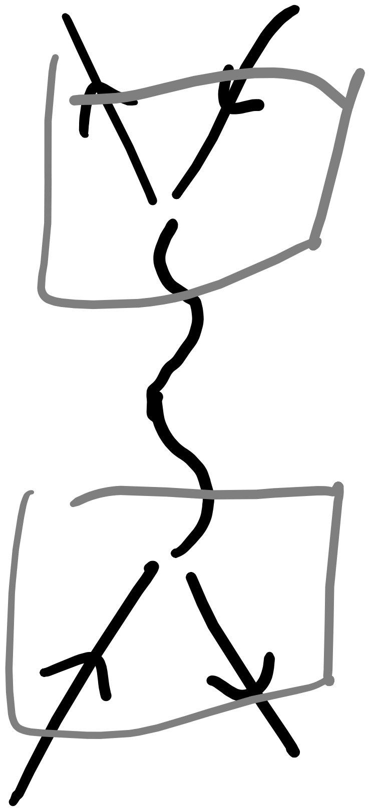

G
F
=
F
G





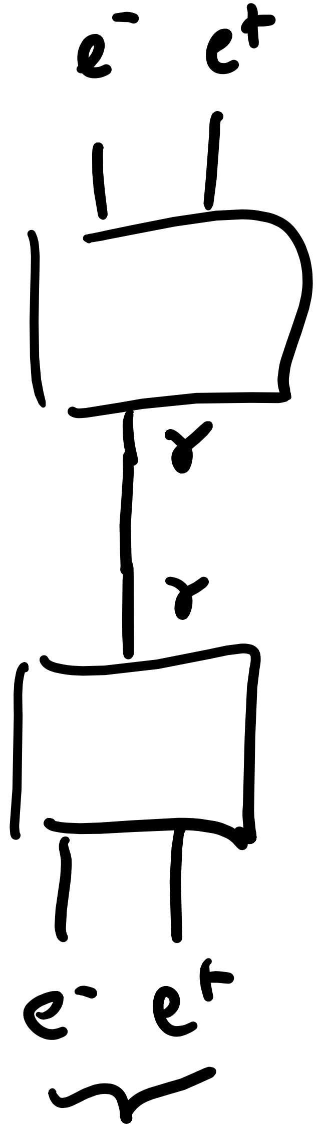
$y, t = F(x, s)$

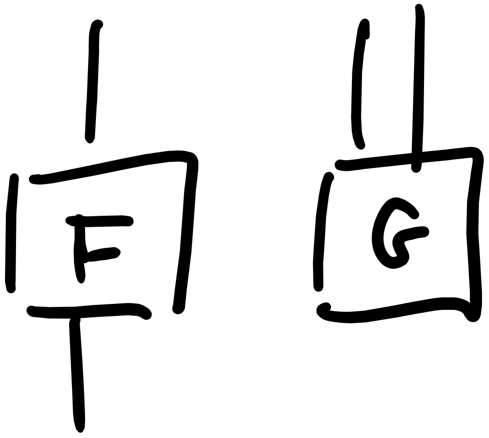
subj to $t = s$



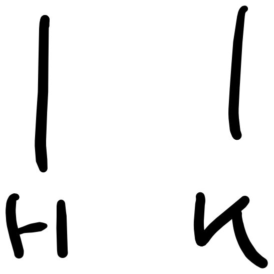
$\approx$

$(\neq)$

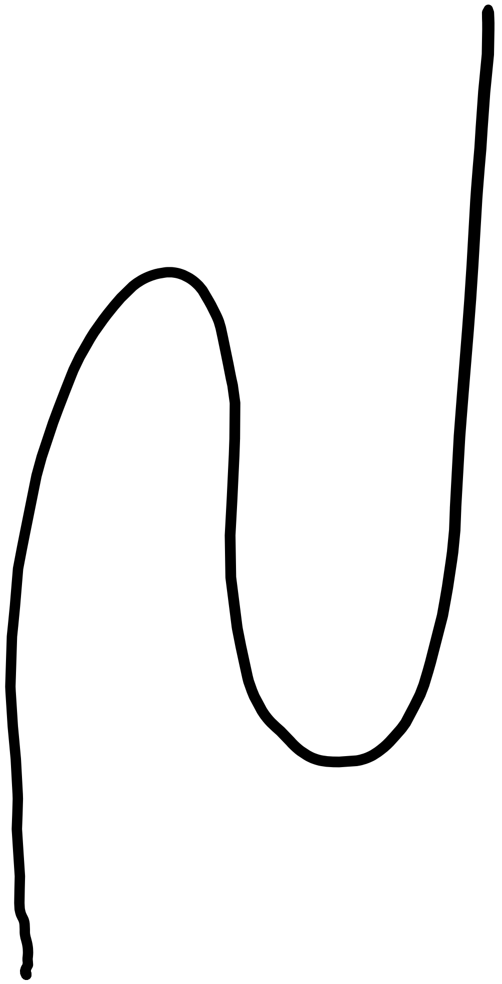




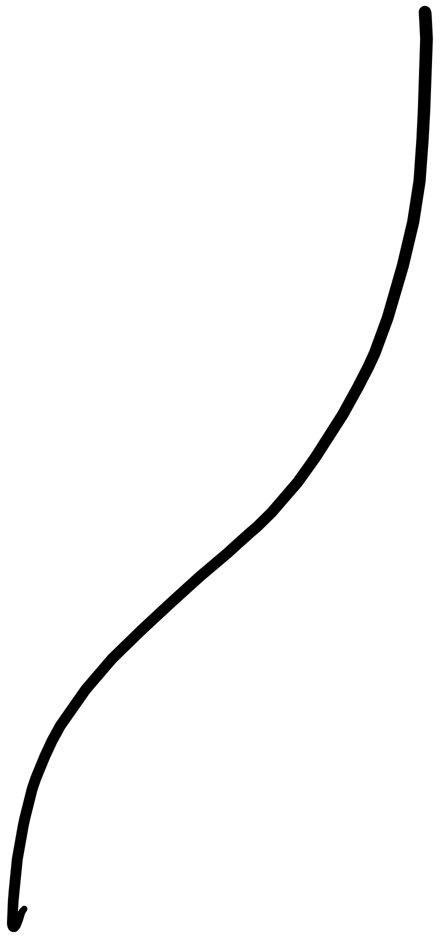
$$= F \otimes G$$

