

C241 Homework Assignment 5

1. Label each step of the following logical derivations with its justification. A justification may be one of the **BOOLEAN IDENTITIES** in Section 3.3, or a derived identity from the textbook examples. Highlight the subexpression to which the identity is applied.

(a)	$\overline{(p \cdot (q + r))}$	
	$= (\overline{p} + \overline{(q + r)})$	<i>DeMorgan</i>
	$= \overline{p} + (\overline{q} \cdot \overline{r})$	
(b)	$\overline{((p \cdot q) \Rightarrow s)}$	
	$= \overline{(\overline{(p \cdot q)} + s)}$	
	$= \overline{(\overline{p \cdot q})} \cdot \overline{s}$	
	$= (p \cdot q) \cdot \overline{s}$	
	$= (p \cdot q \cdot \overline{s})$	
(c)	$\overline{(p + q + (\overline{p} \cdot \overline{q} \cdot r))}$	
	$= \overline{((p + q) + (\overline{p} \cdot \overline{q} \cdot r))}$	
	$= (\overline{(p + q)}) \cdot (\overline{\overline{p} \cdot \overline{q} \cdot r})$	
	$= (\overline{(p + q)}) \cdot (\overline{\overline{p}} + \overline{\overline{q}} + \overline{r})$	
	$= (\overline{(p + q)}) \cdot (p + q + \overline{r})$	
	$= (\overline{(p + q)}) \cdot ((p + q) + \overline{r})$	
	$= (\overline{(p + q)}) \cdot (p + q) + (\overline{(p + q)}) \cdot \overline{r}$	
	$= 0 + (\overline{(p + q)}) \cdot \overline{r}$	
	$= (\overline{p + q}) \cdot \overline{r}$	
	$= (\overline{p} \cdot \overline{q}) \cdot \overline{r}$	
	$= \overline{p} \cdot \overline{q} \cdot \overline{r}$	

2. Use **only** the BOOLEAN IDENTITIES to prove that DeMorgan's Laws are valid for three variables:

$$(a) \overline{(p + q + r)} = (\bar{p} \cdot \bar{q} \cdot \bar{r})$$

$$(b) \overline{(p \cdot q \cdot r)} = (\bar{p} + \bar{q} + \bar{r})$$

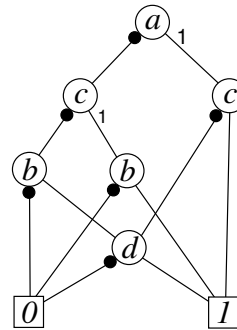
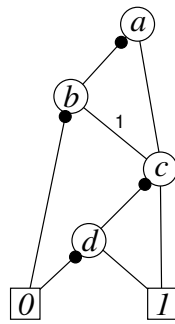
3. Define $x \oplus y$ to be $x\bar{y} + \bar{x}y$. Use boolean algebra to prove

(a) $x \oplus y = \bar{x} \oplus \bar{y}$

(b) $x(y \oplus z) = xy \oplus xz$

(c) $\overline{(x \oplus y)} = \bar{x} \oplus y$

4. The ROBDDs below represent two terms over variables a , b , c and d . In the one on the left, the variables are ordered $[a, b, c, d]$; and on the right the ordering is $[a, c, b, d]$. Determine whether or not the terms represented by these two graphs are equivalent.



5. Use boolean algebra to synthesize the DNF for $(a + b)(c + \overline{a\overline{d}}) + \overline{bcd}$.

6. A standard deck of cards has 52 cards consisting of 13 cards in each of four *suits*: \spadesuit , \heartsuit , \diamondsuit , \clubsuit . In each suit, cards have *face values* from $\{1, 2, \dots, 13\}$, each card having a different face value. A *hand* is a set of five cards from the deck. A hand is called a *flush* if all five cards are of the same suit. A hand is called a *straight* if the five cards are sequential in value, for instance, $\{3\heartsuit, 4\spadesuit, 5\diamondsuit, 6\diamondsuit, 7\heartsuit\}$.
- (a) How many different flushes are there in a standard deck?
- (b) How many different straights are there in a standard deck?

7. Definition 4.1 (p. 63) states that for finite sets A and B

$$|A \cup B| = |A| + |B| - |A \cap B|$$

- (a) Write a corresponding formula for the union of three finite sets.
- (b) Write a corresponding formula for the union of four finite sets.
- (c) Write a formula for the union of n finite sets.

SUPPLEMENTAL PROBLEM. (Amorous Cockroaches) COMMENT: *This is neither a calculus question nor a logic puzzle.*

At last! You've gotten out of the dorms and into an apartment. Life can begin. You get moved in and a few days later are looking for a brownie pan. When you open the cabinet door, you see a 12-inch by 12-inch pan, just perfect for your cooking needs. That's the good news.

Unfortunately, there are four cockroaches in the pan, one at each corner. Now you may not have known this, but cockroaches are very amorous creatures, and will always move directly toward the object of their affection. Each of the four in your pan is attracted to the one in the adjacent corner, going counter-clockwise. So all four cockroaches simultaneously start walking toward the one that attracts them. As a result, they start spiraling toward the center of the pan (See the diagram below).

QUESTION: *How far will the cockroaches travel?*

