

# Quantum in Pictures Lecture Series

Lecturer: Stefano Gogioso

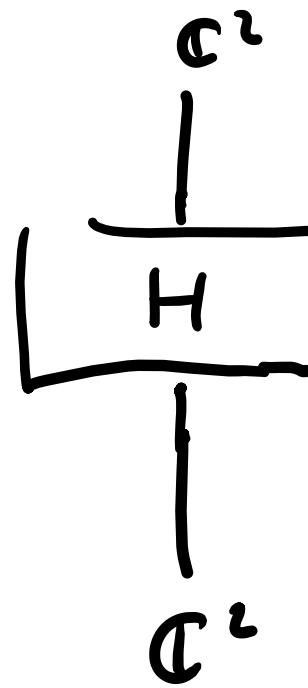
Mon 26 June 2023 – Morning Lecture



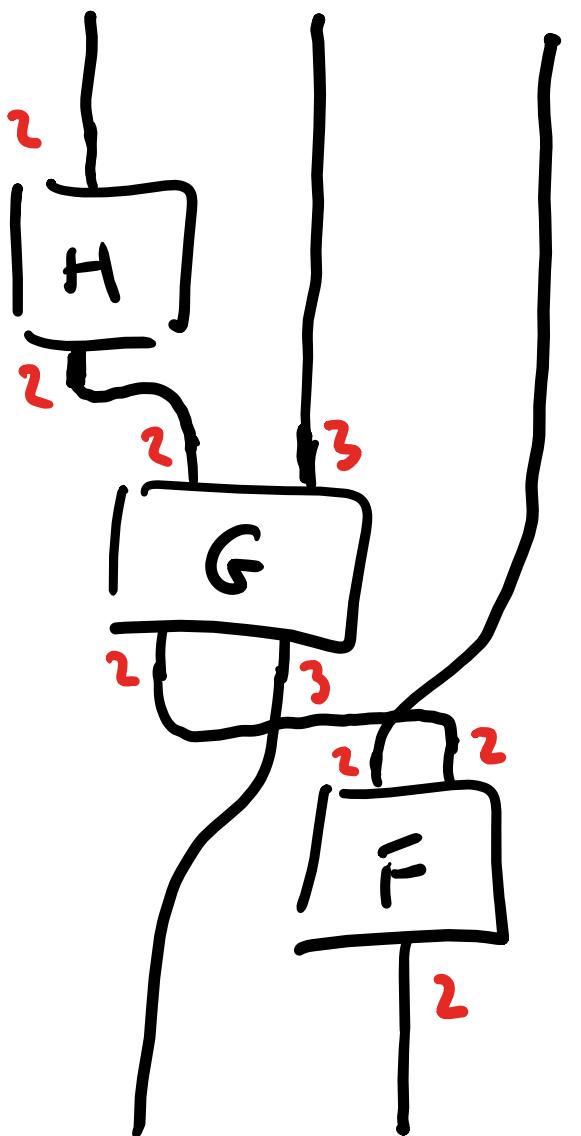
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$$|c\rangle = \begin{pmatrix} 1 \\ c \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



