B501, Fall 2024

Assignment 5: Regular languages

1. (14%) For each of the following languages over $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ show that it is basic, by giving regular expressions and/or using closure properties of the basic languages. Do not use recognition by DFAs or NFAs. Assume that $\star$ is an extra regular expression denoting Σ^*.

(i) The set of strings with no consecutive a's.

Solution. The set of strings with two consecutive a's is denoted by the regular expression $\star\mathtt{aa}\star$ and is therefore basic. The given language is its complement, and is therefore basic as well.

(a) $L = \{w \in \Sigma^* \mid \#_a(w) \text{ even and } \#_b(w) \text{ odd }\}$

(b) The language L consisting of strings with no substring $\mathtt{aaaa}$ or $\mathtt{bbbb}$.

2. (16%) Suppose that $L \subseteq \Sigma^*$ is basic. Use closure under set and language operations to show that the following languages are also basic.

(a) $L' = \{w \in L \mid |w| \text{ is even } \}$

(b) $\tilde{L} = \{x_1 \cdot y_1 \cdot \cdots \cdot x_n \cdot y_n \mid n \geqslant 0, x_i \in L,\ y_i \notin L\}$

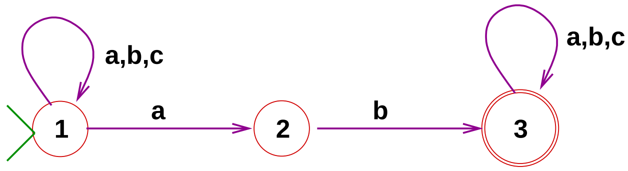
3. (10%)

(a) Given a language K describe an infinite collection $L_1, L_2, \ldots$ of basic languages whose union is K.

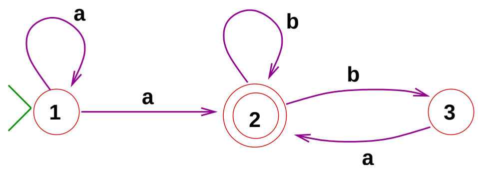
(b) Given a language K describe an infinite collection of basic languages whose ***intersection*** (i.e. the strings that are in all of them) is K.
[Hint: This problem is dual to the previous one. But in place of the union of trivial finite languages, consider here the intersection of trivial co-finite languages.]

4. (10%) Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ and $f : \Sigma^* \to \Sigma^*$ the function that for input w yields the string obtained by duplicating each a. E.g. $f(\mathtt{baaca}) = \mathtt{baaaacaa}$. Prove that if L is a regular language, then so is $\{f(w) \mid w \in L\}$. [Hint: Think of a regular expression for L.]

5. (20%) Convert the following NFA into an equivalent DFA.



6. (20%) Convert the following NFA into an equivalent regular expression. Exhibit all stages of the conversion.



7. (10%) A ***CNFA*** (conjunctive NFA) C (over alphabet Σ) is like an NFA, except that a string w is accepted by C if *every* state p such that $s \xrightarrow{w} p$ is accepting. Prove that a language is recognized by a CNFA iff it is regular. [Hint: When is a string w *not* accepted by C?]