# What is Computation?
# (How) Does Nature Compute?

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November 1st, 2008

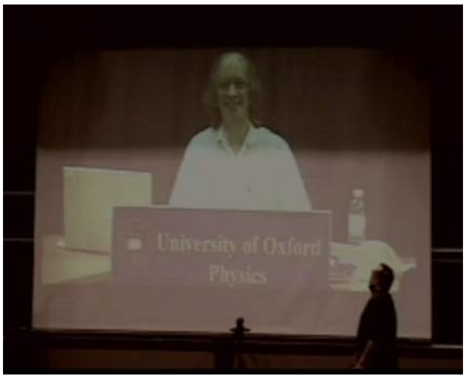


Figure 1: Introducing David Deutsch

**Adrian German** (addressing the audience[1]): Our next guest is the recipient of the 2005 Edge of Computation Science Prize and will be speaking to us via video-link[2] from the University of Oxford. He pioneered the field of quantum computation and was the first person to formulate a description for a quantum Turing machine and to specify an algorithm designed to run on a quantum computer. He is also a proponent of the many-worlds interpretation of quantum mechanics and his new book entitled "The Beginning of Infinity" is due to appear early next year[3]. Ladies and gentlemen it is with great pleasure that I am asking you now to join me in welcoming Prof. David Deutsch. (Applause.) And with

[1]Talk at the 2008 Midwest NKS Conference held October 31-November 2, 2008 on the Bloomington campus of Indiana University. Transcript by Adrian German (who was one of the co-chairs of the conference—the other conference chair being Hector Zenil).

[2]Videoconference made possible through the kind and generous assistance of Angie Day, Ian Campbell and Stig-Topp Jorgensen at the University of Oxford and Steve Egyhazi at Indiana University Bloomington. Videostream available from the conference website `http://www.cs.indiana.edu/~dgerman/2008midwestNKSconference` under the "What's New?" tab, along with the videos for the other keynote talks at that conference.

[3]The book, entitled "The Beginning of Infinity: Explanations That Transform the World" was published in March 31, 2011 by Viking (a member of the Penguin Group).

your permission I'd like to add just one more thing. In one of his books[4] Colin Bruce writes:

> "David Deutsch is to be respected for the courage of his convictions as regards many-worlds. Asking some scientists if they really believe in parallel worlds is a bit like asking a modern theologian if he really believes in miracles; all you discover is that physicists can duck and weave with the best of them. Deutsch does not try to hide behind words or philosophical cop-outs but acknowledges that yes, parallel versions of our world are just as real as our own, including copies in which he himself exists but is doing different things at this moment."

So, David, may I please say how happy and grateful we are because in this Universe it looks like you are, in fact, going to give the talk!

**David Deutsch** (laughs): Thank you. (Applause.) OK, so it seems we're asking ourselves today "What is Computation?" and either "Does Nature Compute?" or "*How* Does Nature Compute?" And there's an amazing fact that motivates both of these questions and indeed motivates every other foundational question about computation as well. It is this: if you take any physical variable whatsoever, for example "who is going to be the next president of the United States", to take a topical example or, another one is "the mean temperature of the Earth's atmosphere as a function of time" and ask how that variable depends on other variables, then the answer will always invariably be a computable function—or, if there's quantum indeterminacy involved then the probability distribution function will be a computable function. This is because the laws of physics refer only to computable functions—either directly or via computable differential equations.

Now, the reason this is amazing is that most mathematical functions are not computable—in fact, the set of computable functions is of measure zero in the class of all mathematical functions, let alone in the class of all mathematical relationships. So, there is something infinitely special about the laws of physics as we actually find them, something exceptionally tractable, prediction-friendly and computation-friendly. That's clearly not accidental. So, there's definitely something there to be explained. But people make different things of it, and *what* they make of it depends on quite deep aspects of their world view. Of course, religious people tend to see divine providence in it and some evolutionists see the signature of evolution namely "apparent design" — but at the level of "laws of nature", to make them computable.

[4] Schroedinger's Rabbits: The Many Worlds of Quantum (2004, pp. 175-6).