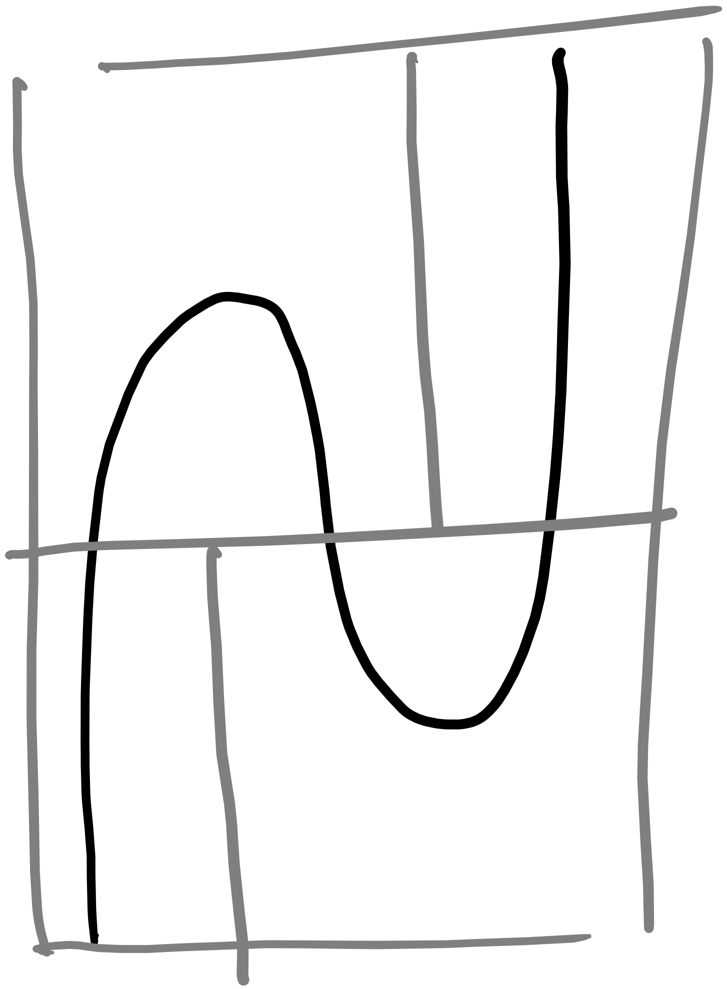


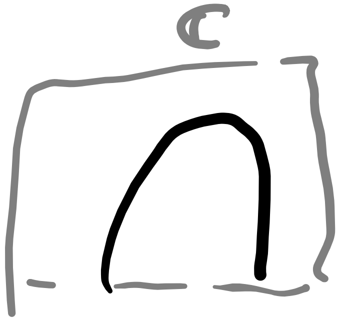
$$= H \otimes K$$



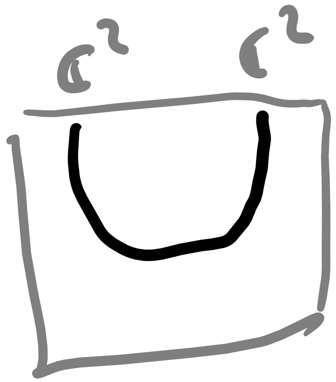
$\overset{\cancel{?}}{=}$ yes!





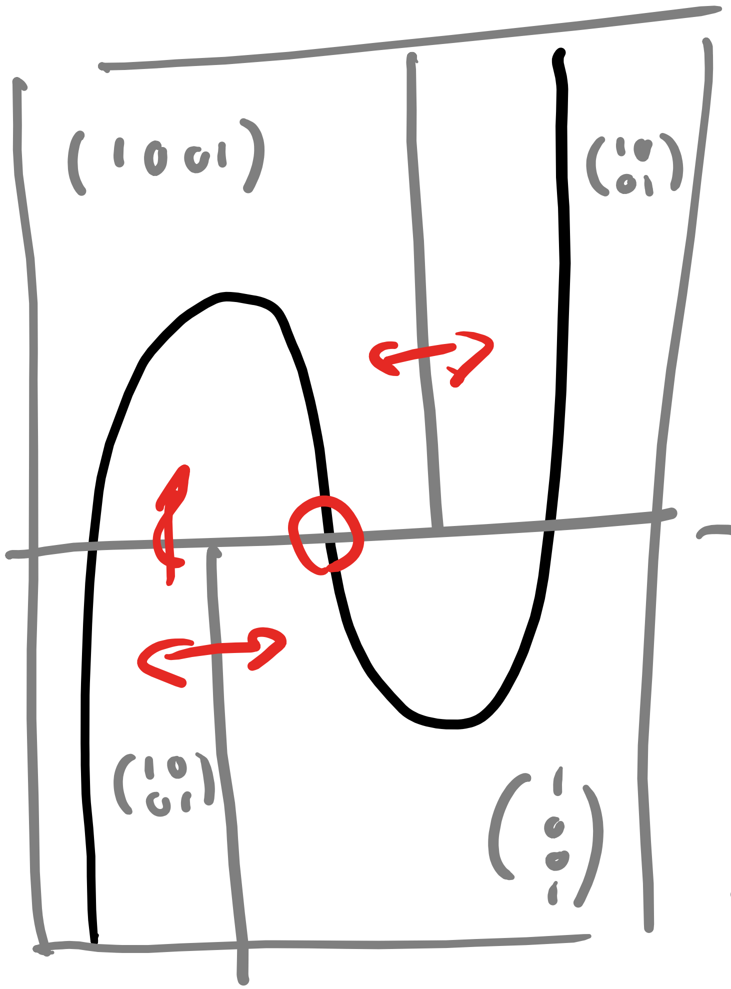


$$\begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \in \mathbb{C}^2 \otimes \mathbb{C}^2$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} =$$

