The Highlights Quiz

Dan-Adrian German
for C290/A590

June 2023

1 Introduction

There are four topics you need to consider becoming familiar with if you want to be able to function in this area of Quantum Information Science (and Engineering). These topics are: probabilities, complex numbers, linear algebra and basic principles of quantum mechanics (that is, physics augmented with maths formalism). In what follows we present a list of problems that aim to activate various competencies in this specific regard. We will work some in class and we will provide references of where answers to these questions can be found.

2 Complex Numbers

1. Consider $x^2 + 5x + 10 = 0$. Solve for x.
2. If $z = 1 + 3i$ and $w = -2 + i$ calculate $z + w$ and zw.
3. For each $z \in \{5 + 10i, 3 - 2i, -3, 2i\}$ calculate $\overline{z}$.
4. For each z in the same set as above calculate $|z|$.
5. Clean up the fraction $\frac{1+i}{2-i}$ then write its complex conjugate.
6. What's the polar form of $z = 5 - 5i$?
7. If $z = e^{-i\frac{\pi}{2}}$ and $w = 4e^{i\frac{\pi}{4}}$ calculate zw.
8. If $z = 2e^{i\frac{\pi}{3}}$ calculate z^4.
9. $z = e^{-i\frac{\pi}{2}}$ and $w = 4e^{i\frac{\pi}{4}}$ calculate $\frac{w}{z}$.

Answers to these problems can be found on pp. 14-20 in this document[1]. In the document you will find much more than just the answers to these problems.

[1] https://legacy.cs.indiana.edu/~dgerman/2020/boot-camp/mathematics_qm_v21-martin-laforest.pdf

3 Probabilities

These problems are from Jim Freericks' EdX class[2] "Quantum Mechanics for Everyone". The class started on June 18 and a certificate costs $69 (and I can help you with the material if you need any help and consider taking that class).

Some of the problems have been adapted. All solutions will be discussed in class and some approaches we will take will combine basic maths with computer science (as we should).

1. Craps is a game played with two dice. You decide to play. Each bet is one chip. The goal is to roll a seven (with two dice). The house pays 4 chips plus your original chip if you win. Is this fair? (If not, define fair).

2. Now the goal is to roll a 2, a 3 or a 12. The house pays 7 chips plus your original chip if you win. Is this fair? If not, what would be fair? Also, can you solve the problem in more than one way and obtain the same result?

3. A hard way 6 bet says that you will roll a pair of threes before a soft six (that is, a six made of dice with different values) or a seven. So, you win if you roll a pair of threes (a hard six). You lose if you roll a soft six or a seven. If none of these happen you kee rolling the dice. Calculate the probability to win with a hard way six. Then compute the probability in some other way and obtain the same result, to confirm your answer.

4. The house pays 9 chips for a hard way six win. Is this fair?

5. What is the probability for a group of three people to not have anyone with the same birth month? Can you, again, solve it in two different ways?

6. What is the probability for at least two people to have the same birth month in a group of three?

7. Now determine the probability that at least two people have the same birth month in a group of 6 people.

8. How many people do we need for the probability to first be larger than 50% that two people have the same birthday in a group of N people? (Find smallest such N.) What is[3] Wolfram Alpha? Can you use it here?

9. You roll three dice. What is the probability that the sum of the three numbers is not a prime[4] number? How can you verify your answer?

10. State and solve the Cube Factory paradox from QBism[5] by Hans Christian von Baeyer. Prof von Baeyer spoke at a full-day pre-symposium SIGCSE event[6] we organized in 2020. Who is/was Bas van Fraassen?

[2] `https://www.edx.org/course/quantum-mechanics-for-everyone`

[3] `https://www.wolframalpha.com/`

[4] When you roll three dice the prime numbers that you can get are 3, 5, 7, 11, 13 and 17.

[5] `https://legacy.cs.indiana.edu/classes/c290-quantum-dgerman/sum2023/resources/QBism.pdf`

[6] `https://legacy.cs.indiana.edu/~dgerman/quantum-computing-for-undergrads/videos.html`

4 Linear Algebra

The following exercises can be solved with info available in Martin LaForest's "The Mathematics of Quantum Mechanics" (a document prepared expressely for high-schoolers visiting IQC Waterloo during the various summer school programs they offer) on pp. 32-71. I have already provided the link to the document, a bit earlier.

1. Add vectors $v = \begin{pmatrix} i \\ 2 \end{pmatrix}$ and $w = \begin{pmatrix} 3 \\ -100 \end{pmatrix}$.
2. Calculate $\begin{pmatrix} 3 & 3 & -1+2i \\ 1 & -3 & 0 \end{pmatrix} + \begin{pmatrix} 2 & -1 & i \\ 0 & 2+3i & -3 \end{pmatrix}$
3. and $2 \cdot \begin{pmatrix} 3 & 3 & -1+2i \\ 1 & -3 & 0 \end{pmatrix}$
4. Calculate $\begin{pmatrix} 2 & 3 & i \\ 3 & -2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 0 & 12 \\ 3 & -2 \end{pmatrix}$
5. Calculate $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix}$
6. and $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$
7. If $M = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ and $N = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$ calculate MN and NM.
8. Calculate the complex conjugate of $M = \begin{pmatrix} 1 & e^{-i\frac{\pi}{5}} \\ 3-i & 10 \end{pmatrix}$
9. Calculate the inner product of $v = \begin{pmatrix} i \\ 2+i \end{pmatrix}$ and $w = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$
10. Same for $\begin{pmatrix} i \\ i \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.
11. How long is the vector $\begin{pmatrix} 1 \\ -2 \\ i \end{pmatrix}$?
12. Normalize the vector $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$.
13. Is this a unitary matrix: $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Why or why not?

14. Same question for $Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
15. Same question for $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
16. Same question for $R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$
17. What is the cooking matrix?
18. How do you calculate what you need to buy if you want to cook 3 pizzas, 1.5 cakes and a baker's dozen (13) bagels?

5 Quantum Mechanics

Starting on page 74 in Martin LaForest's guide[7].

1. If $|v\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$ and $|w\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ calculate inner products $\langle w|v\rangle$, $\langle v|w\rangle$.
2. Given the quantum state $|\Psi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$ what's the probability of measuring it in the state $|+\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)$ where $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$?
3. What is the Hadamard matrix?
4. Calculate $X|0\rangle$ and $Z|-\rangle$.
5. Calculate $v \otimes w$ where $v = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $w = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
6. Same ask for $v = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $w = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$
7. Calculate $|0\rangle \otimes |0\rangle$, $|0\rangle \otimes |1\rangle$, $|1\rangle \otimes |0\rangle$ and $|1\rangle \otimes |1\rangle$.
8. If Alice has the quantum state $|\Psi\rangle = |0\rangle$ and Bob has the state $|\Phi\rangle = |+\rangle$ what is their combined state?
9. Calculate $X \otimes Z$.
10. Prove Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$

[7] This part could also be named Quantum Information Science, instead of QM.