And cosmologists see anthropic selection effects and computer programmer type people, well, they see either a great computer in the sky in which we're all simulations, like in the movie "The Matrix" or that the Universe itself is a computer—and either way that what we perceive as physical phenomena are actually just virtual reality: *running programs*. Now, all those conceptions are wrong—because they all share a fatal flaw. They all have other flaws as well, but I want to concentrate on the shared one, which is relevant to our question, namely: "What is Computation?"

The laws of nature are, by definition, inviolable. For instance, you can't make a perpetual motion machine—I hope you're seeing the slide now... On the way here today I saw a headline on the BBC that said "the future of physics is in jeopardy". But that is exactly what can't happen—the future of physics is unchanging, inviolable, invariable. Its only what we do that can change! So, there's something inviolable in the universal truths of the physical laws! And, also, the theorems of mathematics are inviolable. For instance, you can't change which of the two integers is the larger. So in both cases inviolable refers to a fact of the matter, meaning: it can't be argued away just by changing terms or definitions. For instance, you could define the term perpetual motion so that a glass of tap water is a perpetual motion machine, because the molecules are literally in perpetual motion. Nevertheless that wouldn't enable you to use that notion to charge up batteries in a cycle. That's a fact of the matter. And similarly in arithmetic you can redefine the word billion which in British English used to mean $10^{12}$ and was then redefined down to $10^9$ in line with American usage but that renaming doesn't make any actual number bigger or smaller—and that's another fact of the matter, but of a different kind. So, the laws of physics and the truths of mathematics are equally inviolable, they are universal truths and they are both about something objective. So they're alike in those respects but nevertheless there is a well recognized and important difference between scientific and mathematical truths: it is in *what* these fields are about. Mathematics is about absolutely necessary truths. Such truths are all abstract and essentially they are truths about what is or isn't logically implied by particular axioms, but science isn't

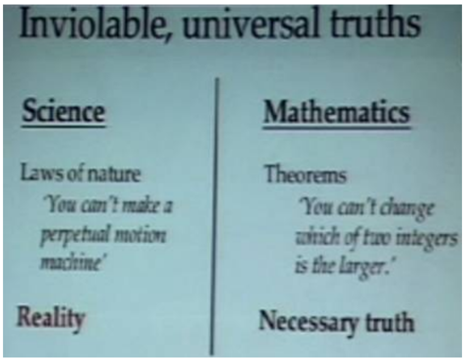


Figure 2: Both science and mathematics are about universal and objective, but distinctly different, types of truths.

about what's implied by anything. It's about what it is really out there in the physical world. Laws of nature do therefore have to be consistent but unlike mathematical axioms they also have to correspond to reality, so that's the fundamental difference between mathematics and science, between theories and theorems.

Now, what about the laws of computation? By a law of computation I mean any inviolable generalization (any universal truth) about computation such as: that there exists no computer that could reliably detect whether a program will eventually halt, or not, or whether it would do *any* given thing. And in order to understand what computation is, the most basic question we have to address first is into which of those two categories do the laws, the inviolable laws of computation fall? Are they absolutely necessary truths or are they determined by the laws of physics?

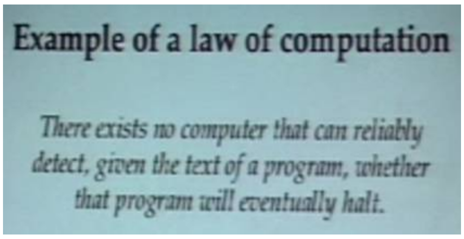

Figure 3: By law of computation I mean...

Now the answer is that they're determined by the laws of physics but there's been a lot of confusion in regard to that and to explain what's going on I have to look at this in a slightly broader context. The context here is that the theory of computation which is a branch of physics was pioneered by mathematicians. That is not unusual, several other important branches of theoretical physics were also started by mathematicians for instance geometry and probability theory and various theories involving the infinite and the infinitesimal (i.e., calculus) and indeed in many of those cases the same confusion that I am going to describe here did arise. In terms of this distinction between reality and necessary truth the confusion keeps arising because mathematicians tend not to have a very firm grip on the reality.