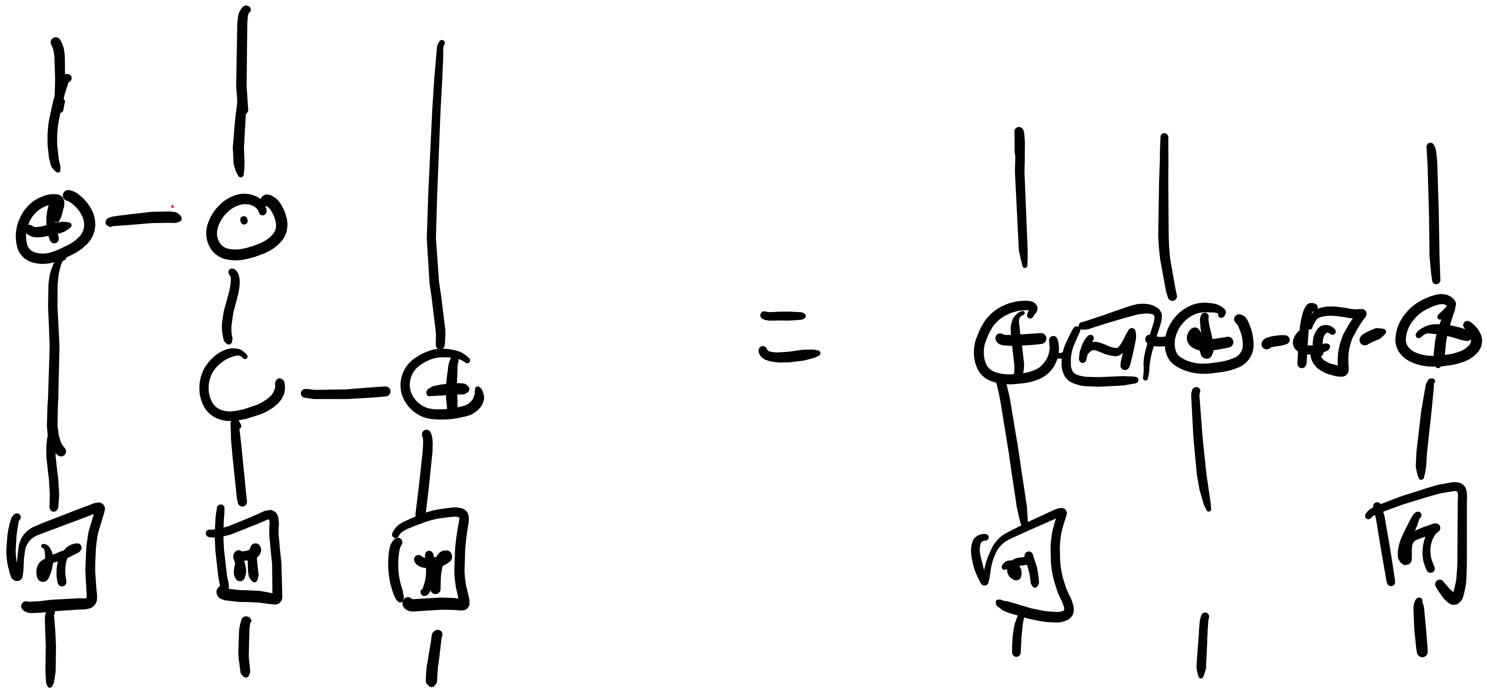


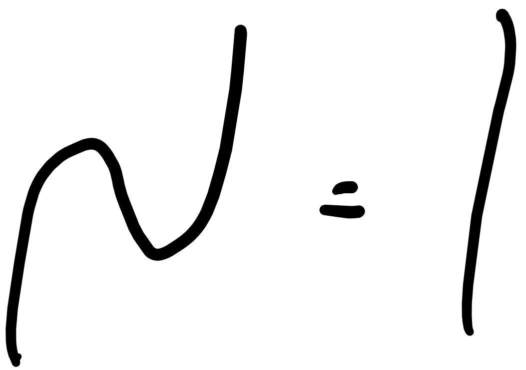
$$\begin{pmatrix}1&0&0&1\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix} = \left(\begin{matrix}1&0\\0&1\end{matrix}\;\;0\;\;0\;\;\begin{matrix}1&0\\0&1\end{matrix}\right)$$

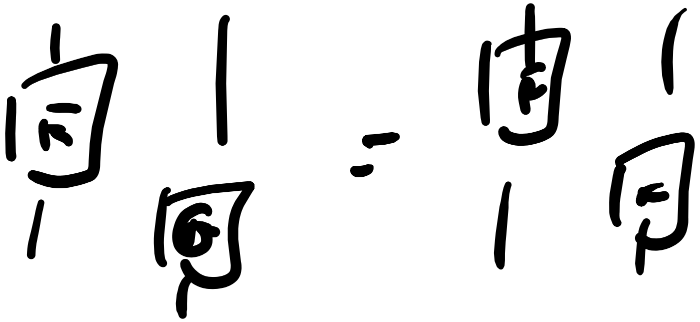
$$\cdot$$

$$\begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\begin{pmatrix}1\\0\\0\\1\end{pmatrix} = \begin{pmatrix}\begin{matrix}1\\0\\0\\1\end{matrix} & 0\\ 0 & \begin{matrix}1\\0\\0\\1\end{matrix}\end{pmatrix}$$

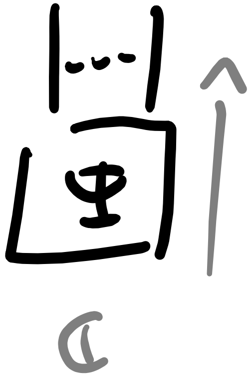
$$= \begin{pmatrix}1&0\\0&1\end{pmatrix}$$



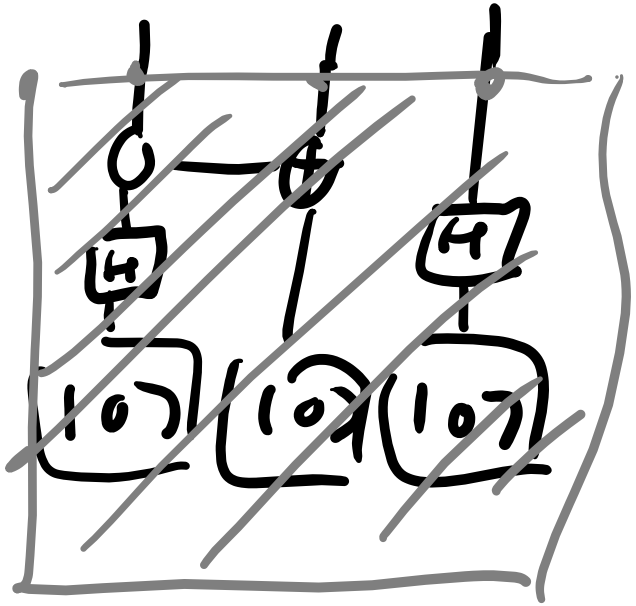




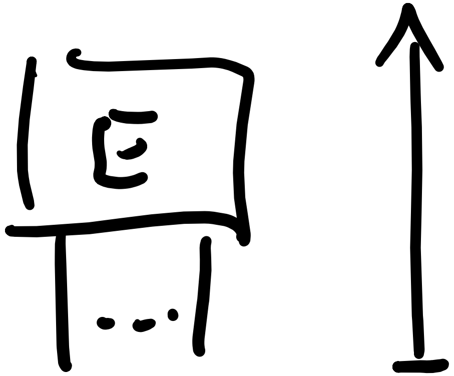
systems



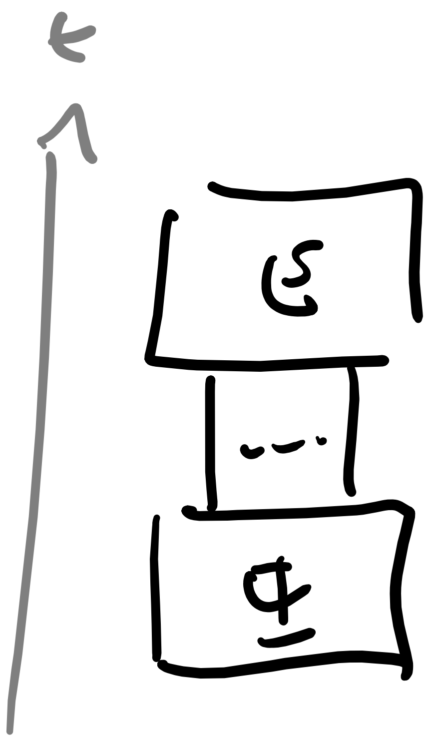
State



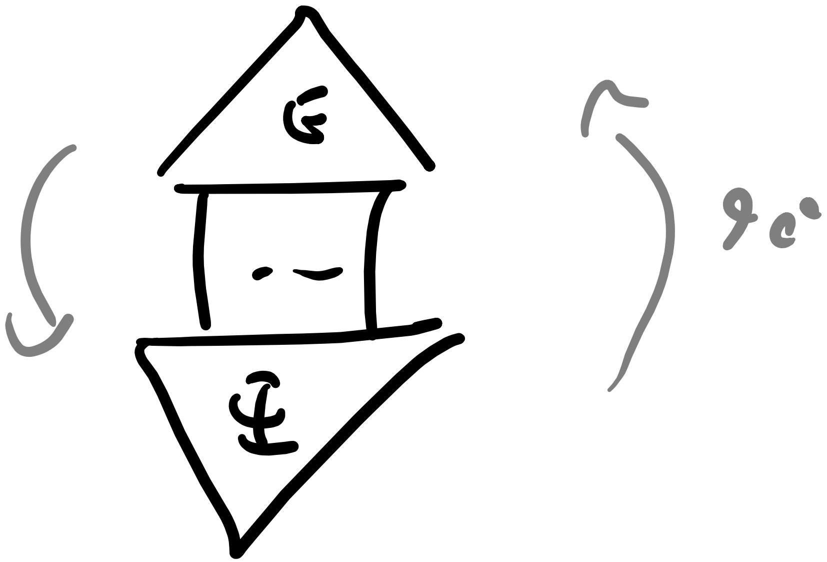
$\langle E | \Psi \rangle \in \mathbb{C}$