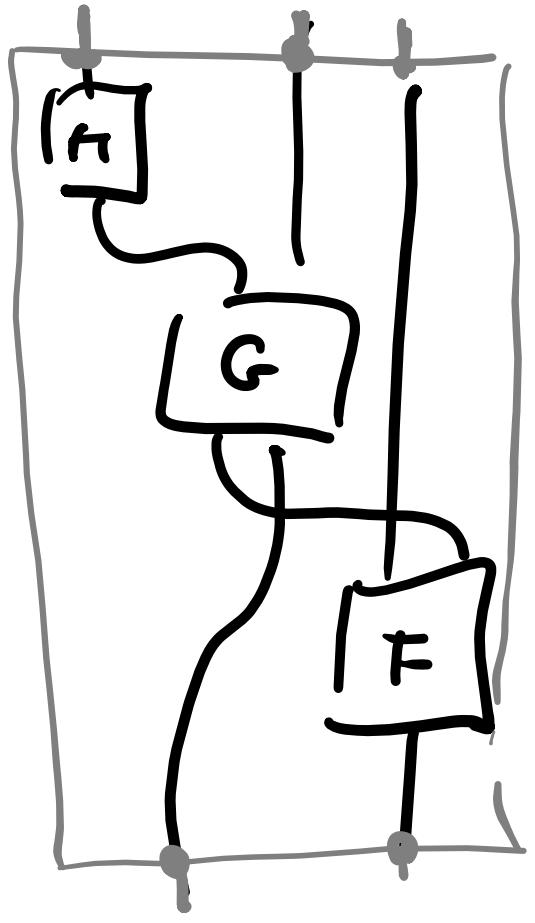
$\uparrow^t$



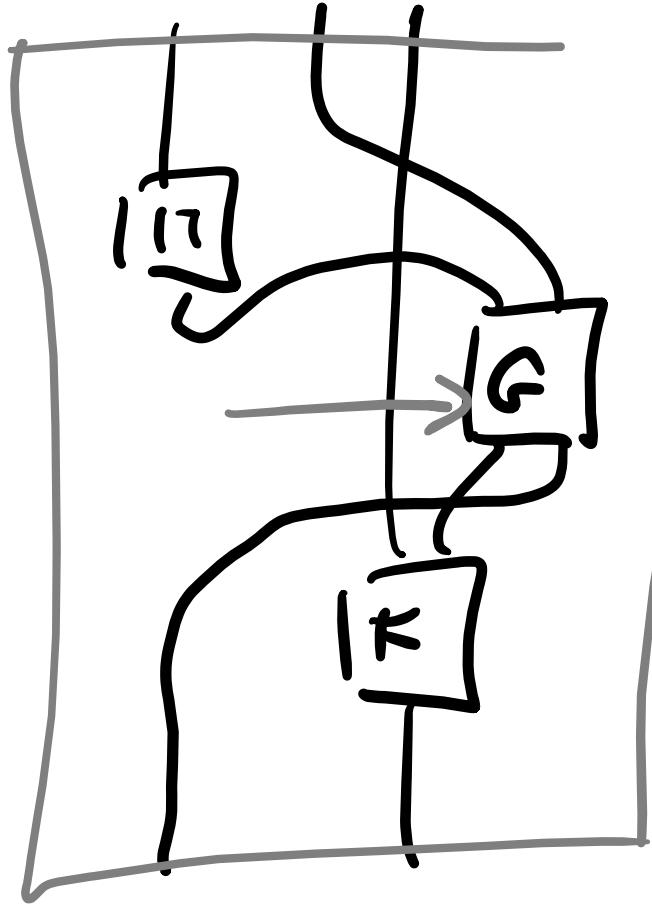
| A

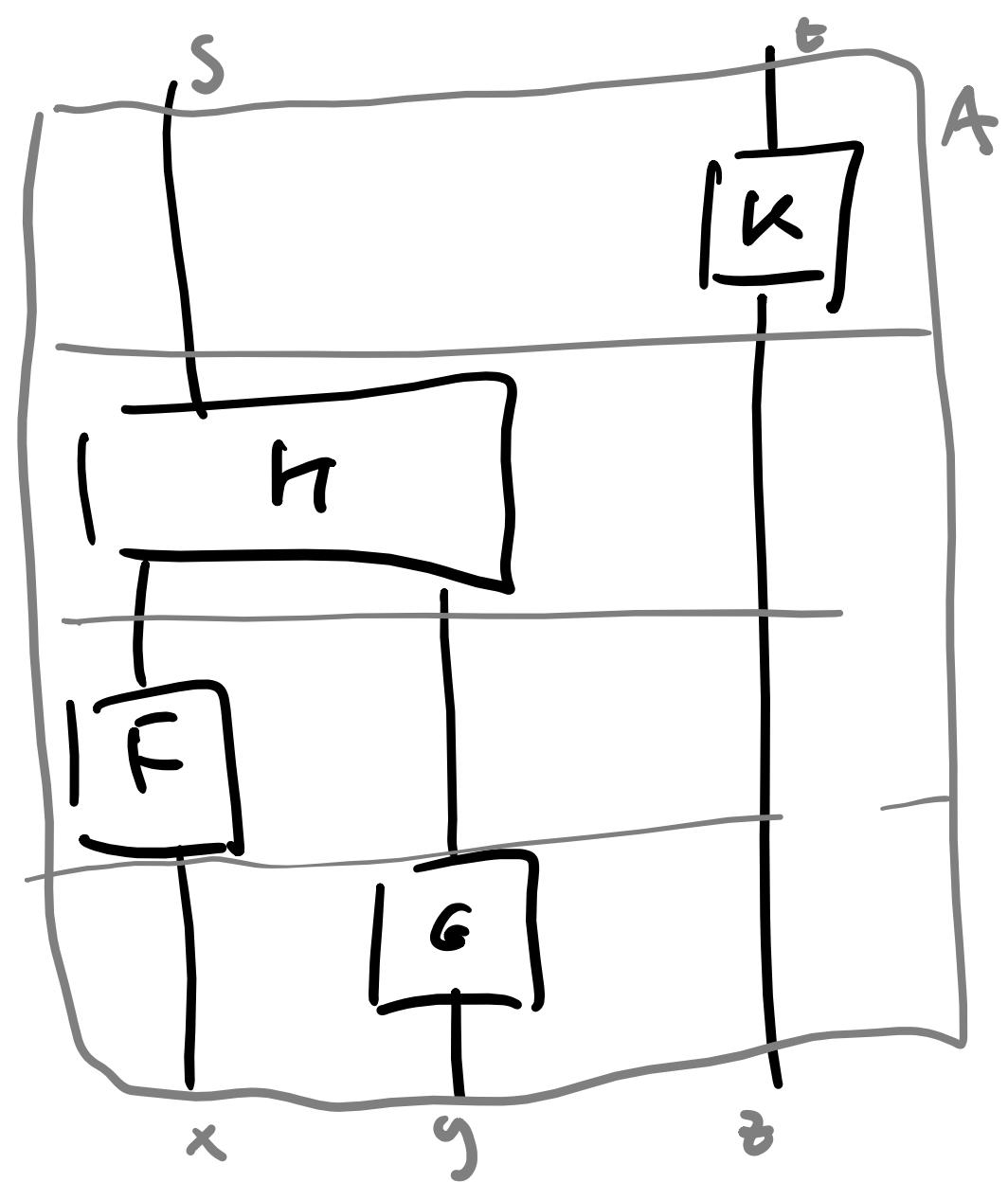
→  
| B

| A   t = 1  
X | AFB



=





def  $A(x, y, z) :$

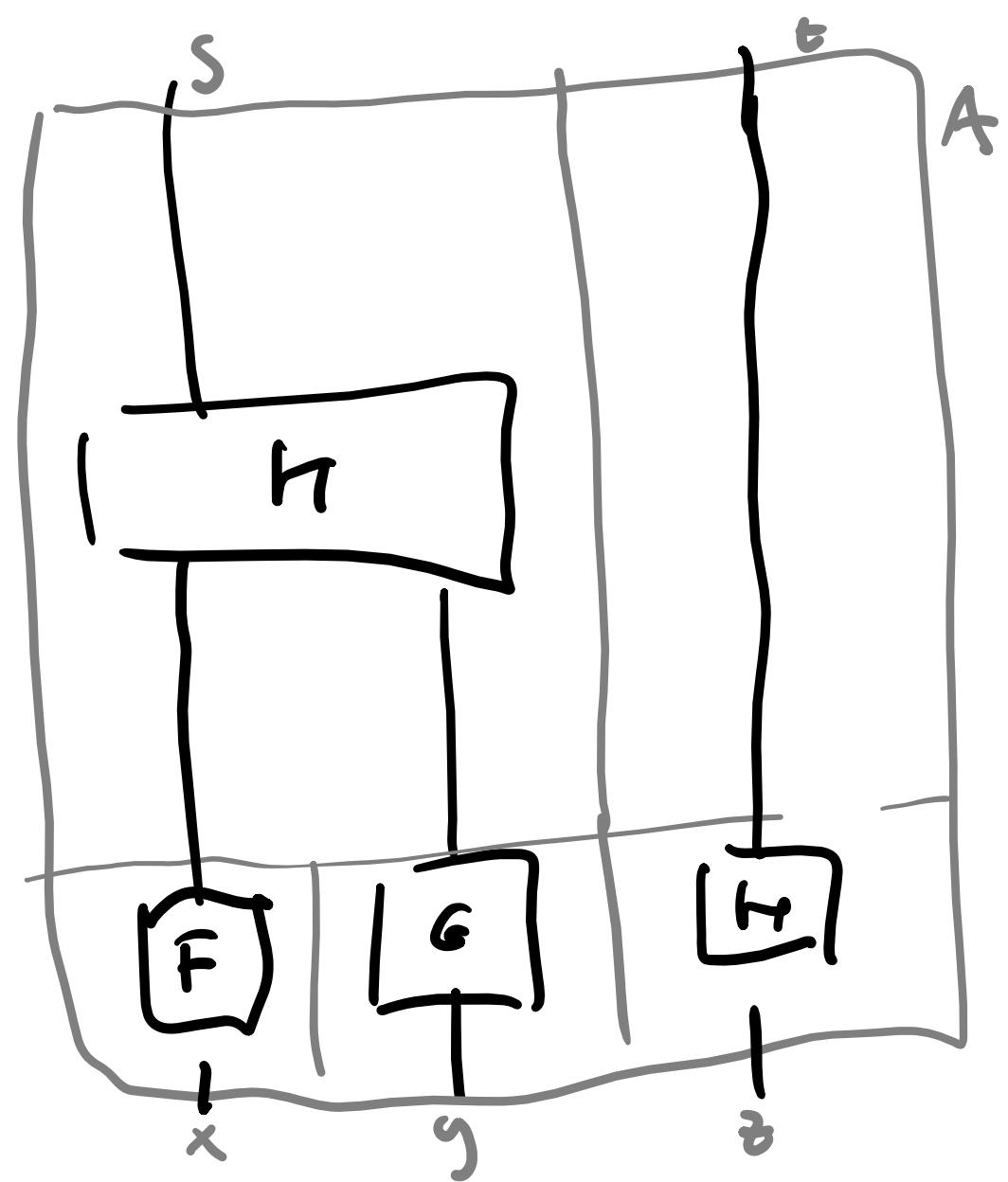
$$-y = G(y)$$