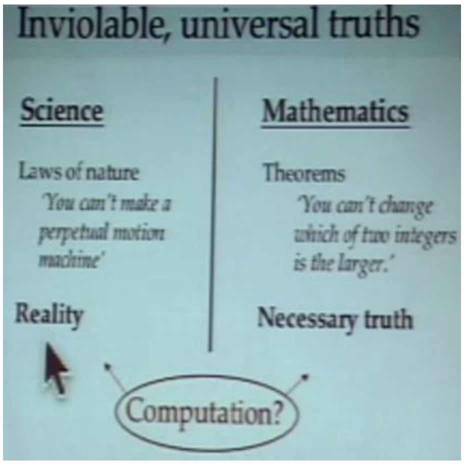

Figure 4: Into what category of truths do the inviolable laws of computation fall?

Here's what happens: take a mathematical idea, say—the idea of infinity. Mathematicians realized centuries ago that they really can work with infinity—they can arrogantly define, say, what an infinite set is, as one that can be placed in a one-one correspondence with a proper set of itself, and then they can prove theorems about such sets and about further abstract structures if they can consistently define them (in terms of such infinite sets) and sometimes they can then use those mathematical structures and theorems to formulate new scientific theories or to make existing theories more precise. For instance—as calculus was used by Newton and Leibnitz to formulate scientific theories about things like instantaneous velocity and other rates of change. OK—so far so good, but notice that the concept of instantaneous velocity in physics and in common sense doesn't involve anything infinite and indeed nothing physical is either infinite or infinitesimal in, say, the smooth motion of a projectile; and on the other hand there's already an informal conception of what infinite does mean. It means bigger than anything merely big, or more of something than can be quantified even in principle, more of something that can be addressed in any sequence however long. In that conception the essence of finiteness is not about sets or mappings, it's about what computer theorists call effectiveness and what physicists call measurability or preparability, durability—and those two conceptions of infinity the mathematical and the physical draw the distinction between finite and infinite at completely different places.

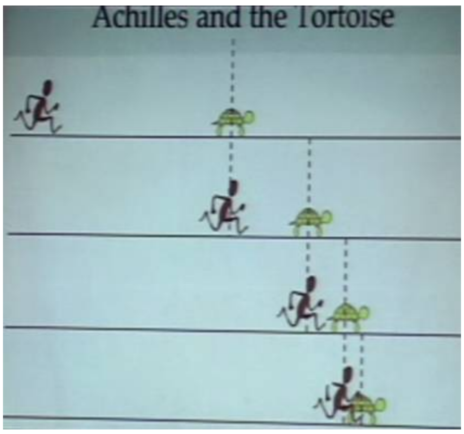


Figure 5: According to Simplicius, Diogenes the Cynic said nothing upon hearing Zeno's arguments—but stood up and walked, in order to demonstrate the falsity of Zeno's conclusions.

And that is essentially how Zeno of Elea in his famous paradox managed to conclude that Achilles will never overtake the tortoise, if the tortoise has a head start—because by the time Achilles reaches the point where the tortoise is now, the tortoise will have moved on a little, and by the time he reaches that point, it will have moved on a little further—and so on *ad infinitum*. And thus, the catching-up process requires Achilles to perform an infinite number of catching-up steps which—as a finite being—he presumably cannot do.

Presumably, but did you see what Zeno did just there? He just presumed that a particular mathematical notion that happens to be called "infinity" faithfully captures the distinction between finite and infinite, and was thus relevant to a particular situation in physics. And he was simply wrong. So, he succeeded only in confusing himself and there's nothing more to his paradox than that mistake. The truth is that what Achilles can or can't do cannot be deduced from mathematics, or *a priori*, in any way. It depends entirely on what the relevant laws of physics say. If they say that he'll overtake the tortoise in a given time, then he'll overtake it. If that happens to involve an infinite number of steps of the form "go to where the tortoise is now" then he'll do an infinite number of such steps. If it involves his passing through an uncountable infinity of points in space then that's what he does—but nothing physically infinite has happened. And, by the way, what did happen didn't happen in steps!