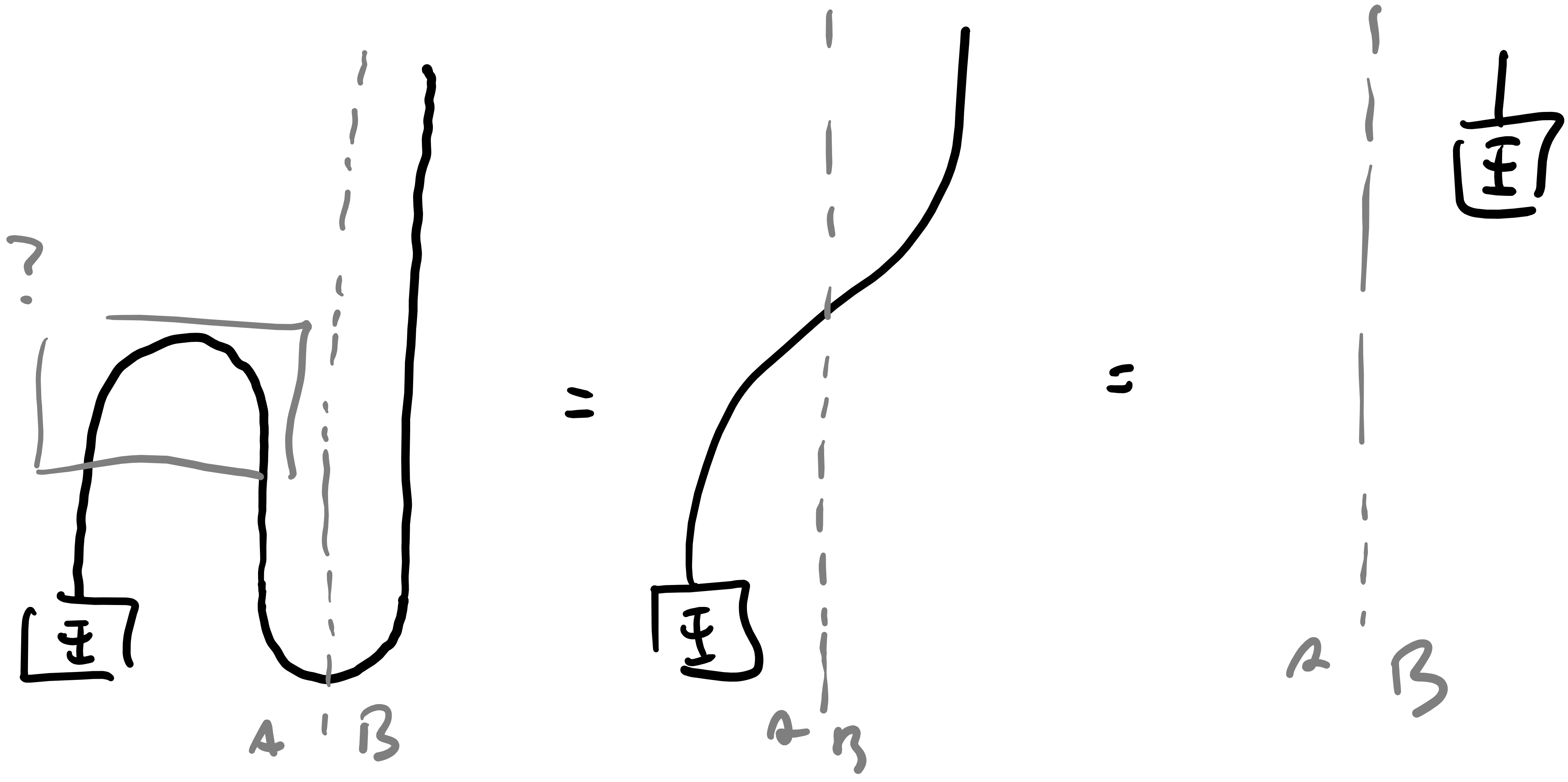


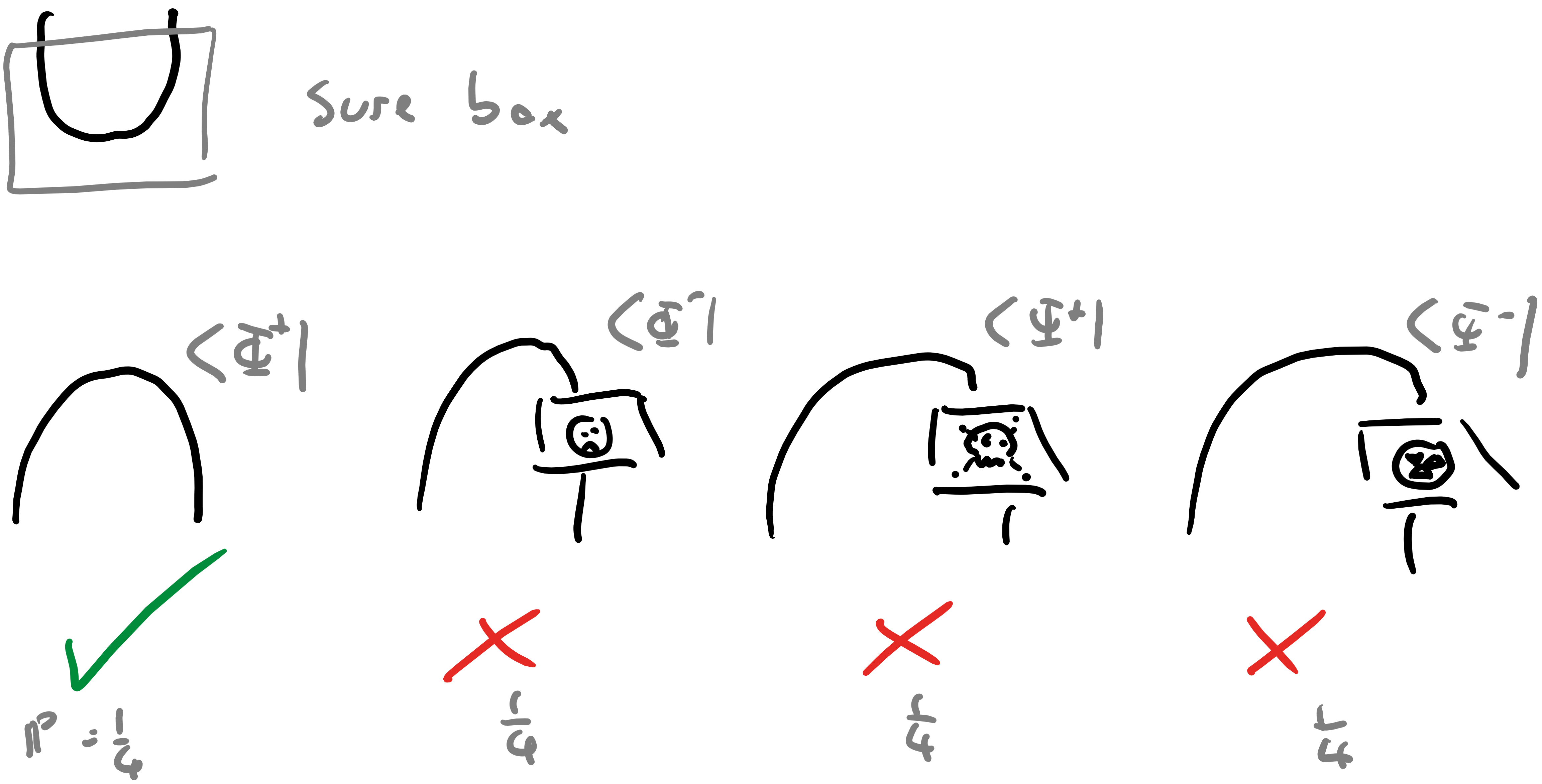
$|\Psi\rangle$

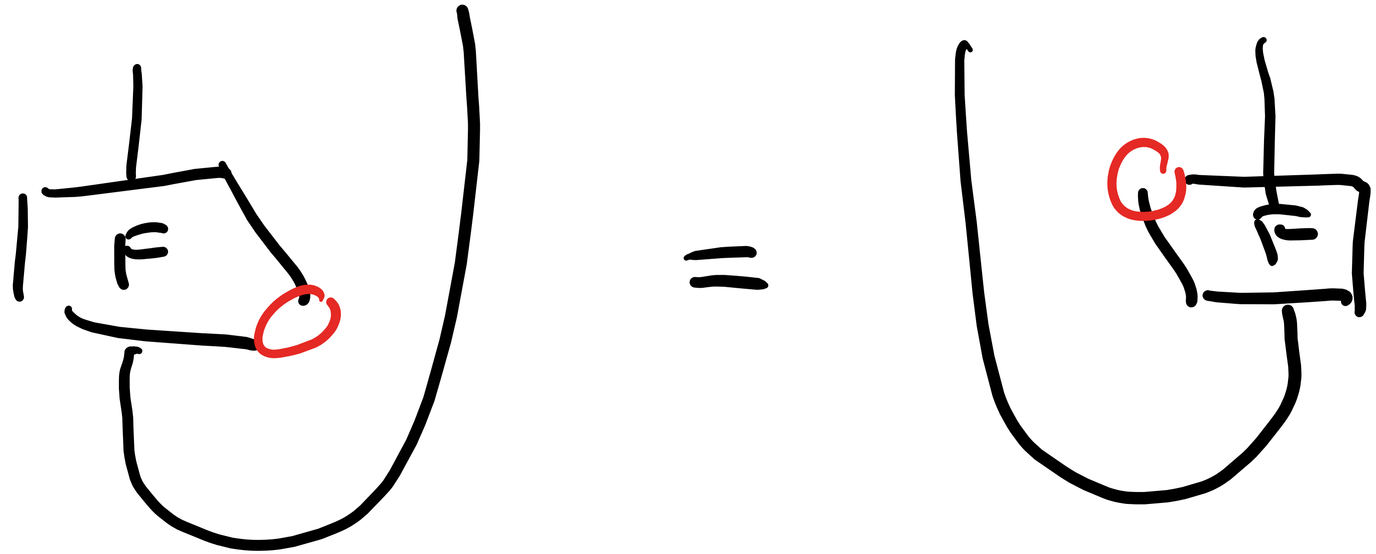