$$-x = F(x)$$

$$s = H(x, y)$$

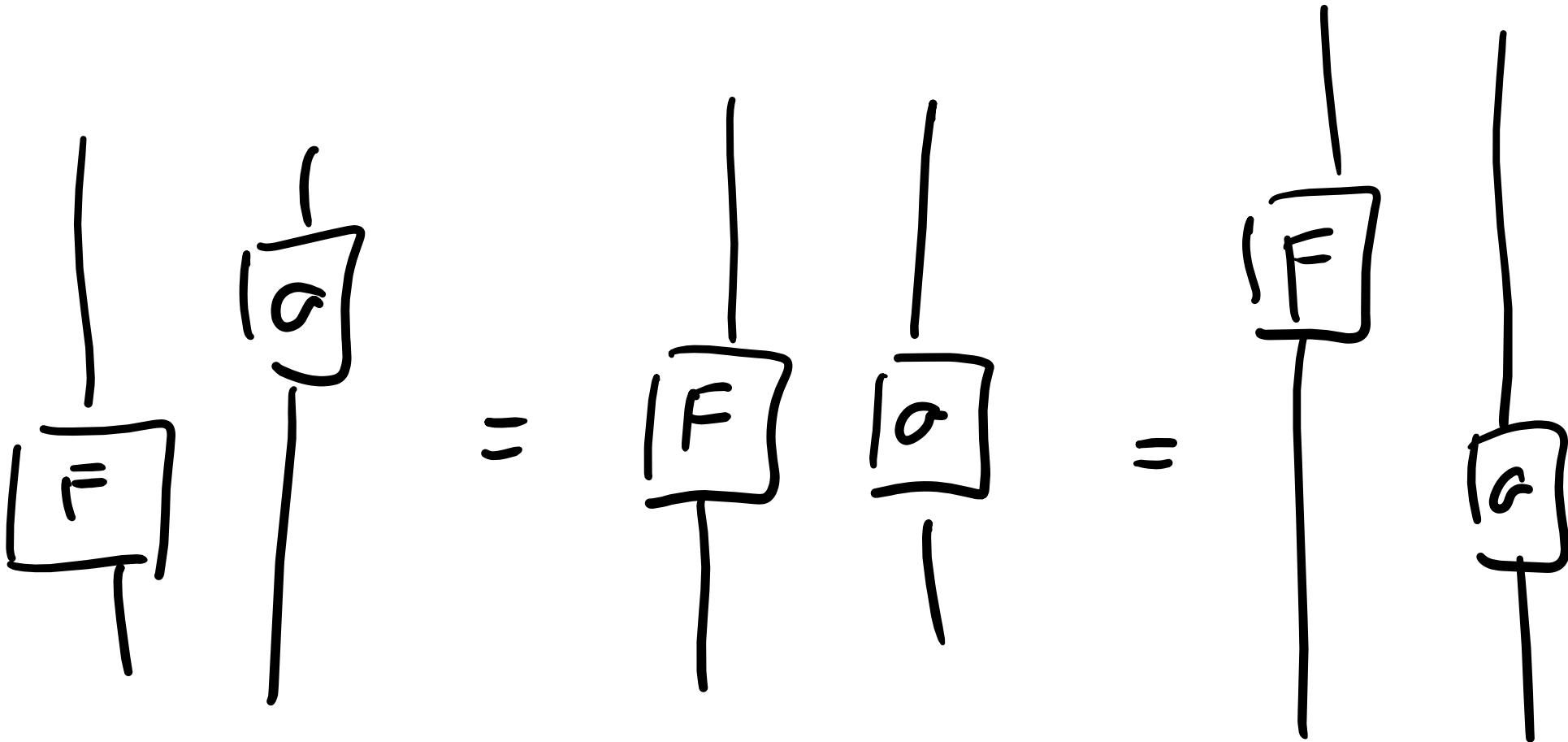
$$t = K(z)$$

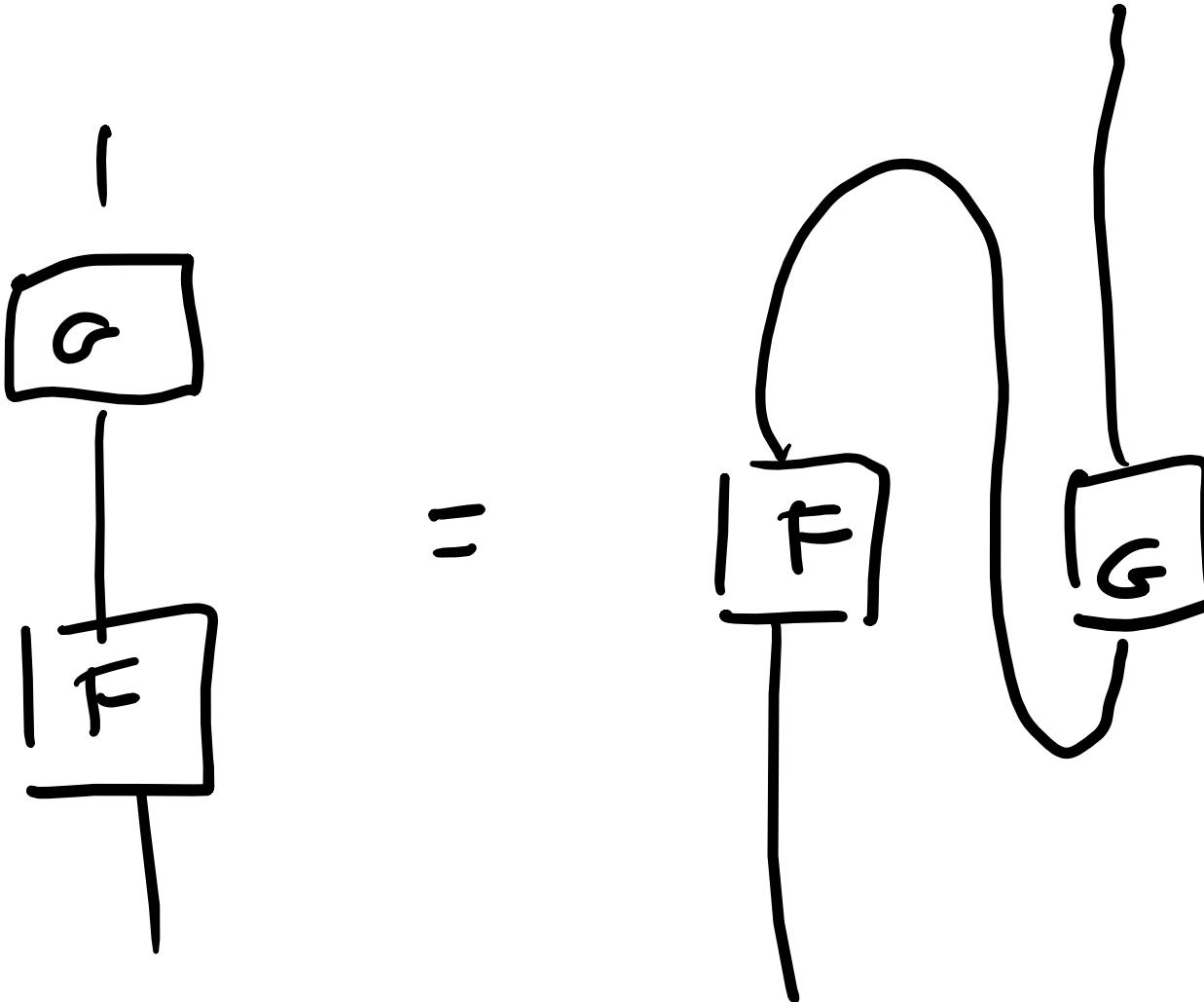
return  $s, t$

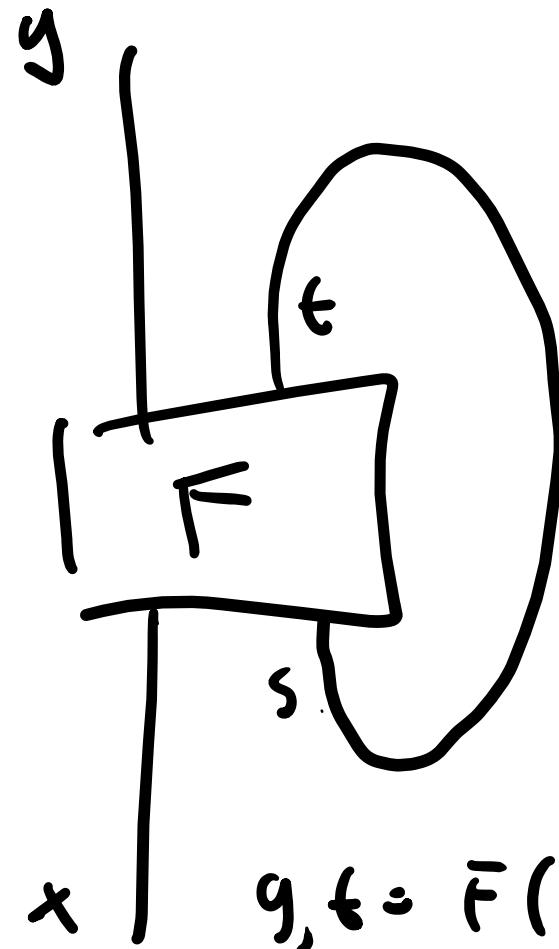
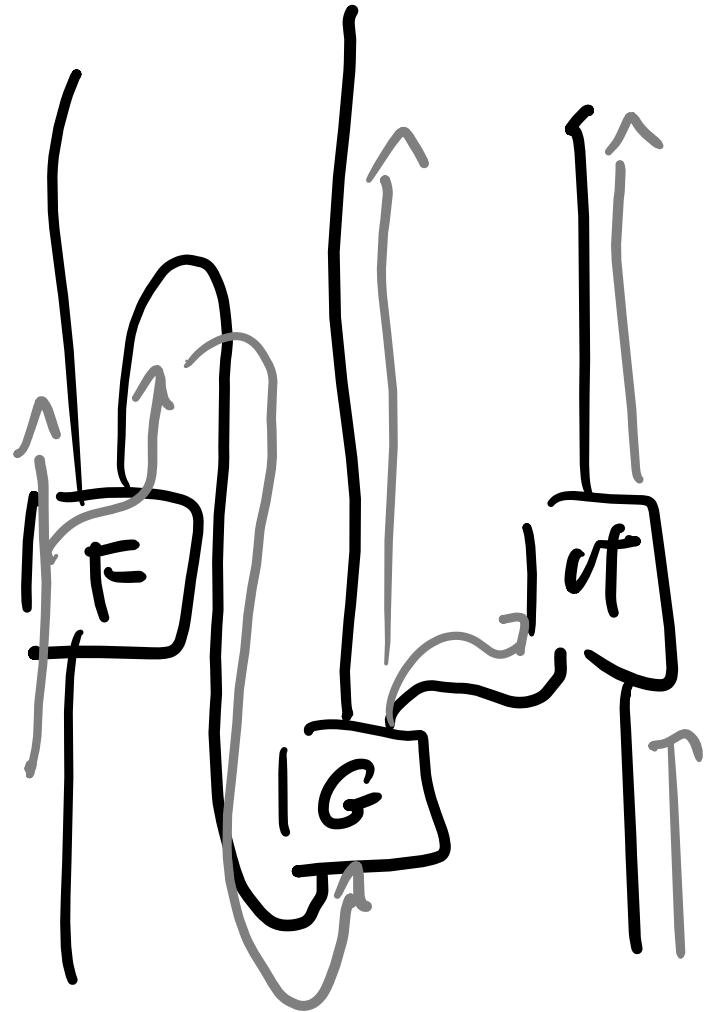


def  $A(x, y, z)$ :

