B501, Fall 2024

Assignment 8: Parse-trees and Dual-Clipping

Practice problems and problem-parts are labeled in red.
Assigned problems and problem-parts are labeled in green.

1. (30%) For each of the following languages L construct a CFG G that generates it. Then give a parse tree for the string indicated, as well as the left-most derivation for that parse-tree.

(i) $L = \{(\mathtt{ab})^n(\mathtt{ba})^n \mid n \geqslant 0\}$ **ababbaba**.

Solution. (without the parse-tree.) $S \to \mathtt{ab}S\mathtt{ba} \mid \varepsilon$

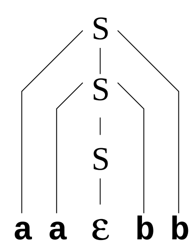
$S \Rightarrow \mathtt{ab}S\mathtt{ba} \Rightarrow \mathtt{abab}S\mathtt{baba} \Rightarrow \mathtt{ababbaba}$

(a) $L = \{\mathtt{a}^n\mathtt{b}^k \mid n \leqslant k\}$ **aabbb**

(b) $L = \{\mathtt{a}^n\mathtt{b}^k \mid k \leqslant n \leqslant 2k\}$ **aaabb**

A. Consider the CFG $S \mid \mathtt{a}S\mathtt{b} \mid \varepsilon$. So the degree is $d = 3$ and the number of nonterminals is m = 1. Take some string in $\mathcal{L}(G)$ of length $\geqslant d^m = 3$, exhibit its parse-tree, and extract the strings v_0, x, y, z, v_1 predicted by the Dual-Clipping Theorem.

Solution.



Taking the two lower occurrences of S, we have $v_0 = \mathtt{a}$. $x = \mathtt{a}$, $y = \varepsilon$, $z = \mathtt{b}$, and $v_1 = \mathtt{b}$.

2. (20%) Consider the following CFG G. (It generates the language $\{\mathtt{c}^i\mathtt{a}^n\mathtt{c}^k\mathtt{d}^k\mathtt{b}^{i+n}\}$.)

$$\begin{aligned} S &\to \mathtt{c}S\mathtt{b} \mid P \\ P &\to \mathtt{a}P\mathtt{b} \mid M \\ M &\to \mathtt{c}M\mathtt{d} \mid \varepsilon \end{aligned}$$

(a) Identify the clipping constant $k = d^m$ for G.

(b) Construct a parse-tree of G for the string **caaccddbbb**.

(c) For the top two P's identify the partition $v_0 \cdot x \cdot y \cdot z \cdot v_1$ stated in the Dual-Clipping Theorem.

(d) Repeat for the bottom two occurrences of M.

3. (20%) Show that $L = \{\mathtt{a}^i\mathtt{b}^{2i}\mathtt{a}^i \mid i \geqslant 0\}$ is not a CFL.

4. (20%) Show that $L = \{\mathtt{a}^i\mathtt{b}^j\mathtt{a}^i \mid i \geqslant j \geqslant 0\}$ is not CF. [Hint: Use dual-pumping]

5. (10%) The *reverse* of a language L is $L^R = \{w^R \mid w \in L\}$.
Give an algorithm that converts a CFG G generating L to a CFG H generating L^R. Your algorithm should produce the desired conversion, but you need not prove that it does.