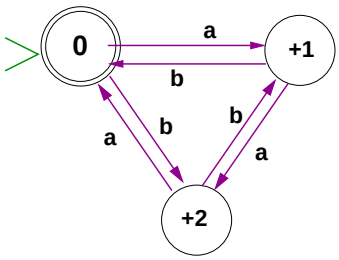
Automata

Sep 2022

Most DFA-diagrams have states labeled with text, recording the development of the DFA. You may disregard these labels.

1. (Practice) Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Construct a 3-state automaton that recognizes the language
   $$w \in \Sigma^* \mid \#_a(w) \equiv_3 \#_b(w).$$

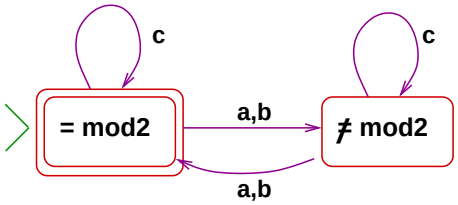
   Solution.



2. For each of the following languages construct an automaton that recognizes it.

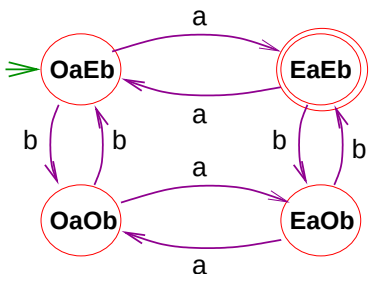
   (a) (Practice) $\{w \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^* \mid \#_a(w) = \#_b(w) \mod 2\}$

   Solution.



   (b) $\{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid \#_a(w)$ **odd and** $\#_b(w)$ **even** $\}$

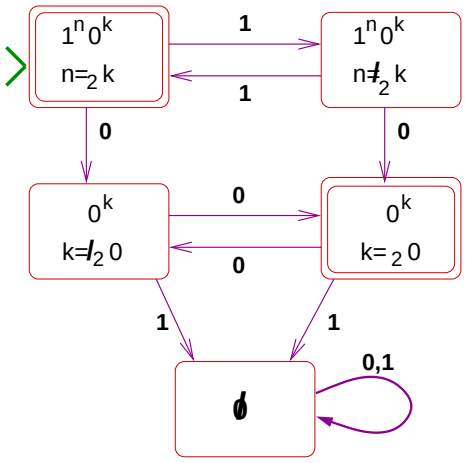
   Solution.



(c) (Practice)

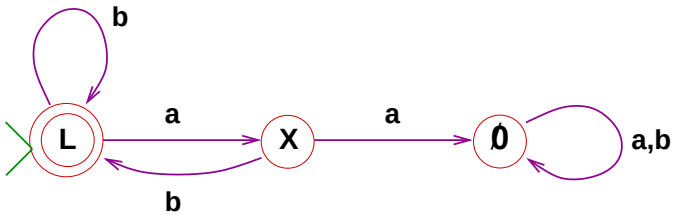
$\{1^n 0^m \mid n = m \mod 2\}$.

Solution.



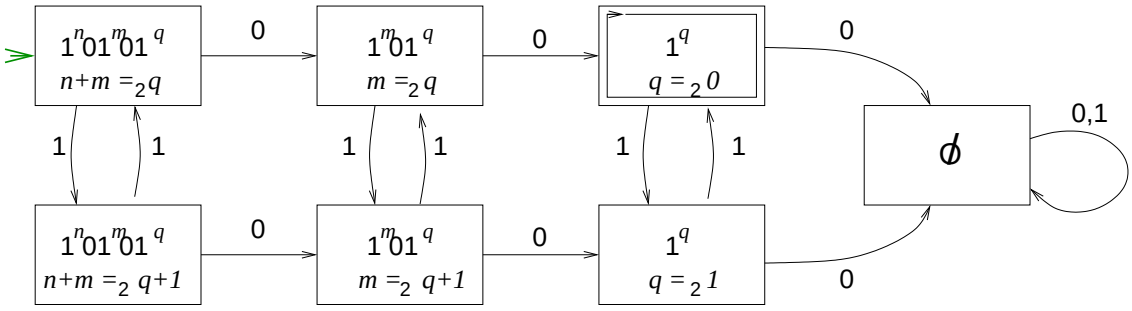
(d) (Practice) $\{w \in \{a, b\}^* \mid$ every substring aa in w is followed by a b$\}$

Solution.



