

C241 Homework Assignment 2

1. Ex. 2.1-1 Let $A = \{1, 2\}$ and $B = \{2, 3, 4\}$. Which of the following relations from A to B are functions?

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|-----|----------------------|-----|----------------------|
| (a) | $\{(1, 3), (2, 4)\}$ | (d) | $\{(1, 3), (2, 5)\}$ |
| (b) | $\{(1, 3), (1, 4)\}$ | (e) | $\{(2, 2), (1, 4)\}$ |
| (c) | $\{(1, 3), (1, 3)\}$ | | |

2. Ex. 2.1-2 Is $\{(1, 2), (2, 3)\}$ a function

(a) from $\{(1, 2)\}$ to $\{(2, 3)\}$?

(b) from \mathbb{N} to \mathbb{N} ?

(c) from $\{1, 2\}$ to \mathbb{N} ?

(d) from $\{1, 2, 3\}$ to $\{2, 3\}$?

(e) from $\{1, 2, 3\}$ to $\{1, 2, 3\}$?

3. Ex. 2.1-3 Let $A = \{1, 2\}$ and $B = \{2, 3, 4\}$.

- (a) List a relation that is an injective function from A to B .
- (b) List a relation that is a surjective function from B to A .
- (c) List two bijections from B to B .

4. Ex. 2.1-8 *Prove:* If $f: A \rightarrow B$ and $g: B \rightarrow C$ are surjections (“onto functions”), then $g \circ f$ is onto.

5. Ex. 2.1-8 If $f: X \rightarrow Y$ is a surjection (onto) and $g: Y \rightarrow Z$ is an injection (one-to-one). Is $g \circ f$ always a surjection? Is it always an injection? If “yes” give a proof; and if “no” provide a counterexample.

SUPPLEMENTAL PROBLEM. **The Farmer's Dilema**

A farmer is taking a head of cabbage and a goat to market. He brings along his faithful dog for company. On the road to the market there is a swift river that must be crossed. A small boat is tied to a tree on the bank of the river for this purpose. The boat is so small that it can carry the farmer and *just one* other thing, the cabbage, the goat or the dog. This is a problem: the farmer knows that, if left alone with the cabbage, the goat will eat it. And if left alone with the goat, the dog will eat it. Can the farmer safely get everything across the river? Or does he have to return home and leave the dog behind?