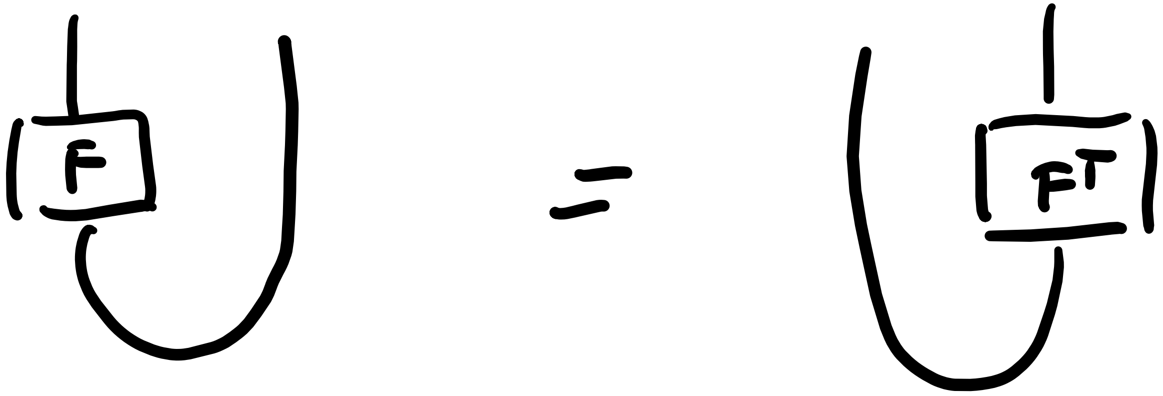
$\langle \mathcal{G} | \mathcal{I} \rangle \in \mathbb{C}$

