Quantum in Pictures - IU Lecture Series

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Period: 26-30 June 2023

Morning Lectures

Morning lectures are based on the book Quantum in Pictures, which they follow closely.

1. **Wires and Boxes.** In this first lecture, we introduce the idea of diagrammatic reasoning, with special reference to compositional structure of quantum processes. We use naïve quantum teleportation as a motivating example.
2. **A World of Spiders.** In this second lecture, we introduce spiders, the fundamental building blocks of digital quantum information and quantum computing (QC). We use spiders to talk about gate-based QC, measurement-based QC, and to provide a complete picture of quantum teleportation.
3. **Keeping Einstein Happy.** In this third lecture, we introduce relativistic causality and the idea that certain quantum processes are fundamentally non-deterministic. We also explain how diagrams can be used to compute probabilities for the outcomes of quantum computations and protocols (Chapter 6 of the book).
4. **Quantum vs Ordinary Particles.** In this fourth lecture, we extend our diagrammatic language to explicitly handle both quantum systems and classical systems. We talk about measurements, decoherence, the no-broadcasting principle and quantum key distribution.
5. **Everything just from Pictures.** In this fifth lecture, we finally complete the set of diagrammatic rules necessary to phrase all of quantum information using pictures. We talk about non-locality, the Quantum Approximate Optimization Algorithm, and the simulation of quantum dynamics.

Afternoon lectures

Afternoon lectures cover the advanced mathematical background for the corresponding morning lectures. The book Picturing Quantum Processes provides a solid overall introduction to these topics, and specialised references are listed for each lecture. Of interest are also the summary paper "ZX-calculus for the working quantum computer scientist" and the list of ZX-calculus publications on zxcalculus.com.

1. **String Diagrams for Monoidal Categories.** In this first lecture, we talk about string diagrams for monoidal categories, in a variety of flavours of interest for quantum theory. We talk about planar, braided and symmetric monoidal structure; traces, closure, compact-closure and dual objects; higher categories. This is mostly based on the classic "A survey of graphical languages for monoidal categories", but the recent "An Introduction to String Diagrams for Computer Scientists" also provides an excellent reference. Time permitting, we will briefly discuss connections to 3D-TQFT from "Modular categories as representations of the 3-dimensional bordism 2-category" and a generalisation of string diagrams to associative n-categories from "High-level methods for homotopy construction in associative n-categories".
2. **Frobenius and Hopf algebras.** In this second lecture, we talk about Frobenius algebras and Hopf algebras, the algebraic structures behind spiders. We talk about the connection with quantum observables and finite-dimensional C*-algebras developed in "A new description of orthogonal bases" and "Categorical formulation of quantum algebras". We discuss generalisations to infinite dimensions from "H-star algebras and nonunital Frobenius algebras", "Infinite-dimensional Categorical Quantum Mechanics" and "Towards Quantum Field Theory in Categorical Quantum

Mechanics". Time permitting, we will briefly discuss generalisations to Hopf algebras described by "Quantum and Braided ZX calculus".

3. **Terminality, Causality and Probabilistic Enrichment.** In this third lecture, we talk about categorical structures modelling causality and non-determinism in quantum information theory and beyond. We explain how the categorical requirement of "terminality" relates to the non-signalling condition, based on "Environment and classical channels in categorical quantum mechanics", "Causal categories" and "Terminality implies non-signalling". We then introduce probabilistic structure into the picture, based on "Categorical Probabilistic Theories", and discuss how diagrams can be used to reason about tests and instruments in quantum information theory (with reference to foundational work on Operational Probabilistic Theories by the Pavia school, e.g. "Probabilistic theories with purification" and "Entanglement and thermodynamics in general probabilistic theories").

4. **The CPM and CP* constructions.** In this fourth lecture, we talk about the CPM construction from "Dagger Compact Closed Categories and Completely Positive Maps" and "Environment and classical channels in categorical quantum mechanics", the canonical categorical foundation for the description of quantum-classical processes. We will then introduce the CP* construction, generalising the CPM construction to finite-dimensional C* algebras and completely positive maps between them, based on "Categories of Quantum and Classical Channels". We will briefly discuss the categorical axiomatisation of CPM and CP* categories from "Axiomatic description of mixed states from Selinger's CPM-construction" and "Axiomatising Complete Positivity". Time permitting, we will briefly discuss "Higher-order CPM Constructions" and their application to the modelling of hypothetical post-quantum theories featuring higher-order interference and hyper-decoherence (based on "Density Hypercubes, Higher Order Interference and Hyper-Decoherence" and "Hyper-decoherence in Density Hypercubes").

5. **Bialgebras and Representation Theory.** In this fifth lecture, we complete the set of rules defining the ZX calculus, based on the seminal "Interacting Quantum Observables" and the following series of papers: arXiv:1705.11151, arXiv:1706.09877, arXiv:1812.09114, arXiv:1903.06035, arXiv:1911.06752. Further references of interest are the PhD theses of Miriam Backens, Harny Wang and Renaud Vilmart. We explain the connection between bialgebras and quantum dynamics, based on "A Diagrammatic Approach to Quantum Dynamics" and the lecturer's PhD thesis. We discuss the significance of bialgebras for quantum algorithms based on representation theory (cf. "The Topology of Quantum Algorithms" and "Fully graphical treatment of the quantum algorithm for the Hidden Subgroup Problem") and to the study of Mermin-type non-locality arguments (cf. "Strong Complementarity and Non-locality in Categorical Quantum Mechanics" and "Generalised Mermin-type non-locality arguments"). Time permitting, we will briefly discuss two important variations of the ZX calculus, the ZH Calculus and the ZXW Calculus.