OK, well, the distinction between finite and infinite is also at the heart of the theory of computation. For instance, an effective algorithm is defined as one that halts after a finite number of steps, where a step has to be defined according to a finite list of rules. And a rule has to be finitely executable. And so on. Now these requirements were implemented in the classical theory of computation which was pioneered by Alan Turing but they date back to mathematical requirements set by the mathematician David Hilbert in 1900 with the intention of formalizing the concept of mathematical proof. In fact, classical computations are essentially the same things as proofs in Hilbert's and Turing's sense because every valid proof is a computation of the conclusion from the premises and every correctly executed computation is a proof that the output is the result of the given operations on the input. Hilbert had required finiteness conditions, he had required that proofs use only a fixed, finite set of rules of inference and executing a proof had to take only a finite number of elementary steps and the steps themselves had to be finite—so you recognize that from the theory of computation, that's exactly where finiteness, effectiveness came from in the theory of computation.

Now Hilbert contemptuously ridiculed the idea that his finiteness requirements were substantive ones, but do you see that he was thereby making exactly the same fundamental mistake as Zeno was? He was assuming that a particular mathematical distinction between finite and infinite in regard to steps and axioms and so on was self-evidently the one that corresponds to doable, the effective, thinkable and provable in physical objects—such as the brain of mathematicians. Fortunately, Hilbert's intuition about finite and infinite unlike the one that Zeno tried to implement did match physics (and as far as we know even quantum physics) so his conceptual mistake didn't do all that much harm at first and Turing's implementation of it was therefore enormously successful and fruitful. But the point is that if the laws of physics were kind of different from what we currently think they are then so might be the set of mathematical truths that we then would be able to prove, and so might be operations that we'd be able to use to prove them with. The laws of physics that we know happen to afford a privileged position to such operations as `AND` and `OR` and `NOT` and to the concatenation of functions and to individual bits of information. But if instead they were based on, say, Turing machines as elementary objects instead of points and so on, and if the laws of motion depended on functions like "does this halt?" instead of on differential equations then one could compute using operations which with our physics we call non-computable. Well, perhaps, the functions that seem natural and elementary to us would be non-computable in that physics. Similarly it's not just a distinction between computable and non-computable that depends on the laws of physics but also a very important distinction between simple and complex. We now know, because of quantum computation, that Turing's and Hilbert's conceptions of what is simple and complex is not reflected in real physics. Quantum computation drives a coach and horses through the intuitive notion of simple or elementary operation and it makes some intuitively complex things simple.

It might be objected that quantum computation—therefore—isn't real computation, it's just physics, just engineering, and it might be argued that the logical possibilities that I have just been describing (that would enable exotic forms of physics which would then enable exotic forms of computation) likewise don't address the issue of what a proof really is, or what a computation really is. So, more precisely the objection would go something like this: "Under suitable laws of physics we would be able to compute non-Turing computable functions but that wouldn't be genuine computation; and similarly we might be able to establish truth or falsity of undecidable mathematical propositions, but then again that 'establishing' wouldn't be the same as genuinely proving—because then our knowledge of whether the proposition was true or false would forever depend on our knowledge of what the laws of physics really are. If we discovered one day that the real laws of physics are different we might have to change our mind about the proof, too, and its conclusion and so it wouldn't really be a proof (so the objection runs) because real proof is independent of physics."