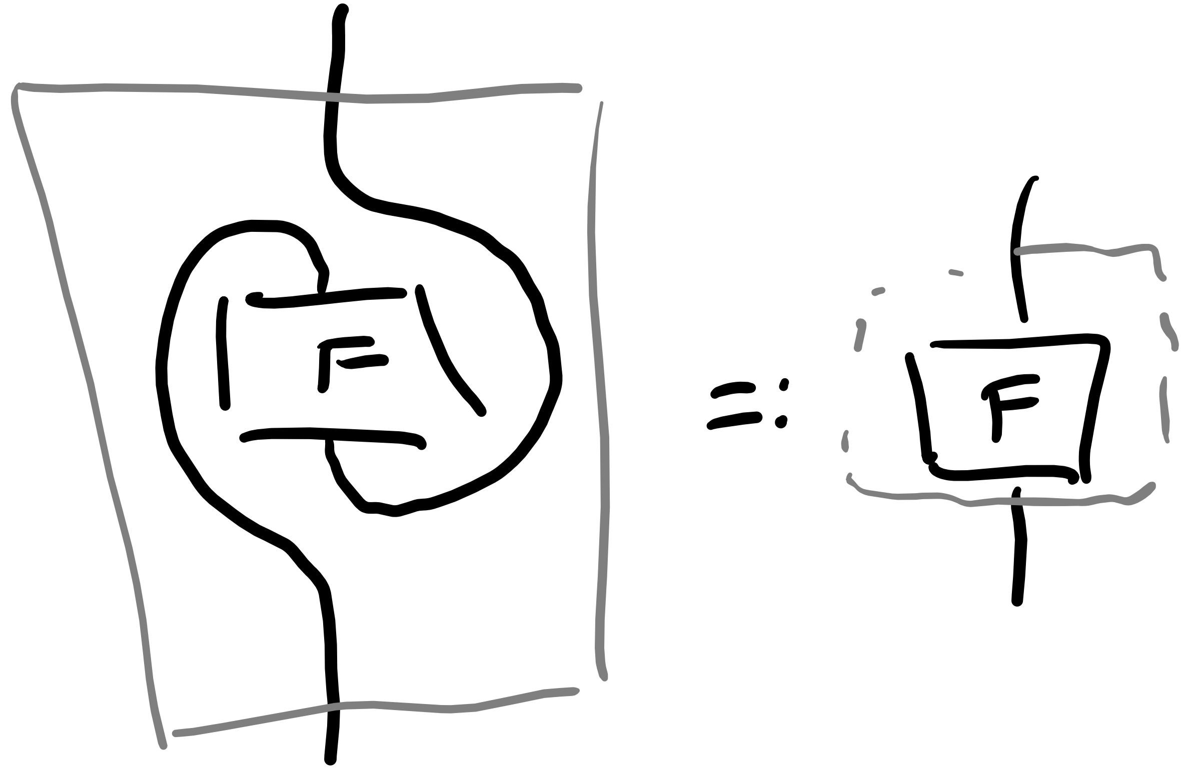


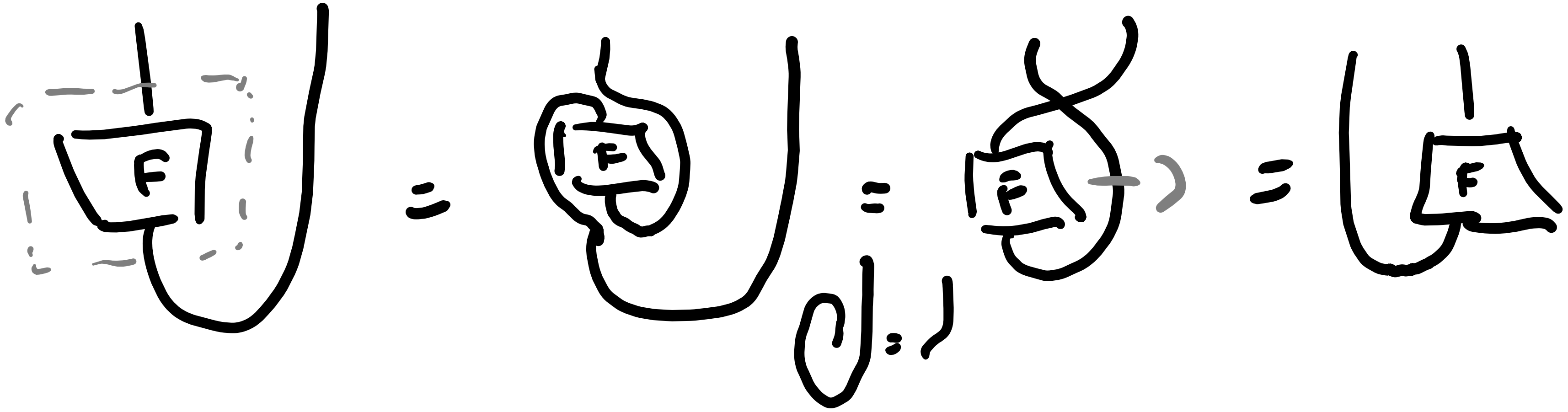

?
A
B
=
A
B
=
A
B

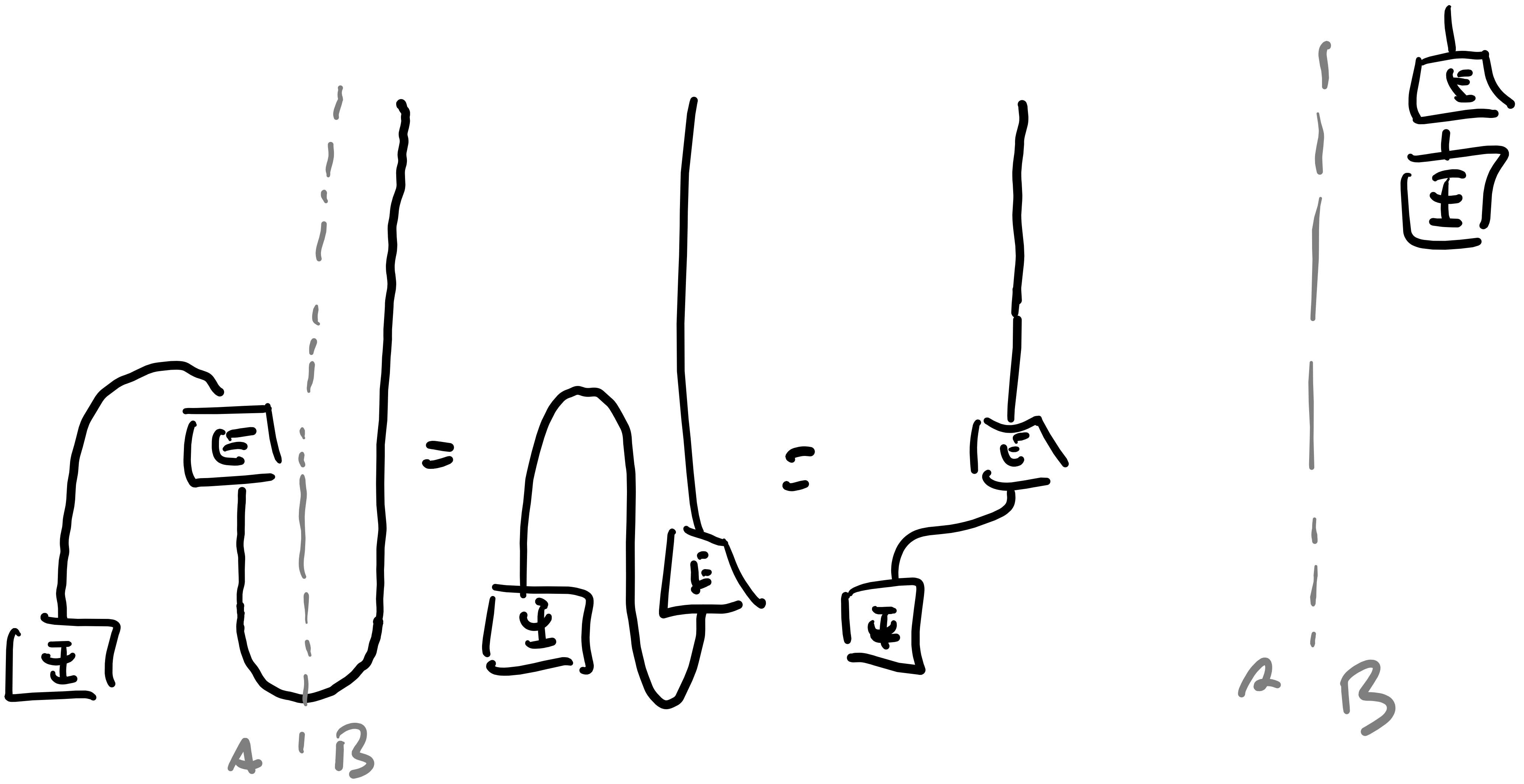

Sure box
$\langle \Phi^+|$
$\langle \Phi^-|$
$\langle \Psi^+|$
$\langle \Psi^-|$
$P = \frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$