(e) $\{1^n\,0\,1^m\,0\,1^q \mid n+m \equiv q \mod 2\}$

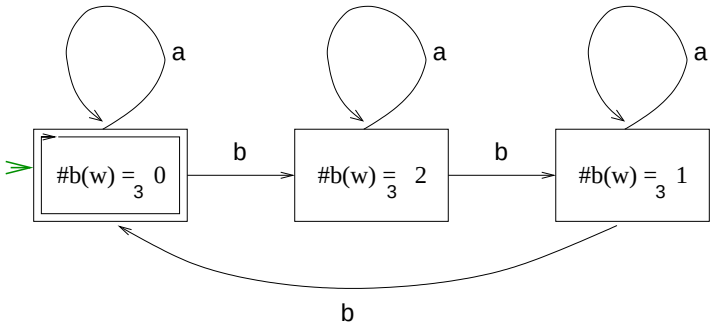
Solution.



3. For a decimal numeral $w \in \{0,1,2,3,4,5,6,7,8,9\}^*$ let $[w]_{10}$ be the number denoted by w and $\Sigma(w)$ the sum of its digits.
For example, for $w = 124$ $[w]_{10} = 124$ and $\Sigma(w) = 7$.
It is know that $[w]_{10} \equiv \Sigma(w) \mod (3)$.

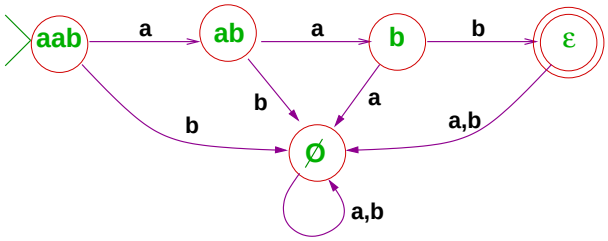
   Construct an automaton that recognizes the language
   $\{w \in \{0,1,2,3,4,5,6,7,8,9\}^* \mid [w]_{10}$ **is divisible by 3** $\}$.

Solution.



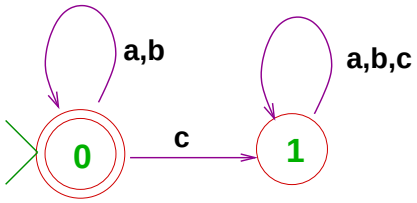
4. (Practice) $L = \{\mathtt{aab}\}$, $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

Solution.



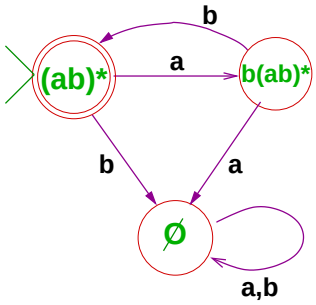
5. $L = \{\mathtt{a}, \mathtt{b}\}^*$, $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$.

Solution.



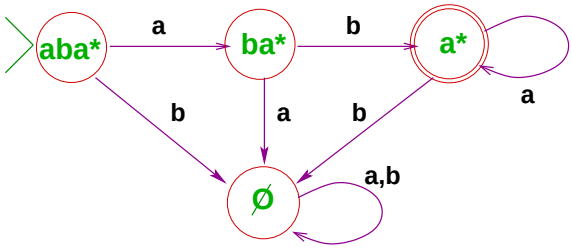
6. (Practice) $L = \{\mathtt{ab}\}^*$

Solution.



7. (Practice) $L = \{\mathtt{aba}^n \mid n \geqslant 0\}$

Solution.



8. (a) Construct an automaton that recognizes the language

$$L = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid \#_b(w) \text{ is divisible by } 3\}$$

(b) Give the computation-trace of your automaton for the strings **aba** and **bbab**.