

Homework Zero

A201/A597/I210
Spring Semester 2005

Due at the end of lecture on Jan 20

Abstract

Please try to solve the problems below. Do your best and submit your answers in writing. You're encouraged to work in groups and discuss the problems but you need to prepare the write-up all by yourself. As an analogy assume you're sportswriters¹ for one of the major newspapers in the business (Sports Illustrated, New York Times, The Chicago Tribune, IDS²) and you are allowed to watch a game of basketball together³, and discuss it with each other, but when you write your report for the newspaper you're working for, you each write your own, from scratch, and trying to be as original and authentic as you could possibly be. Otherwise the newspaper will terminate your employment. In our case the Computer Science Department⁴ and the School of Informatics⁵ clearly specify the rules of academic honesty and academic integrity, so please read the documents and make sure you understand them and comply with them.

Also posting solutions or major hints on the bulletin board is not allowed.

1 Does Canada have a 4th of July?

Consider the following excerpt from Grosswirth and Salny on intelligence:

"I admit that mine is a personal and untested theory, but I believe that over the years, we have been culturally conditioned to abandon thinking as unnecessary. Everything—news, information, entertainment, medical care, food, merchandise—is provided in neat packages, predigested, pre-conditioned. Just add water, batteries not included. We've lost the habit of thinking for ourselves. [...] Intelligence implies the ability to think, and an IQ score is an index of a potential. If your scores are lower than you would have liked, you are making a mistake if you believe there is nothing you can do about [it]. It has been said before, but it bears repeating: Use it or lose it. You don't need any teachers, counselors, psychologists, or gurus to help you expand your intelligence. All you need is the desire to do

¹Or, more general, just *pundits*.

²You name it.

³Same goes for a presidential debate, for example.

⁴<http://www.cs.indiana.edu/Academics/integrity.html>

⁵<http://www.informatics.indiana.edu/courses/honesty.asp>

so and the initiative to practice. Ask questions. Understand everything, even if it has to be explained several times or in a new way. Give free rein to your curiosity. Most important of all, free yourself of the mental boundaries I mentioned earlier.”

Now take a look at the problems listed below. You may find them tricky, and they are. What makes them tricky is that they rely on the likelihood that your thinking is conditioned to move in only one direction with respect to the information given. Two problems are described in detail, the rest are listed at the end for your exercise.

1. The Scottish Name Gambit

This should be done orally; it's too easy to solve when you see it in print. Ask your victim to pronounce M-A-C-T-A-V-I-S-H. After he or she has done so, try M-A-C-C-A-R-T-H-Y; then M-A-C-D-O-U-G-A-L. You can use any other “Mac” names, as long as they are legitimate. After each name has been pronounced in turn, ask the sucker to pronounce M-A-C-H-I-N-E-S. As you can see, that spells *machines*, but if your victim doesn't answer with “MacHines,” it's safe to assume that he or she has fallen into the trap before. By spelling out the “Mac” names first, you will have effectively preconditioned the listener's thinking, leading it straight down a predetermined—and totally incorect—path.

2. The Time Gambit

One more supposedly trick question, this one to demonstrate how it isn't even necessary for you to set the person up; you can simply take advantage of the preconditioning that already exists. Punctuate the following: TIME FLIES I CANT THEYRE TOO FAST. Most people will stick in the missing apostrophes immediately, but then they're stymied. Look how simple it is, once you free up your thinking (the quotation marks are a little added refinement to aid comprehension): “Time flies.” “I can't. They're too fast.” Still doesn't make sense? Look again. The problem is that you're already familiar with the cliché, “Time flies”. In that expression, *time* is a noun and *flies* is a verb, and your conditioning leads you to assume that it has to be that way. It doesn't. Read the punctuated sentence again, but this time consider *time* as the verb and *flies* as the noun. Tricky? Only if you insist on retaining the strictures placed on your mind, on your native intelligence.

3. Practice Questions (with Answers)

- a) Under international law, if a plane crashes in the middle of the Atlantic, where would the survivors be buried?
- b) How many months have twenty-eight days?
- c) What is the next letter in this series? O T T F F S S E N.
- d) I have two coins that add up to 55 cents. One of them is not a nickel. What are the two coins?
- e) Divide 100 by $1/2$ and add 10. What do you have?
- f) A snail is climbing out of a well. The well is twenty feet deep. Every day the snail climbs up three feet and every night he slips back two feet. How many days will it take to get out of the well?