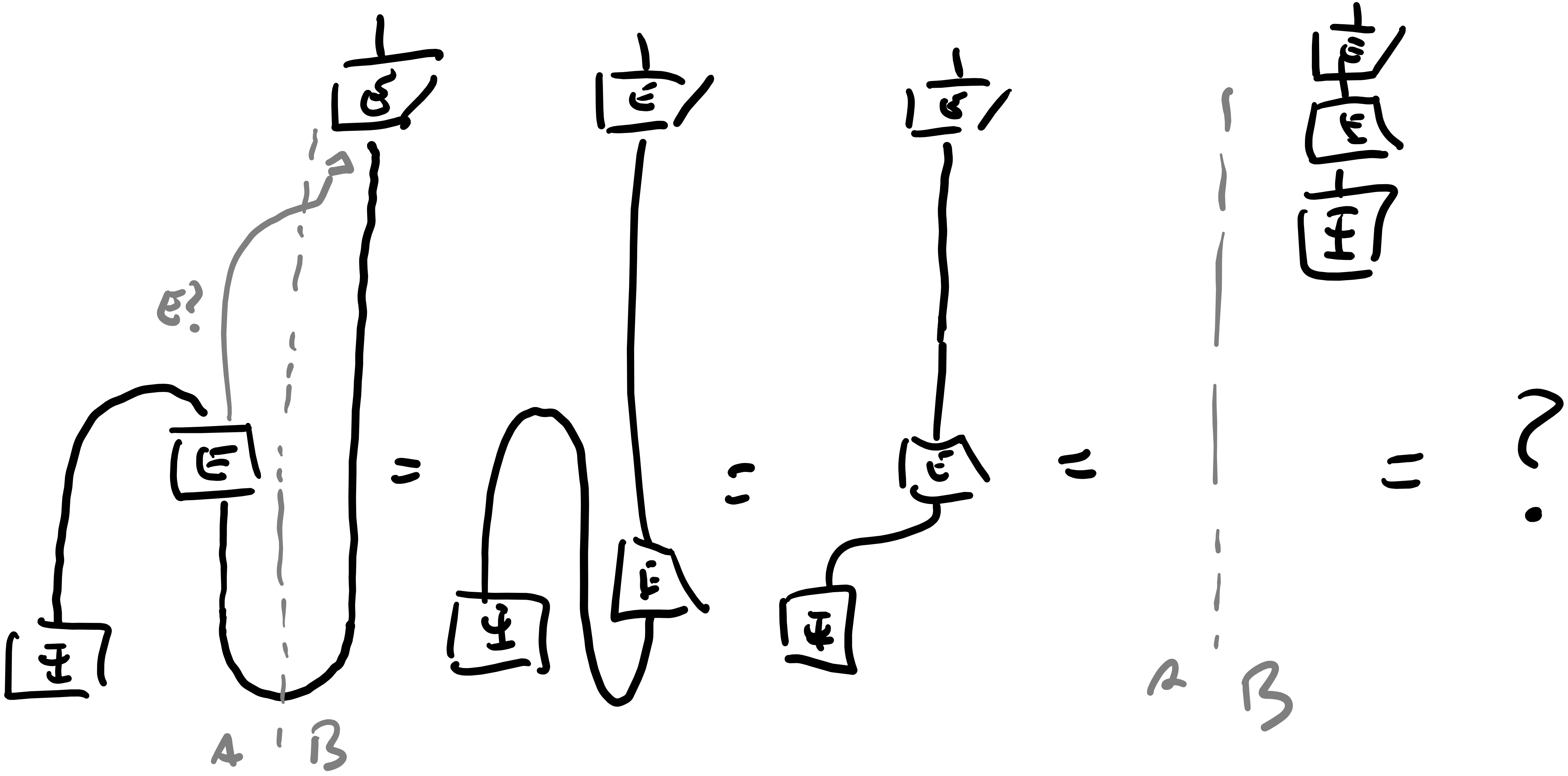

F
=


F
=
F^T

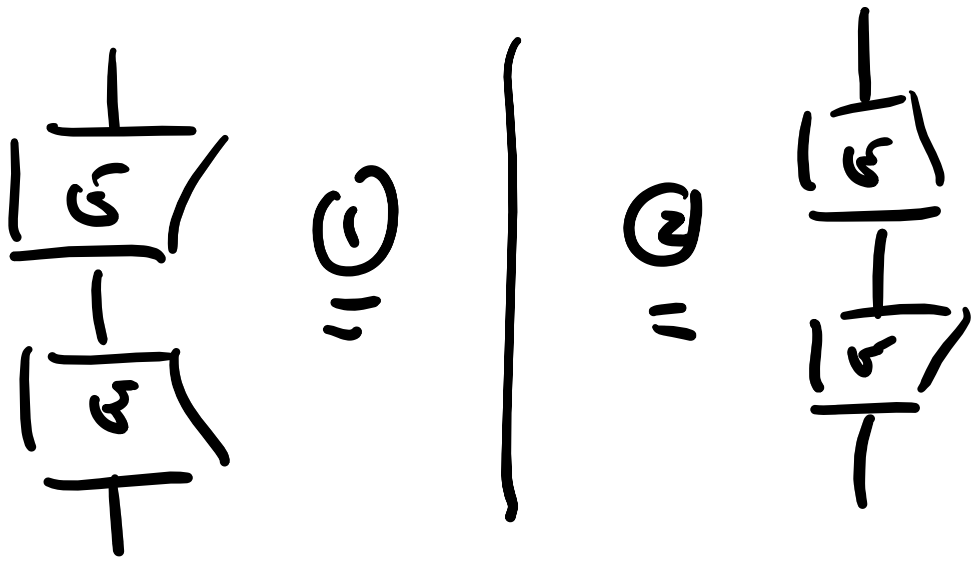

F
=:
F

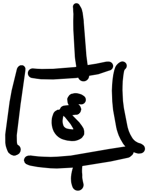

F
=
F
=
F
=
F
=:

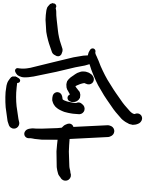

A
B
A
B


e?
A
B
=
=
=
=
?
A
B

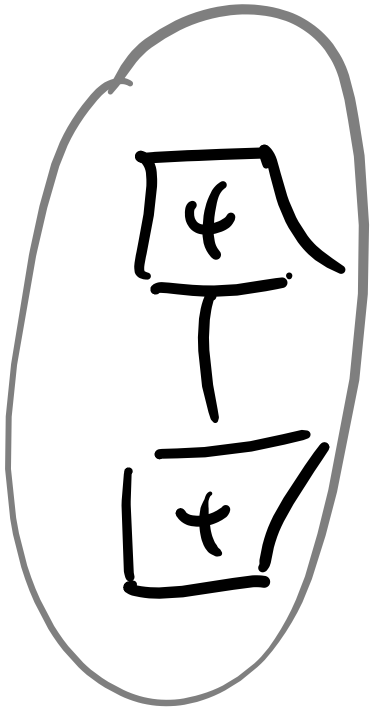
$G \in \mathbb{C}^{n \times m}$ $\quad$ $G^\dagger \in \mathbb{C}^{m \times n}$



