

C241 Homework Assignment 1

1. (Exercise 1.1-1) List the following sets:
 - (a) $\{2^i \mid i \in \mathbb{N} \text{ and } 0 \leq i \leq 8\}$
 - (b) $\{i^2 \mid i \in \mathbb{N} \text{ and } 0 \leq i \leq 8\}$
 - (c) $\{2k + 1 \mid k \in \mathbb{N}\}$
 - (d) $\{m \mid 23 < m < 29 \text{ and } m \text{ is a prime number}\}$

2. (Exercise 1.1-2) Let $A = \{a, b\}$; let $B = \{1, 2, 3\}$; let $C = \emptyset$; and let $D = \{a, b, c, d\}$. List the following sets:

- | | |
|--------------------------|------------------------------|
| (a) $A \cup B$ | (f) $A \cup C$ |
| (b) $A \cap B$ | (g) $A \cap D$ |
| (c) $A \times B$ | (h) A^3 |
| (d) $\mathcal{P}(A)$ | (i) $\mathcal{P}(\emptyset)$ |
| (e) $B \times \emptyset$ | (j) $(D \cap A) \times B$ |

3. Let $A = \{a, b\}$, $B = \{1, 3, 5\}$, and $C = \{\oplus, \otimes\}$.

(a) List the set $A \times B$.

(b) List the set $A \times (B \times C)$.

(c) List the set $A \times B \times C$.

4. (Exercise 1.1-3) Let $A = \{a, b\}$; let $B = \{1, 2, 3\}$; and let $E = A \times B$. List the following sets:

(a) $\{(x, y, y) \mid (x, y) \in E\}$

(b) $\{(x, x) \mid x \in E\}$

(c) $\{(y, z) \mid (x, y) \in E \text{ and } z \in B\}$

5. List the first twelve elements of the set

$$F = \{f_k \in \mathbb{N} \mid f_1 = 1, f_2 = 2, \text{ and for any } k \geq 1, f_{k+2} = f_k + f_{k+1}\}$$

That is, list $\{f_1, f_2, \dots, f_{12}\}$

6. Let $V = \{1, 2, 3\}$. List the set $\{xyz \mid x, y, z \in V\}$ of words in V^+ .

7. (Exercise 1.3-1) Let $V = \{a, b, \$\}$. For each of the following languages $L_i \subseteq V^+$, list enough elements to make it clear what each contains.

(a) In language L_1 each word has exactly one $\$$ and equally many a s as b s.

(b) In each word of language L_2 , a s and b s alternate with any number of $\$$ s mixed in.

(c) In each word of language L_3 , no a occurs next to a b .

(d) $L_4 = \{u^{\wedge}\$^{\wedge}v \mid u \in \{a\}^+ \text{ and } v \in \{\$, b\}^+\}$

(e) $L_5 = \{a^{k^{\wedge}}\$^{\wedge}b^k \mid k \in \mathbb{N}\}$

8.

What does this STMT program compute? Trace its execution by hand for a few small values of A and B . Place a statement in the empty assertion at the end of the program saying what condition holds at that point. Explain why the program satisfies your assertion when it reaches the `end`.

```
{  $x = A \in \mathbb{W}$  and  $y = B \in \mathbb{W}$  }  
begin  
while  $x \neq y$  do  
  if  $x < y$   
    then  $y := y - x$   
    else  $x := x - y$   
  end  
end
```

{ }

(Optional) Write and test a program in the language of your choice (preferably Scheme) that performs the same computation as the given program above.

SUPPLEMENTAL PROBLEM. A small island is ruled by a benevolent queen. One morning, a proclamation is posted in the central common:

—To All My Subjects—

It has come to my attention that one or more husbands in my realm are unfaithful to their wives. I hereby decree that any woman who learns her husband is unfaithful must shoot him at the stroke of midnight.

Her Highness, The QUEEN

It's a small island. Every wife knows who all the unfaithful husbands are, *but does not know whether or not her own husband is faithful.*

On the third night, shots ring out. How many husbands are/were unfaithful?