Assignment 11: Computable reductions

This assignment contains solved practice problems, numbered in red.
The assigned problems and sub-problems are numbered in green.

A. The problem **COMMON-ACCEPT** asks whether a given pair (M_0, M_1) of Turing acceptors accept a common string.

A decidable certification for **COMMON-ACCEPT** has $c \vdash (M_0, M_1)$ iff c is a pair (t_0, t_1) where t_0 and t_1 are accepting traces of M_0 and M_1, respectively, for the same input string. So **COMMON-ACCEPT** is SD.

(i) Define a computable reduction of **ε-ACCEPT** to **COMMON-ACCEPT**.
Solution. Let ρ map an instance M of **ε-ACCEPT** to the instance (E, M) of **COMMON-ACCEPT**, where E is an acceptor for the singleton language $\{\varepsilon\}$. Then M accepts ε iff $\{\varepsilon\} = \mathcal{L}(E) \subseteq \mathcal{L}(M)$, i.e. ρ is a reduction. ρ is computable trivially.

(ii) Conclude that **COMMON-ACCEPT** is not decidable. (This cannot be proved by invoking Rice's Theorem as we stated it, because the instances are here ***pairs*** of acceptors.)
Solution. Since **ε-ACCEPT** is undecidable and computably-reducible to **COMMON-ACCEPT** it follows that the latter is undecidable as well.

1. The problem **SUBLANG** asks whether a given pair (M, M') of Turing acceptors satisfies $\mathcal{L}(M) \subseteq \mathcal{L}(M')$.

(a) Define a computable reduction of **ε-ACCEPT** to **SUBLANG**.

(b) Define a computable reduction of **ε-NONACCEPT** to **SUBLANG**.

(c) Conclude that neither **SUBLANG** nor its complement are SD. (You may use the fact that **ε-NONACCEPT** is not SD, as proved in class).

2. Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. For Σ-languages L, L' define $L \oplus L' =_{\text{df}} \{\mathtt{a}\} \cdot L \cup \{\mathtt{b}\} \cdot L'$.

(a) Define computable reductions $\rho : L \leqslant_c L \oplus L'$ and $\rho' : L' \leqslant_c L \oplus L'$.

(b) Suppose L is SD but not decidable.
Prove that $L \oplus \bar{L}$ is neither SD nor co-SD.