B501, Fall 2024 © Daniel Leivant 2024

## **Assignment 7: Context-free languages**

Solved practice problems numbered in red, assigned problems in green.

- 1. (25%) For each grammar below describe in words the language it generates.
  - (i)  $S \to aSb \mid bSa \mid \varepsilon$

**Solution.** The set of strings  $w\tilde{w}$  where  $\tilde{w}$  is the flip of w, i.e. with a and b interchanged.

(a)  $S \to SS \mid a$ 

 $n \leq q$ .

- (ii)  $S \to aS | cT, T \to aT | cR, R \to aR | \varepsilon.$ Solution. The strings of the form  $a^i ca^j ca^k$  where  $i, j, k \ge 0$ .
- (b)  $S \to aA \mid bB$ ,  $A \to aA \mid bA \mid a$ ,  $B \to aB \mid bB \mid b$ .
- 2. (75%) For each of the following languages give a CFG that generates it.
  - (i) L = {a<sup>n</sup>b<sup>2n</sup> | n ≥ 0} Solution. S → aSbb | ε
    (a) L = {a<sup>n</sup>b<sup>p</sup>c<sup>q</sup> | n = p+q}
    (ii) L = {a<sup>n</sup>x | x ∈ {a,b}\*, |x| = n} Solution. S → aSa | aSb | ε
    (b) L = {a<sup>n</sup>x | #<sub>a</sub>(x) = n}
    (iii) L = {a<sup>i</sup>b<sup>i</sup>c<sup>j</sup>d<sup>j</sup> | i, j ≥ 0} Solution. S → LR, L → aLb | ε, R → cRd | ε
    (c) L = {a<sup>p</sup>b<sup>p+2q</sup>}
    (d) L = {a<sup>n</sup>b<sup>m</sup>c<sup>p</sup>d<sup>q</sup> | m+n = p+q, n ≥ q}. Hint: L = {a<sup>q+i</sup>b<sup>m</sup>c<sup>m+i</sup>d<sup>q</sup> | m,q,i ≥ 0}
    (e) L = {a<sup>n</sup>b<sup>m</sup>c<sup>p</sup>d<sup>q</sup> | m+n = p+q}. Hint: This is the union of the previous language and a similar language with