C241 Homework Assignment 4

1. Which of the following formulas are tautologies and which are contradictions? Which of the formulas are logically equivalent to each other?

(a) $p \wedge (q \vee r)$

(b) $(\neg p \wedge r) \Rightarrow (q \vee r)$

(c) $(p \wedge q) \vee (p \vee r)$

(d) $\neg(r \Rightarrow q \wedge r)$

(e) $\neg(p \Rightarrow (q \Rightarrow p)$

(f) $((p \Rightarrow q) \vee (r \wedge s \vee t)) \vee (p \wedge \neg q)$

2. Use a truth table to show that

(a) $\neg(p \vee q)$ is *not* logically equivalent to $(\neg p \vee \neg q)$, and

(b) $\neg(p \wedge q)$ is *not* logically equivalent to $(\neg p \wedge \neg q)$.

Remember these facts.

3. Consider the logical operation defined below:

P	Q	$P \downarrow Q$
F	F	T
F	T	T
T	F	T
T	T	F

Show that '$\downarrow$' can be used to implement (in the sense of Prop. 3.2) all of the operations of Definition 3.1.

4. Let P stand for the proposition "Sue says it." Let Q stand for the proposition "Sam saw it." Let R stand for the proposition "Sid did it." Express the following sentences as formulas involving the logical connectives. If there is more than one way to translate a sentence, use truth tables to explain any differences in the meaning among these translations.

(a) Sid did it, Sam saw it, and Sue says it.

(b) If Sid did it, Sam saw it.

(c) Sid did it only if Sam saw it.

(d) Sue says it only if Sid did it, and Sam saw it.

(e) If Sue says it implies Sam saw it, Sid did it.

5. Determine whether the following proposition is a tautology.

$$(a \vee b \Leftrightarrow c) \wedge (d \vee e) \Leftrightarrow ((a \vee b \Leftrightarrow c) \wedge d) \vee ((a \vee b \Leftrightarrow c) \wedge e)$$

6. Show whether the following pairs of formulas are equivalent.

(a) $(p \Rightarrow q) \Rightarrow r$ and $p \Rightarrow (q \Rightarrow r)$

(b) $p \Rightarrow (q \Rightarrow r)$ and $(p \wedge q) \Rightarrow r$

(c) $(p \wedge q) \Rightarrow r$ and $(p \Rightarrow r) \wedge (q \Rightarrow r)$

7. For each of the following propositions, give the DNF under the variable ordering $\langle a, b, c \rangle$.

(a) $a \vee (\neg a \wedge \neg b)$

(b) $a \Rightarrow (b \Leftrightarrow c)$

(c) $(\neg b \wedge c) \wedge (\neg a \Rightarrow \neg c) \wedge (c \wedge (\neg b \vee \neg a))$

(d) $(a \Rightarrow b) \Leftrightarrow (b \Rightarrow c)$

8. Reduce the following boolean expressions to simpler terms

(a) $xy + (x + y)\overline{z} + y$

(b) $x + y + \overline{(\overline{x} + y + z)}$

(c) $yz + wx + z + [wz(xy + wz)]$

9. Write the truth tables for the following logical formulas and state whether each is a tautology, a contradiction, or neither (a contingency).

(a) $P \wedge (Q \vee R)$

(b) $(P \wedge \neg P) \Rightarrow Q$

(c) $P \Rightarrow (Q \vee \neg Q)$

No Supplemental Problem this week.