①  isometry $G^\dagger G = I$

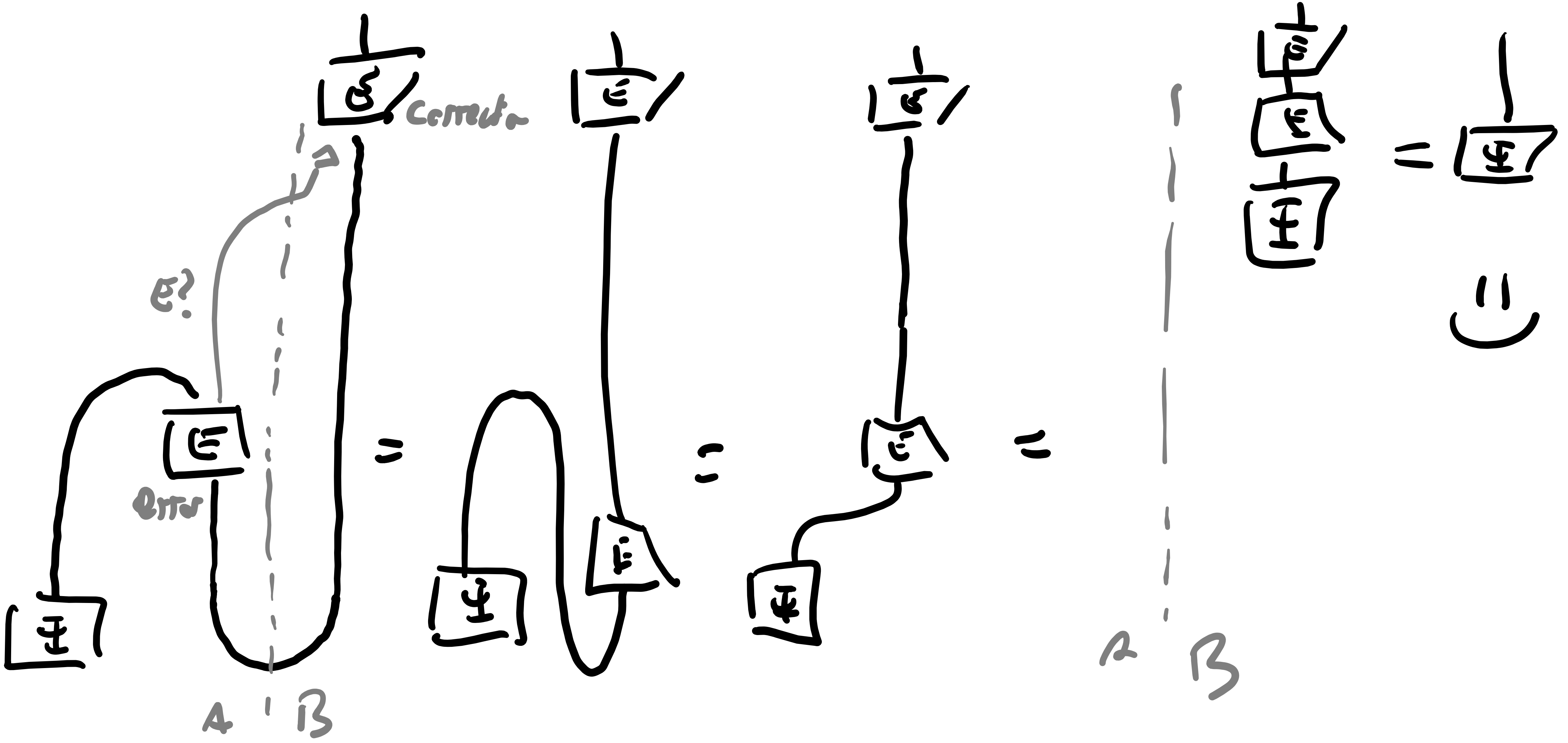
① ②  unitary $G^\dagger G = I = G G^\dagger$

$\langle\psi|\psi\rangle = 1$

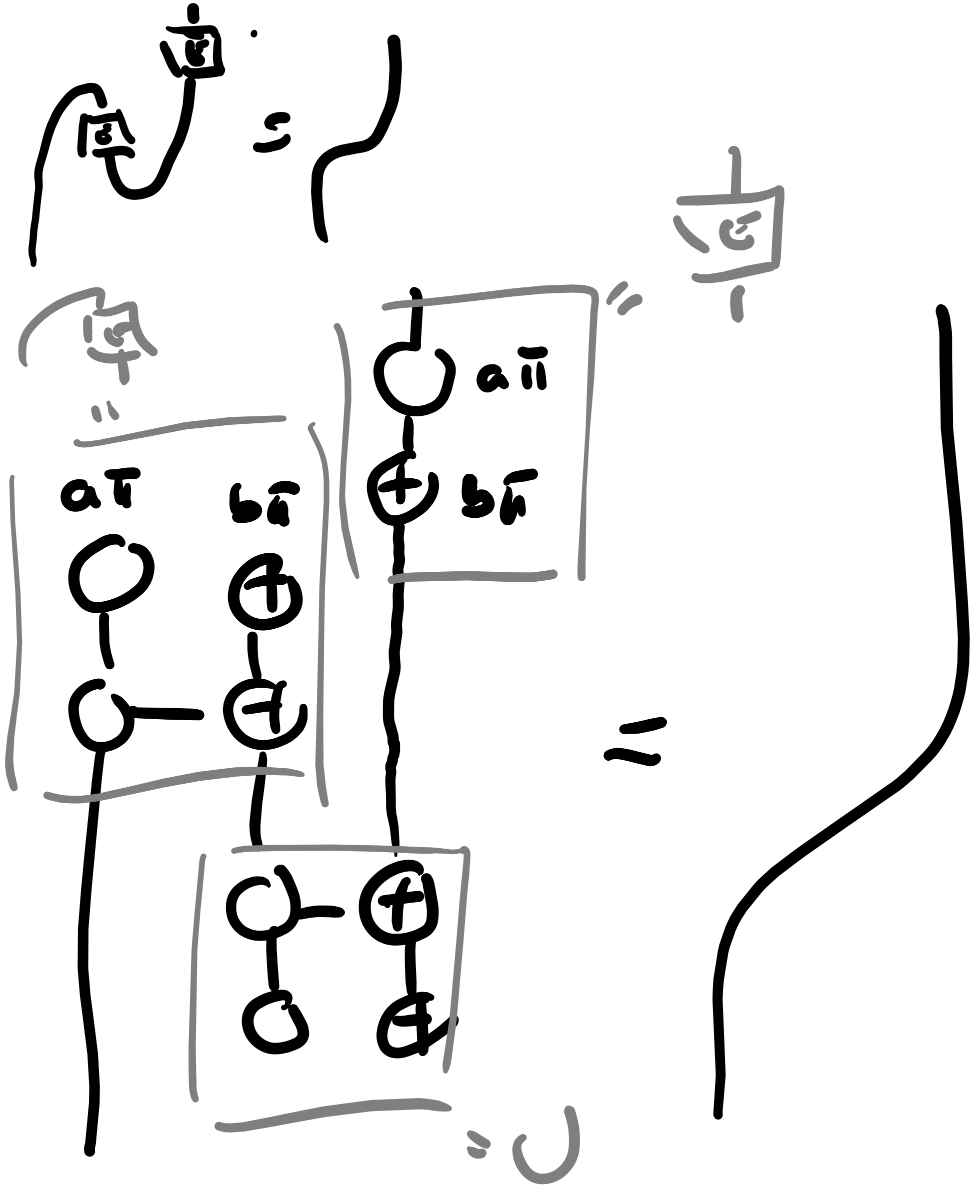


$\leftrightarrow \langle\psi|$

$\leftrightarrow |\psi\rangle$



Corrector
E?
Error
A
B
=
=
=
A
B
=



aπ
bπ
aπ
bπ