# B561 – Selected Solutions for Assignment 2

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## Selected Solutions

(7) Let $X$ be a subset of $R$ and let $r$ and $s$ be relations on $R$ (i.e., r is the relation $r(R)$ and $s$ is the relation $s(R)$). Prove or disprove the following equalities.

(a) $\pi_X(r \cap s) = \pi_X(r) \cap \pi_X(s)$

(b) $\pi_X(r \cup s) = \pi_X(r) \cup \pi_X(s)$

(c) $\pi_X(r - s) = \pi_X(r) - \pi_X(s)$

(a) False. Counterexample: $r = \{(x, y)\}, s = \{(x, z)\}$ and $R = \{A, B\}, X = \{A\}$. Then $\pi_X(r \cap s) = \emptyset$ but $\pi_X(r) \cap \pi_X(s) = \{(x)\}$.

(b) True. $\pi_X(r) = \{t \mid \exists v \in r \text{ such that } \pi_X(\{v\}) = \{t\})\}$ and $\pi_X(s) = \{t \mid \exists v \in s \text{ such that } \pi_X(\{v\}) = \{t\}\}$. Hence, $\pi_X(r) \cup \pi_X(s) = \{t \mid (\exists v \in r \text{ such that } \pi_X(\{v\}) = \{t\}) \text{ or } (\exists v \in r \text{ such that } \pi_X(\{v\}) = \{t\})\} = \{t \mid \exists v \in r \text{ or } \exists v \in s \text{ such that } \pi_X(\{v\}) = \{t\}\} = \{t \mid \exists v \in r \cup s \text{ such that } \pi_X(\{v\}) = \{t\}\} = \pi_X(r \cup s)$. Note that the "or" and "and" connectives are indeed those from propositional logic and that one could write $\vee$ and $\wedge$ instead.

(c) False. Use counterexample from (a).

(8) Let $r$ and $r'$ be relations on $R$, and let $s$ be a relation on $S$. Prove or disprove:

(a) $(r \cap r') \bowtie s = (r \bowtie s) \cap (r' \bowtie s)$

(b) $(r - r') \bowtie s = (r \bowtie s) - (r' \bowtie s)$

(a) The statement is correct. Prove: $(r \cap r') \bowtie s = \pi_{R \cup S}(\{t \mid t \in (r \cap r') \times s \text{ and } \forall A, B \in R \cap S : t[A] = t[B]\}) = \pi_{R \cup S}(\{t \mid t \in (r \times s) \text{ and } t \in (r' \times s) \text{ and } \forall A, B \in R \cap S : t[A] = t[B]\}) = \pi_{R \cup S}(\{t \mid (t \in (r \times s) \text{ and } \forall A, B \in R \cap S : t[A] = t[B]) \text{ and } (t \in (r' \times s) \text{ and } \forall A, B \in R \cap S : t[A] = t[B])\}) = \pi_{R \cup S}(\{t \mid t \in r \times s \text{ and } \forall A, B \in R \cap S : t[A] = t[B]\} \cap \pi_{R \cup S}(\{t \mid t \in r' \times s \text{ and } \forall A, B \in R \cap S : t[A] = t[B]\} = (r \bowtie s) \cap (r' \bowtie s)$.

(b) The statement is correct. Proof analogous to (a).

(11) Let $r(R)$ and $s(S)$ be relations where $R \cap S = \emptyset$. Prove

$$(r \bowtie s) \div s = r.$$

Here $\div$ denotes the division operator.

Since $R \cap S = \emptyset$, $(r \bowtie s) \div s = (r \times s) \div s$. Hence we have to show that $(r \times s) \div s = r$. Let $d = (r \times s) \div s$. We know by definition that $d$ is the largest (in terms of cardinality) relation instance such that $d \times s \subseteq r \times s$ (e.g., see the textbook). But then $d$ must be $r$.

(12) Let $r$ be a relation on schema $R$ and let $s$ and $s'$ be relations on scheme $S$, where $R \supseteq S$. Show that if $s \subseteq s'$, then

$$r \div s \supseteq r \div s'.$$

Show the converse is false.

We use the formula for the division from problem (13)(a). Since $s \subseteq s'$, we clearly have that$\pi_{R'}((\pi_{R'}(r) \bowtie s) - r) = \pi_{R'}((\pi_{R'}(r) \times s) - r) \subseteq \pi_{R'}((\pi_{R'}(r) \times s') - r) = \pi_{R'}((\pi_{R'}(r) \bowtie s') - r)$. But then$\pi_{R'}(r) - \pi_{R'}((\pi_{R'}(r) \bowtie s) - r) \supseteq \pi_{R'}(r) - \pi_{R'}((\pi_{R'}(r) \bowtie s') - r)$ and hence, $r \div s \supseteq r \div s'$.

The converse is false. Counterexample: $\{(r_1, s_1), (r_1, s_2), (r_2, s_1)\}, s = \{(s_1)\}, s' = \{(s_2)\}$. Now we have that $r \div s = \{(r_1), (r_2)\}$ and $r \div s' = \{r_1)\}$, i.e., $r \div s \supseteq r \div s'$ but not $s \subseteq s'$.

(13) Let $r(R)$ and $s(S)$ be relations with $R \supseteq S$ and let $R' = R - S$. Note that $t \in s$ denotes a tuple $t$ in $s$ and the expression $\sigma_{S=t}(s)$ denotes the selection of exactly the tuple $t$ on $s$. Prove the identities

(a) $r \div s = \pi_{R'}(r) - \pi_{R'}((\pi_{R'}(r) \bowtie s) - r)$

By definition, $r \div s = \{t \mid t \in \pi_{R-S}(r)$ and $\forall t_s \in s \; \exists \; t_r \in r$ such that $\pi_S(\{t_r\}) = \{t_s\}$ and $\pi_{R-S}(\{t_r\}) = \{t\}\}$.
Since $R' = R - S$ we have that $\pi_{R'}((\pi_{R'}(r) \bowtie s) - r) = \pi_{R'}((\pi_{R'}(r) \times s) - r)$.
Clearly, $\pi_{R'}((\pi_{R'}(r) \times s) - r) = \{t \mid t \in \pi_{R-S}(r)$ and $\exists t_s \in s \; \forall \; t_r \in r$ one has that $\pi_S(\{t_r\}) \neq \{t_s\}\}$ or $\pi_{R-S}(\{t_r\}) \neq \{t\}\}$. In other words, it is the set of all "disqualified" tuples. Now, $\pi_{R'}(r) - \pi_{R'}((\pi_{R'}(r) \bowtie s) - r) = \{t \mid t \in \pi_{R-S}(r)$ and $\forall t_s \in s \; \exists \; t_r \in r$ such that $\pi_S(\{t_r\}) = \{t_s\}$ and $\pi_{R-S}(\{t_r\}) = \{t\}\} = r \div s$.