- $x, y, z = F(x), G(y), H(z)$
- 
- $s = h(x, y)$
- $t = k(z)$
- return  $s, t$

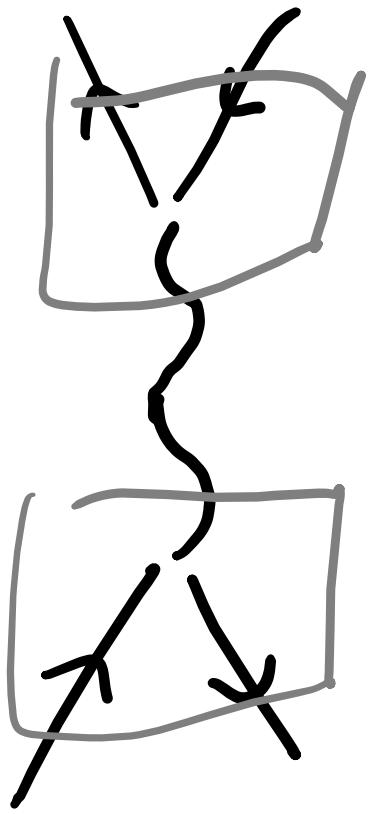




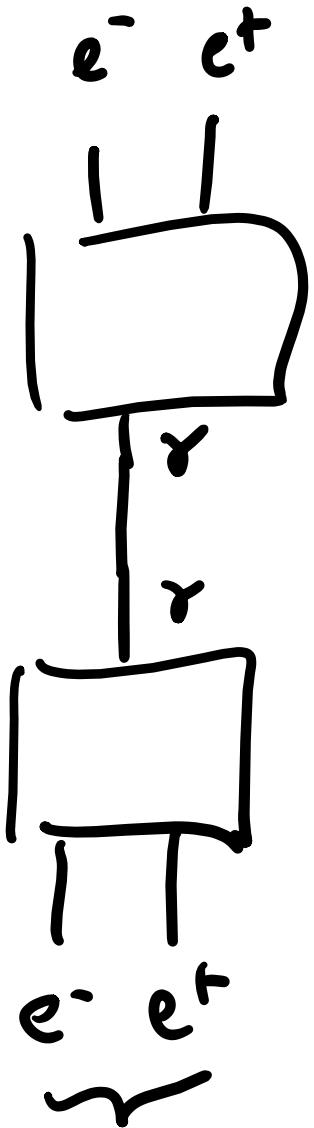


$$g, t = \bar{F}(x, s)$$

s.t.  $t = s$

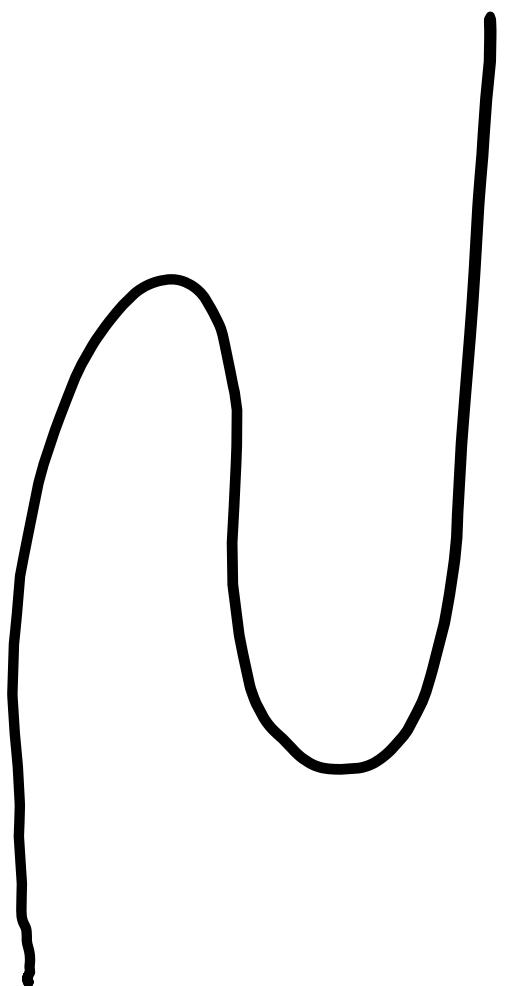


$\approx$   
 $(\neq)$

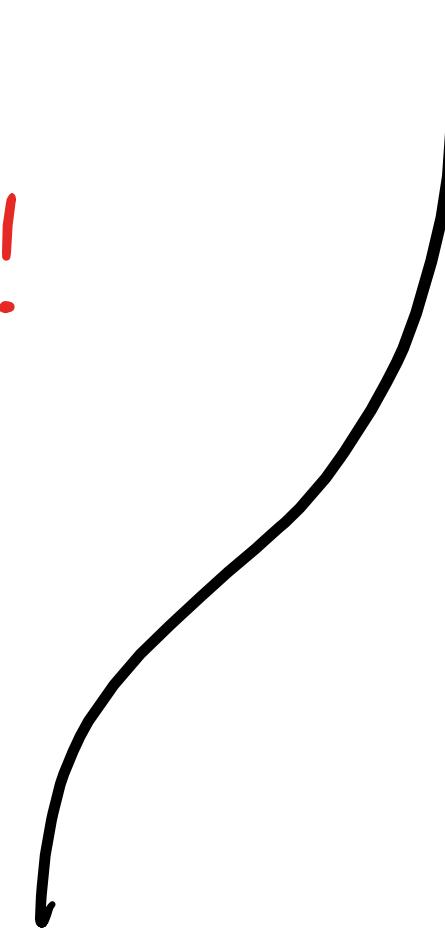


$$\begin{array}{c} | \\ \boxed{F} \end{array} \quad \begin{array}{c} || \\ \boxed{G} \end{array} = F \otimes G$$

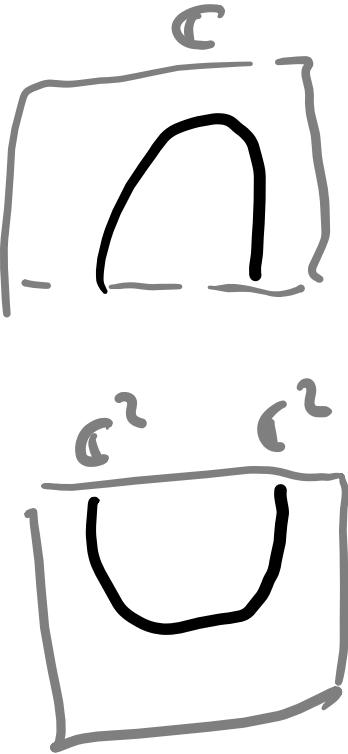
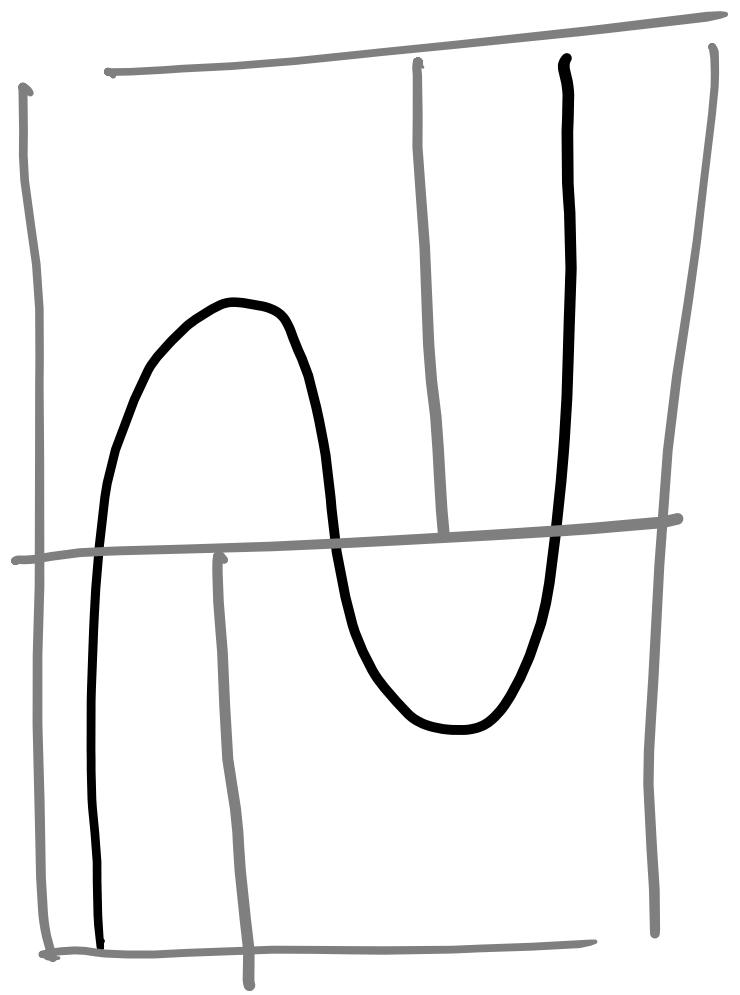
$$\begin{array}{c} | \\ H \end{array} \quad \begin{array}{c} | \\ K \end{array} = H \otimes K$$