- g) A poor but honest knight wants to marry a beautiful princess, and she wants to marry him. The king offers the knight a choice, as follows: He can draw one of two slips of paper from a golden box. One will say “Marriage,” the other “Death”. The princess manages to whisper to her suitor that both slips say “Death”. But the knight and the beautiful princess are wed. How did the knight accomplish this?
- g) A very fast train runs from A to B in an hour and a quarter, but on the return trip it takes 75 minutes under identical conditions. Why? (Answers are provided here for the group as a whole⁶.)

Now you're ready for your Homework Assignment.

2 What is Programming?

2.1 A First Example

You are given two different length strings that have the characteristic that they both take exactly one hour to burn. However, neither string burns at a constant rate. Some sections of the strings burn very fast; other sections burn very slowly. All you have to work with is a box of matches and the two strings.

- a) *Describe an algorithm that uses the strings and the matches to calculate when exactly 45 minutes have elapsed.*
- b) *Same task when each string takes 30 minutes to burn completely.*

Note: this problem serves a dual role. Its solution clearly marks it as an exercise in threads scheduling (pun intended) and/or parallel programming. But there is a more important role it plays: it gives the student that solves the problem (on her own) tremendous confidence.

The satisfaction that comes out of finding the solution to this problem (on one's own) is so strong it can become addictive. Because despite severe limitations and restrictions (how much is really given in the problem?) an elegant solution is immediate. It is also instructive sometimes to compare and contrast the problem above with this one:

Using six identical matches create four equilateral triangles.

One is likely to find more than one solution to this last problem. This would also be a perfect place to differentiate between this type of problems and those of the kind shown before of which we list some more:

⁶a) Nowhere. You don't bury survivors. b) Every month has twenty-eight days; most of them also have more. c) T, for Ten. The sequence is One, Two, Three, etc. d) One of them is a nickel: a 50 cents piece and a 5 cents piece. e) 210. If you divide a number by 1/2 you double it. f) Eighteen days. On the eighteenth day he reaches the twenty-foot level and climbs out; he doesn't have to fall back. g) The poor but honest and clever knight tears up the paper he picks, and offers the other one to the king. Since the untorn one says “Death,” obviously, says the knight, the one he tore up said “Marriage” h) One hour and a quarter is 75 minutes; there is no difference. A few more examples will be given in the next section.

- a) What five-letter English word becomes shorter when you add two letters to it?
- b) What four-letter English word becomes shorter when you add three letters to it?
- c) How many bricks do you need to complete a brick house in England?
- d) Take away my first letter: I remain the same. Take away my fourth letter: I remain the same. Take away my last letter and I remain the same. What am I?
- e) What is the number of letters in the correct answer to this question?
- f) What English word is pronounced incorrectly by more than half of the Harvard and Yale graduates?

These questions don't offer any deep insight into what programming is (which is why we list their answers right away⁷).

And perhaps these last few questions are somewhat humorous. Yet we need to understand that the two types of problems discussed in this section could be equally entertaining and, under the circumstances, quite possibly *just as hard*. We need to remind ourselves constantly that programming is a contact sport.

2.2 The Rest of the Examples

2.2.1 The Captive Queen

A captive queen weighing 195 pounds, her son weighing 90 pounds, and her daughter weighing 165 pounds, were trapped in a very high tower. Outside their window was a pulley and rope with a basket fastened on each end. They managed to escape by using the baskets and a 75-pound weight they found in the tower. How did they escape? The problem is that anytime the difference in weight between the two baskets is more than 15 pounds, someone might get killed. Describe a sequence of steps that gets them down safely.

Notes: at first this is not as clean a problem as the previous one. Issues about the pulley and the basket and many operational details always arise. But that should be a reason to encourage discussion of the problem with the clear goal of abstracting away all the irrelevant details. In any event after a certain amount of clarification the problem becomes just as unambiguous as the preceding one.

This is not a difficult problem (the previous one is). What makes this problem easy is that you can work at it and perhaps accidentally discover a solution. In that respects it's like the Tower of Hanoi problem, only not as hard. The real benefit of this problem, however, lies in the fact that the steps comprising the first two fifths of the solution need to be repeated at the end. So it would be a perfect case of defining a named procedure: a sequence of steps that you want to refer to later, by name.

⁷a) short, b) shoe (SHOrtEr), c) just one (in England and anywhere else the last one completes the house), d) a mailman, e) four (in English, it can be trickier in other languages), and f) "incorrectly" is the word.

2.2.2 A Farmer's Predicament

A farmer lent the mechanic next door a 40-pound weight. Unfortunately, the mechanic dropped the weight and it broke into four pieces. The good news is that, according to the mechanic, it is still possible to use the four pieces to weigh any quantity between one and 40 pounds on a balance scale. How much did each of the four pieces weigh? (Note: You can weigh a 4-pound object on a balance by putting a 5-pound weight on one side and a 1-pound weight on the other).