~~?~~  
=



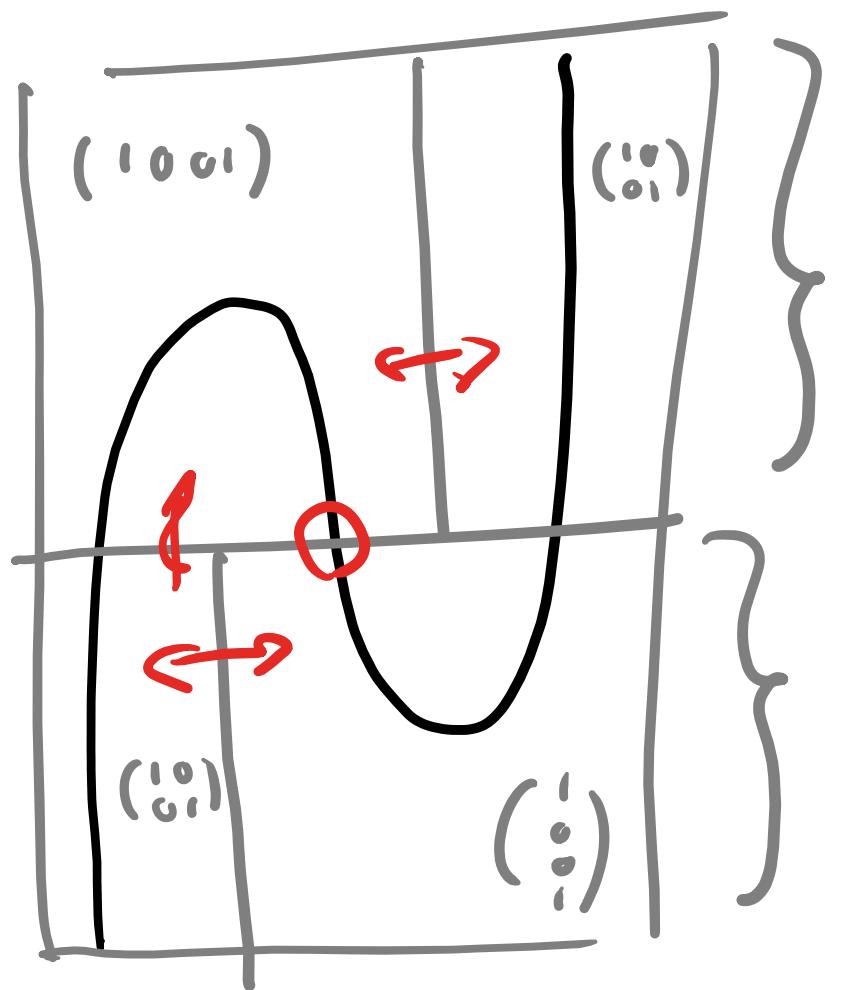
Yes!



$$\begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \vdots \\ \vdots \\ \vdots \end{pmatrix} \in \mathbb{C}^{\tilde{\times}} \otimes \mathbb{C}^{\tilde{\times}}$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} =$$



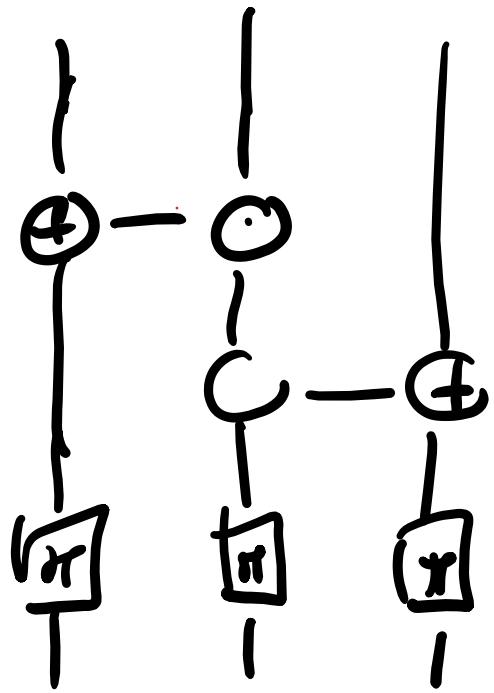
$$(1001) \otimes (10)$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

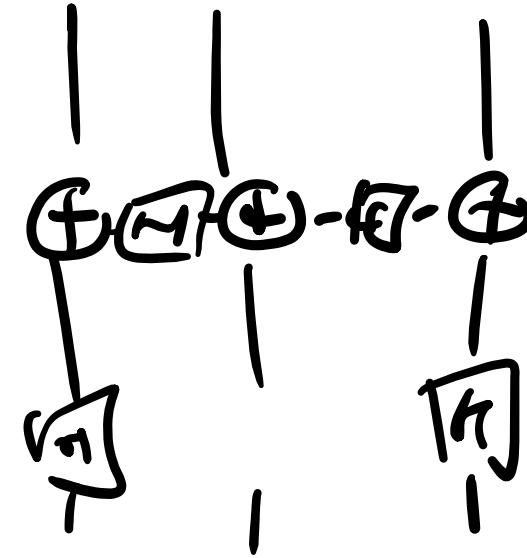
$$(10) \otimes (1)$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

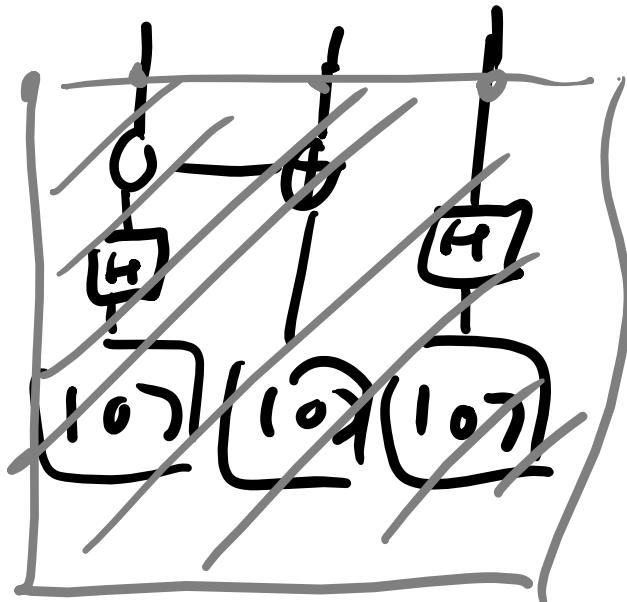


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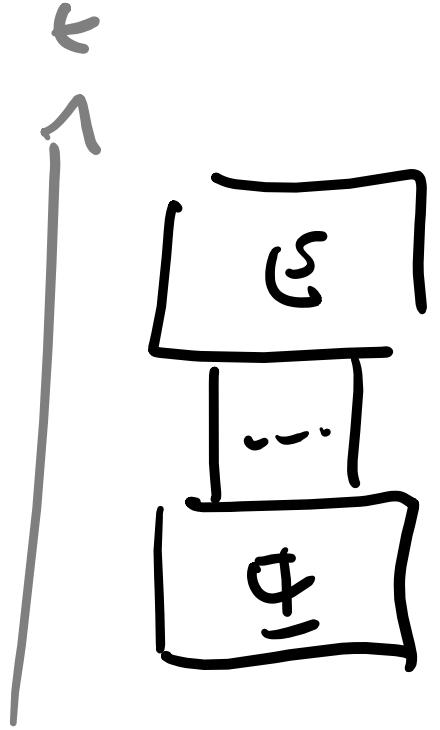


$\sim = \text{由}$

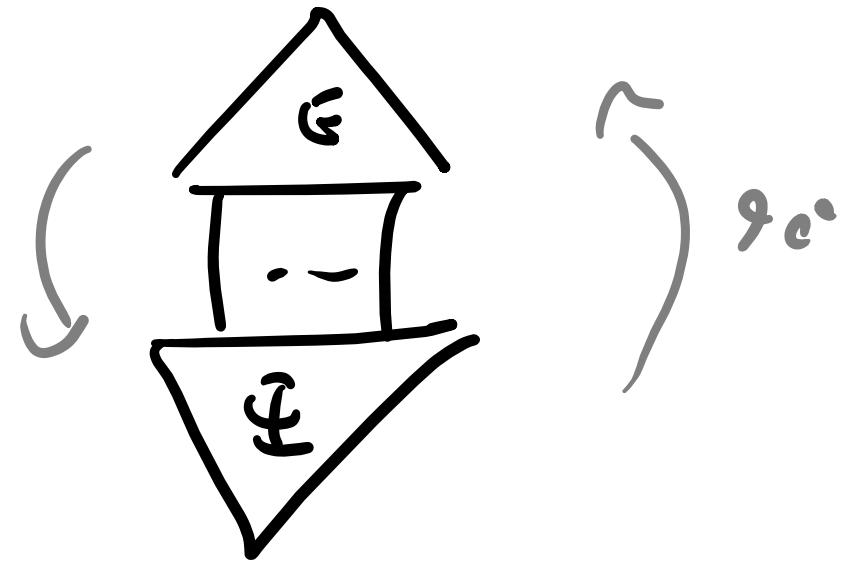
systems  
 $\text{由} \dots \text{由}$   
C  
State

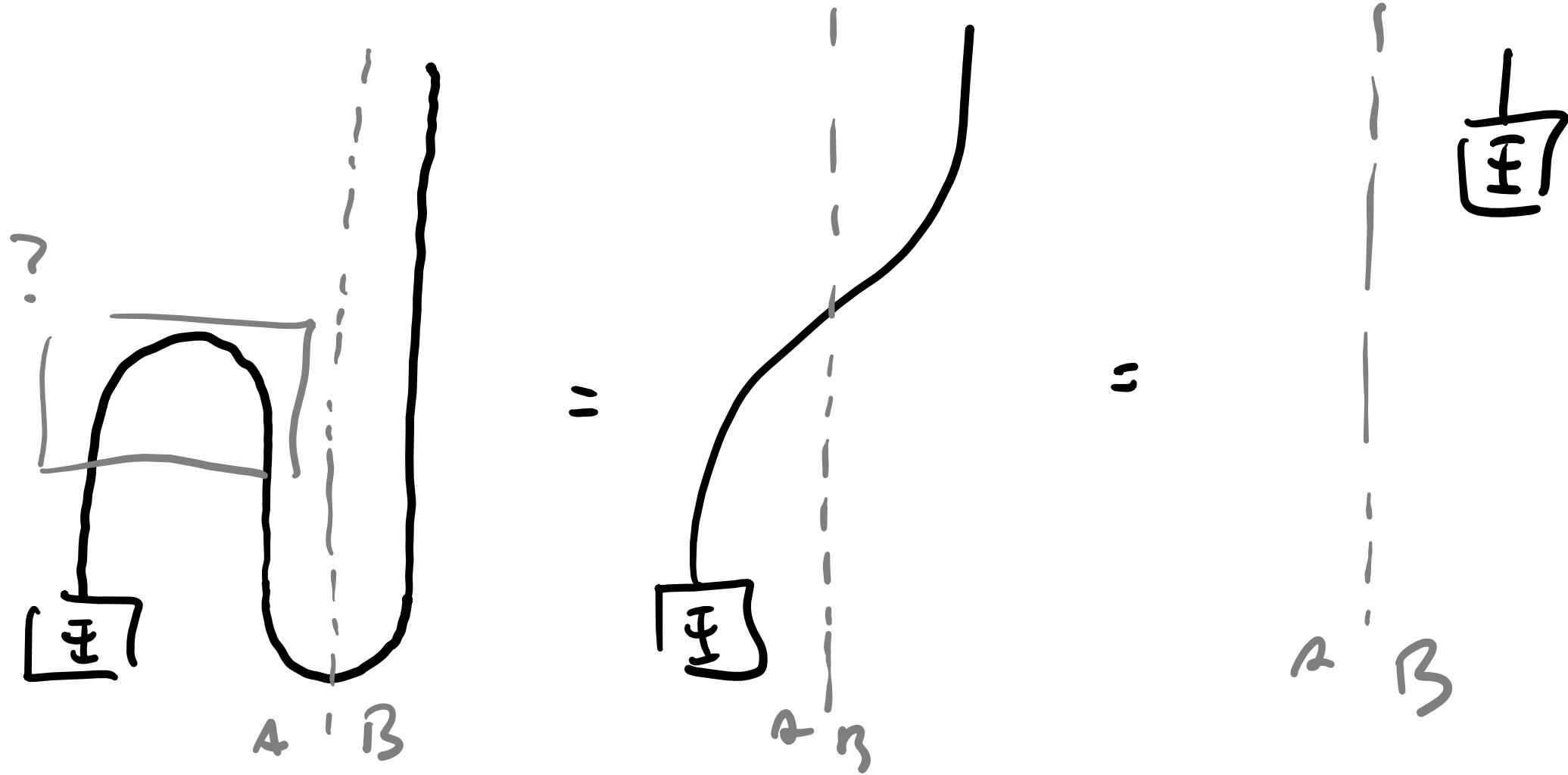


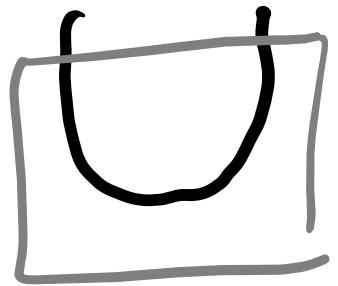
$\langle \text{E} | \Psi \rangle \in \mathcal{C}$



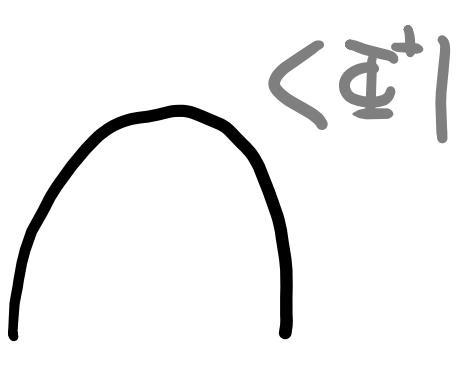
$$\langle \xi | \bar{\psi} \rangle \in \mathbb{C}$$