Notes: both issues of *representation* and of *logic programming* can be brought up. One usually is very creative in approaching this problem. I use it to introduce the balanced ternary system of notation (which is a non-redundant and symmetric positional number system). Once this topic is brought up it's easy to discuss the binary representation of numbers or representation in any other base. So, what are the four numbers? Remember: do the best you can, write your answer up, then turn it in.

2.2.3 Calculating a Test

Given two numbers (identified by the names x and y) can you calculate the largest of the two numbers?

Notes: please don't try to solve this problem by *comparing* the two numbers in any way. You cannot. You don't know the *values* of the two numbers, you only know their *names*. So, how do you do it?

2.2.4 Symbolic Reasoning

This is a short test of your mathematical ability to understand an argument (in a somewhat more formal way). Consider the following instructions and:

- a) use them to calculate some square root by hand⁸, and
- b) to explain if you think the rules are correct, or complete, or not?

Rules: Start by making an initial guess $x_0 > 0$ for $\sqrt{5}$. If we guessed right already (how do we know?) then we are done. But if x_0 is not the square root of 5 then x_0 is either smaller or bigger than the square root we're looking for. When it's smaller we have

$$x_0 < \sqrt{5} \tag{1}$$

and when it's bigger we have

$$\sqrt{5} < x_0 \tag{2}$$

In both cases we multiply on both sides by $\sqrt{5}$ and divide by x_0 to obtain equivalent expressions:

$$\sqrt{5} < \frac{5}{x_0} \tag{3}$$

⁸You can also write a program as discussed in the first week of classes to help out.

$$\frac{5}{x_0} < \sqrt{5} \tag{4}$$

Combining that, regardless of the original guess x_0 we have either:

$$x_0 < \sqrt{5} < \frac{5}{x_0}$$

or

$$\frac{5}{x_0} < \sqrt{5} < x_0$$

Therefore, if we take the average of x_0 and $\frac{5}{x_0}$, namely

$$x_1 = \frac{1}{2} \left(x_0 + \frac{5}{x_0} \right)$$

the result must be closer to $\sqrt{5}$, which is what we're after.

Can you use these rules to calculate $\sqrt{4}$ (which we know to be 2) and $\sqrt{3}$ (which is about 1.7320508075688772)? Do you feel the rules are complete, clear or perhaps there's a mistake or an omission somewhere?

2.2.5 Recursion and Analytical Geometry

Given two distinct points in the plane $A(x_0, y_0)$ and $B(x_1, y_1)$

- a) determine their midpoint⁹
- b) determine a way to draw point by point the *entire* segment AB

Notes: this is first an exercise in (minimal) analytical geometry. Intersections between two straight lines expressed parametrically could also be discussed eventually, but this is not the real aim of this problem. I use it to introduce both recursion and notions of 2D graphics, at the same time.

2.2.6 Repetitive Behavior

Given a sequence of numbers $\{x_1, \dots, x_n\}$ use the formula below

$$S = \sqrt{\frac{\sum x_i^2 - \frac{1}{n}(\sum x_i)^2}{n-1}}$$

to calculate their standard deviation.

Calculate with this formula the standard deviation for the following sequence of 5 numbers: $\{5, 0, 2, 2, 3\}$ and describe the steps you took (and the partial results you obtained) in writing—as part of your assignment.

⁹The point $M(x, y) \in AB$ that's at the same distance from A as it is from B

2.2.7 Magic Squares

This is the last problem of the assignment.

Implement the following procedure to construct magic n -by- n squares; it works only if n is odd. Place a 1 in the middle of the bottom row. After k has been placed in the (i, j) square, place $k + 1$ into the square to the right and down, wrapping around the borders. However, if the square to the right and down has already been filled, or if you are in the lower right corner, then you must move to the square straight up instead. Here's the 5×5 square that you get if you follow this method:

11	18	25	2	9
10	12	19	21	3
4	6	13	20	22
23	5	7	14	16
17	24	1	8	15

Check that the square above is magic.

Calculate the 3×3 and 7×7 magic squares.

3 What is Programming?

Programming is the art of giving precise instructions. An analyst usually solves the problem and outlines a solution for a programmer. The programmer codes the solution in using a programming language. The drudgery is then assumed by the computer.

In this class you will be both analysts and programmers, and we will be using the Java programming language to write our programs. You will learn to solve problems (as analysts) and to code their solutions into Java (as programmers).

The problems above try to give you a glimpse of what the semester will be like (all of it, that's why the assignment is so long) before we really even get started. Do not be disappointed if you find these problems challenging. Use this as a calibrating device, ask for help, and warm up for an exciting semester ahead.