Sure box



$$P = \frac{1}{2}$$



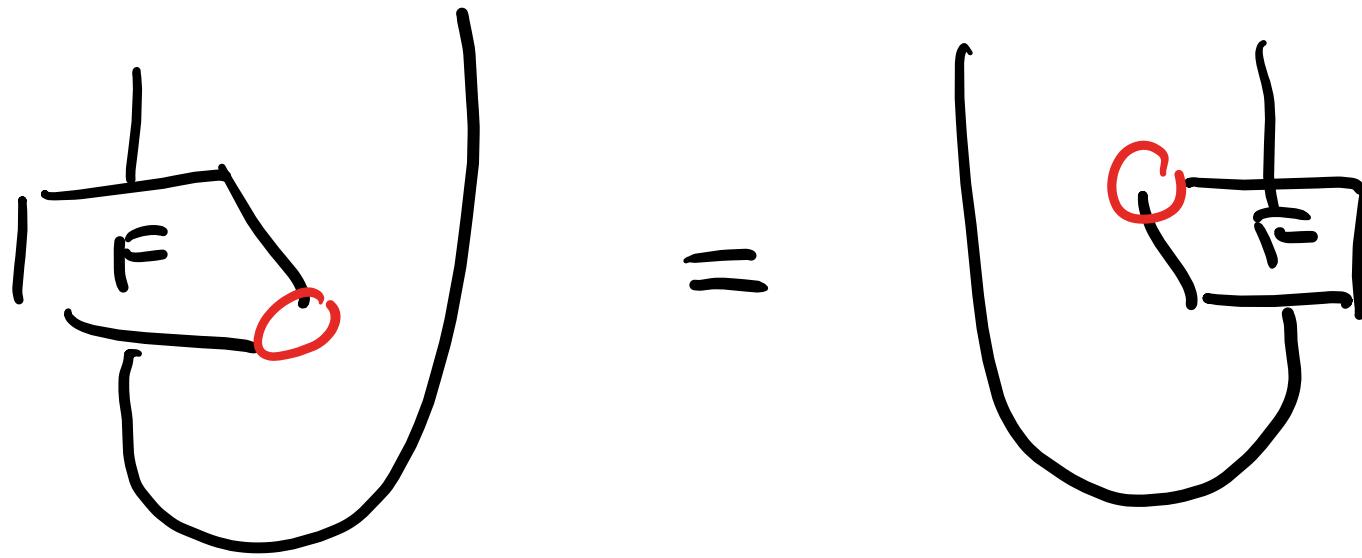
$$\frac{1}{2}$$

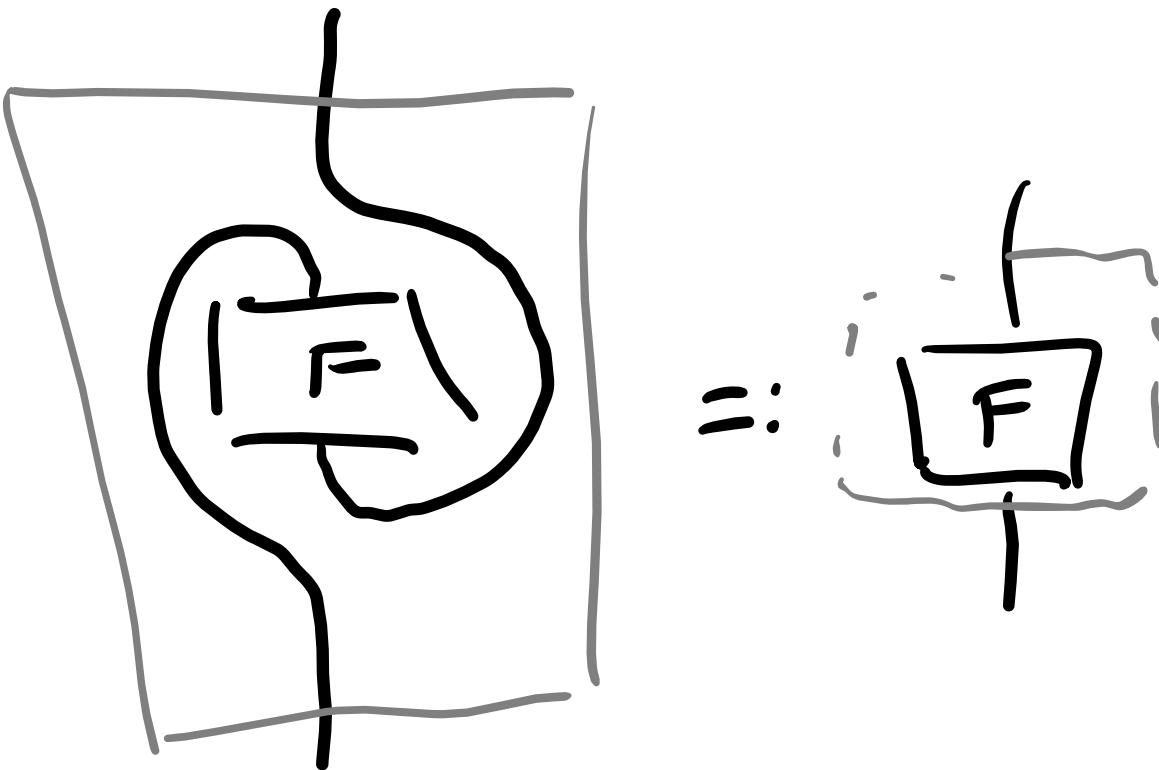


$$\frac{1}{4}$$



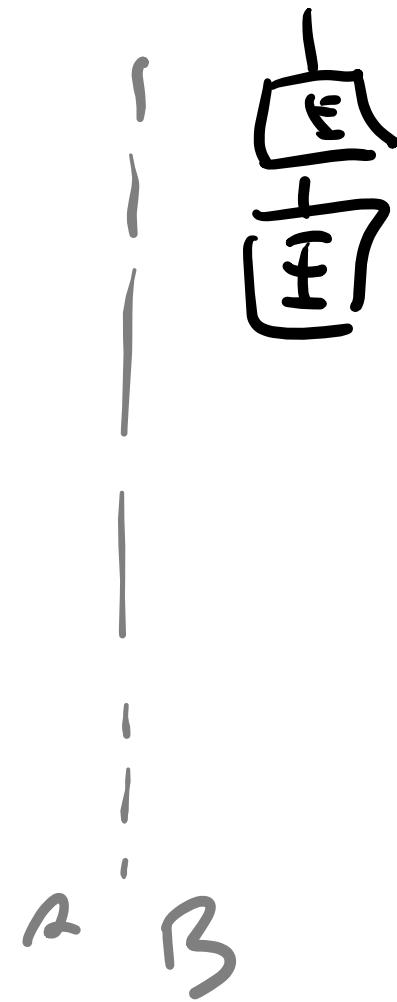
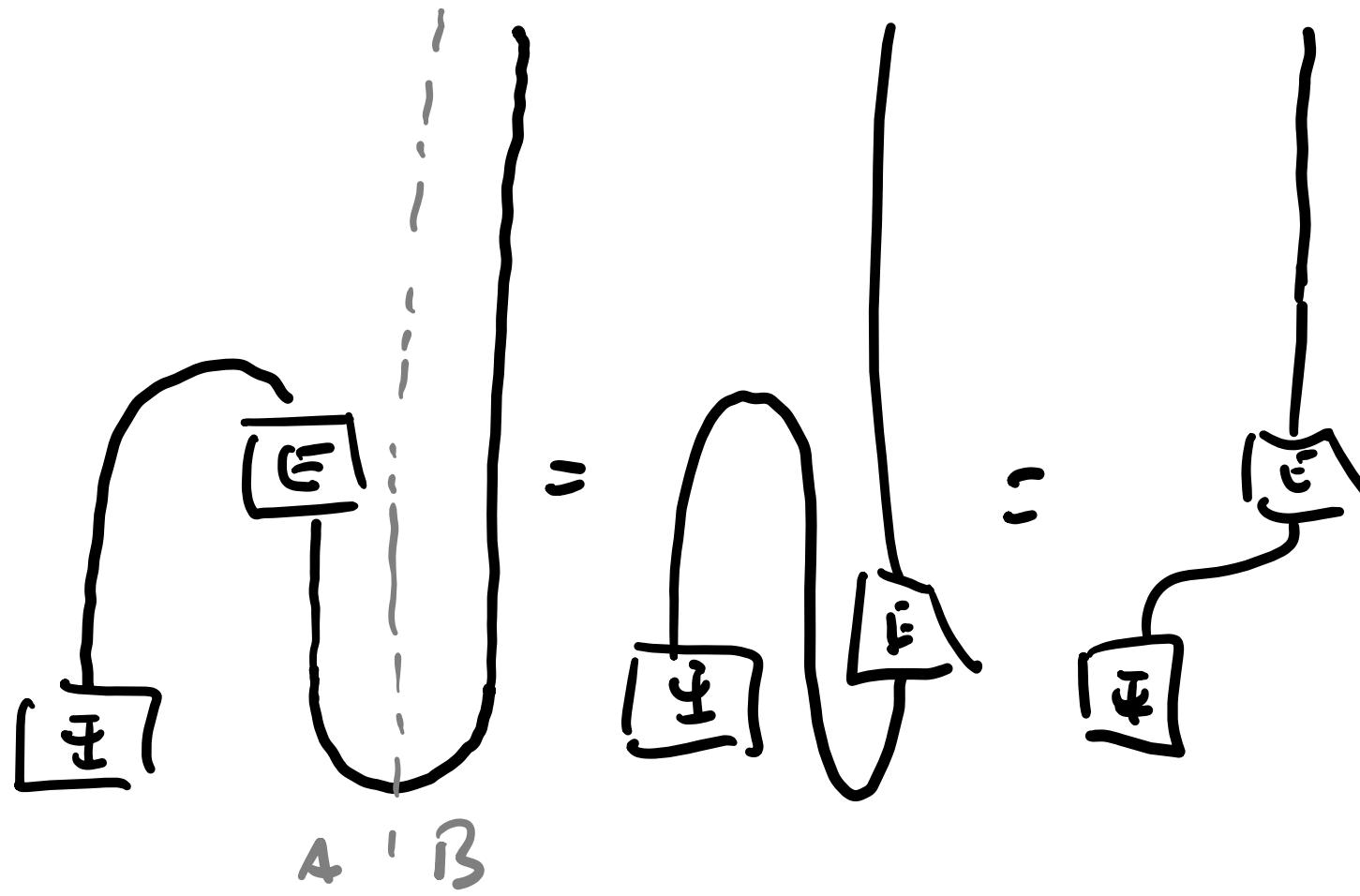
$$\frac{1}{4}$$

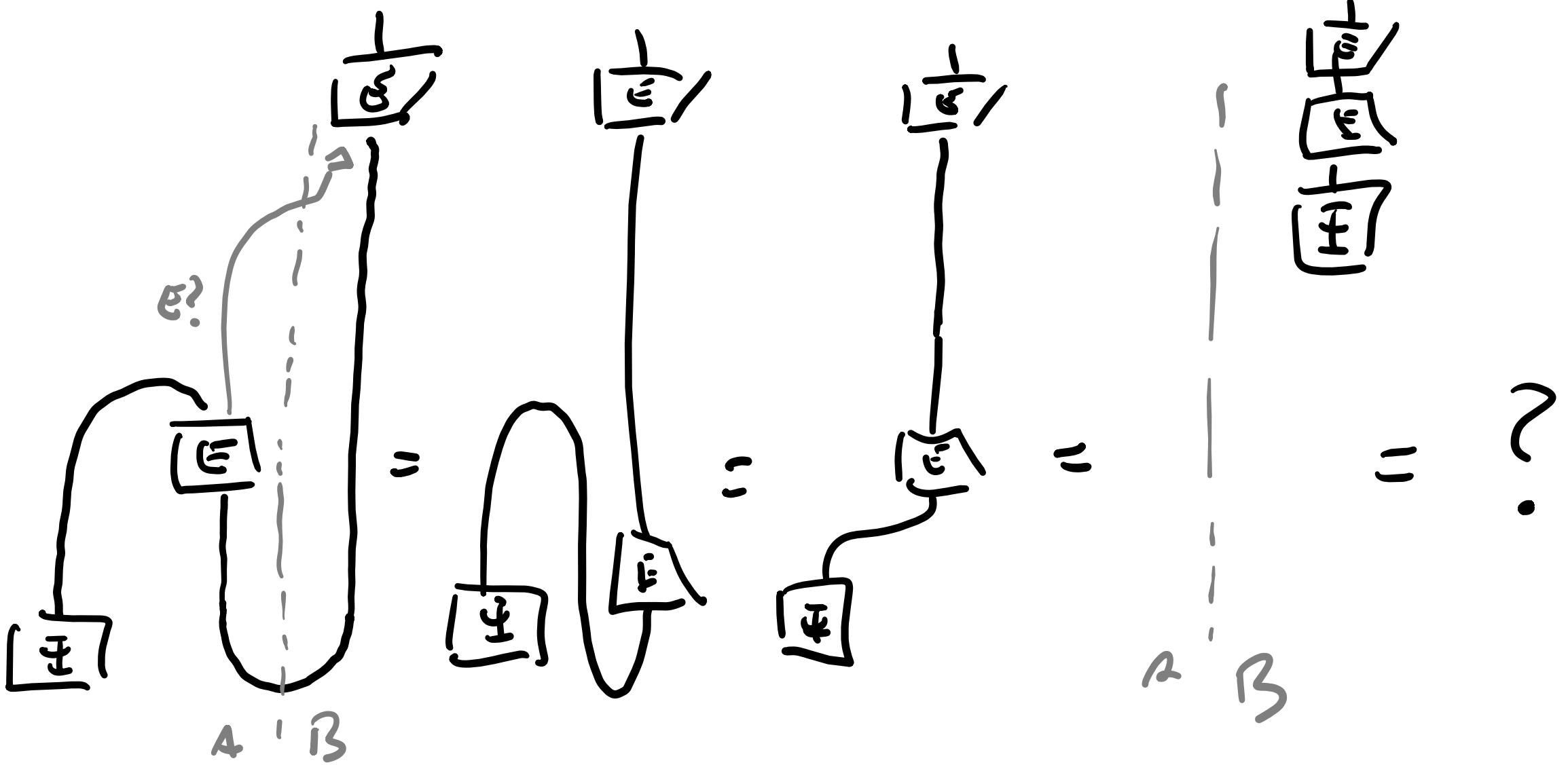




$$\begin{array}{c} \text{!} \quad \vdash \\ \boxed{F} \end{array} \quad = \quad \begin{array}{c} \text{!} \quad \vdash \\ \text{!} \quad \vdash \\ \boxed{F} \end{array} \quad = \quad \begin{array}{c} \text{!} \quad \vdash \\ \text{!} \quad \vdash \\ \boxed{F} \end{array} \quad = \quad \begin{array}{c} \text{!} \quad \vdash \\ \boxed{F} \end{array}$$

A sequence of four diagrams connected by equals signs. The first diagram shows a dashed arrow pointing to a box containing 'F'. The second diagram shows two dashed arrows pointing to a box containing 'F'. The third diagram shows three dashed arrows pointing to a box containing 'F'. The fourth diagram shows a single dashed arrow pointing to a box containing 'F'.





$$\begin{bmatrix} m & | & \alpha \\ \hline \beta & | & n \end{bmatrix} \xrightarrow{\text{flip!}} \begin{bmatrix} m & | & \alpha \\ \hline \beta & | & n \end{bmatrix}$$

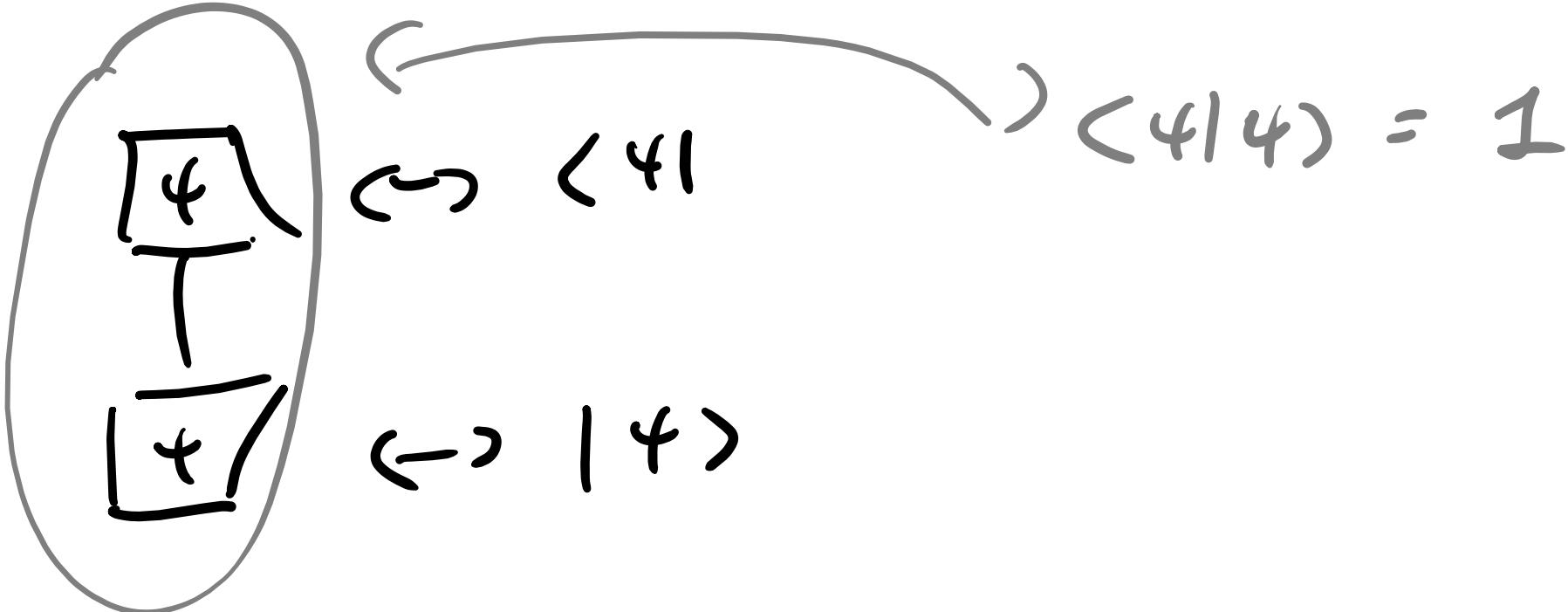
$$\begin{bmatrix} m & | & e \\ \hline \beta & | & n \end{bmatrix}$$

$$\left. \begin{array}{c} \begin{bmatrix} 1 & | & \alpha \\ \hline \beta & | & 1 \end{bmatrix} \\ \stackrel{(1)}{=} \end{array} \right\} \begin{array}{l} (2) \\ = \\ \begin{bmatrix} 1 & | & \alpha \\ \hline \beta & | & 1 \end{bmatrix} \end{array}$$

$$E \in \mathbb{C}^{n \times m}$$

$$G^+ \in \mathbb{C}^{m \times n}$$

$$\begin{array}{ll} (1) & \begin{bmatrix} 1 & | & \alpha \\ \hline \beta & | & 1 \end{bmatrix} \text{ isometry } G^+ G = I \\ (1)(2) & \begin{bmatrix} 1 & | & \alpha \\ \hline \beta & | & 1 \end{bmatrix} \text{ unitary } G^+ G = I = GG^+ \end{array}$$



A hand-drawn diagram illustrating a quantum state decomposition. On the left, a large oval encloses two basis states:  $|4\rangle$  (represented by a square with a diagonal line) and  $|+\rangle$  (represented by a square with a plus sign). Two curved arrows point from these states to the right. The top arrow points from the oval to the expression  $\langle 4|$ . The bottom arrow points from the oval to the expression  $|+\rangle$ . To the right of the oval, a curved arrow points from the expression  $\langle 4|$  to the final result  $\langle 4|4\rangle = 1$ .

$$\langle 4|4\rangle = 1$$