Wrong! That objection is nonsense, because it would apply equally well to any proof. Our knowledge of whether a proposition is true or false always depends on our knowledge about how physical objects behave, be they computers or our own brains! If we changed our minds about what physically a computer or a brain has been doing, and in principle the changing of our minds could be due to changing our minds about what the laws of physics are—that they're not what we thought they were, for instance if we decided that when formulae become sufficiently complex computers or brains are subject to, and there's a different term in the equations of motion that comes in and causes systematic false memories (this is a little bit like Roger Penrose's idea about what happens to the wave function when information becomes more complex than a certain amount)—then we'd be forced to change our opinion about whether we proved something or not and possibly about whether we know it to be true or not. And here I must stress that whether a mathematical proposition is true or false is completely, that is, indeed *completely independent* of physics but proof is 100% physics, proofs are not abstract, there is no such thing as abstractly proving something just as there is no such thing as abstractly calculating or computing something. One can of course define a class of abstract entities and call them proofs, just like you can define a perpetual motion machine to be something else, one can define abstract entities and call them *computations* but those proofs can't do the job of verifying mathematical statements, they're not effective or doable. A mathematical theory of proofs therefore has no bearing on which proofs can or cannot be proved in reality or known in reality and similarly, a theory of abstract computation as well has no bearing on what can or cannot be computed in reality.

So, what is provable or unprovable is determined by the laws of physics in exactly the same sense as "what the angles of a triangle add up to" is determined by the laws of physics. Immanuel Kant thought that Euclidean geometry was self-evidently true—and that's another example of the same misconception that I've been talking about. The truth is that you can define abstract entities and call them triangles and have them obey Euclidian geometry but if you do that you can't then infer anything from that theory about, say, what angle you'll turn through if you walk around the closed path consisting of three straight lines—that thing might not be a triangle as you have defined it. Likewise in probability theory you can define abstract quantities that obey a certain calculus about mutually exclusive alternative events and you can

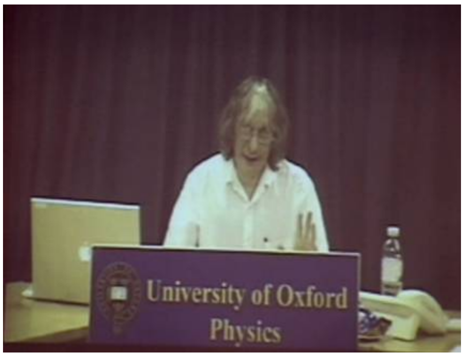


Figure 6: A computation is a physical process in which physical objects (like computers, or slide rules or brains) are used to discover, or to demonstrate, or to harness properties of abstract objects like numbers and equations. Hence the reliability of our knowledge of mathematics, nothing to do with whether propositions are true or false, just our knowledge of mathematics, remains forever subsidiary to our knowledge of physical reality.

call those quantities probabilities but in that case your theory tells you nothing about how you should bet in current events and, of course, in quantum theory real probabilities do not even obey those axioms of the probability calculus that refer to alternative intermediate events.


Figure 7: So, contrary to what Hilbert thought, contrary to what mathematicians since antiquity believed (and continue to believe to this day) proof theory can never be made into a branch of mathematics—nor is it healthful to think of it as a metamathematics, as its sometimes known. Proof theory is a science, and specifically—it is computer science.

So, quite generally: things that happen in reality are governed by the laws of physics, period. And therefore I can now give the definitive answer to the question that is the title of my talk and the title of the conference: "What is (a) computation?" A *computation* is a physical process in which physical objects like computers, or slide rules or brains are used to discover, or to demonstrate or to harness properties of abstract objects—like numbers and equations. *How* can they do that? The answer is that we use them only in situations where to the best of our understanding the laws of physics will cause physical variables like electric currents in computers (representing bits) faithfully to mimic the abstract entities that we're interested in. The reliability of the proof therefore depends on the accuracy with which those physical symbols do indeed mimic the abstract entities of interest. If we changed our mind about what the laws of physics are, we might indeed have to change our mind about whether a proposition we thought we'd proved was really true.

Hence the reliability of our knowledge of mathematics, nothing to do with whether propositions are true or false, just our knowledge of mathematics remains forever subsidiary to our knowledge of physical reality. Every mathematical proof depends absolutely, for its validity, on our being right about the rules that govern the behavior of some physical object like computers, or ink and paper, or our brains. So, contrary to what Hilbert thought, contrary to what mathematicians since antiquity believed and continue to believe to this day, proof theory can never be made into a branch of mathematics nor is it healthful to think of it as a metamathematics, as it's sometimes known. Proof theory is a science, and specifically it is *computer science*. So that's what computers are, that's what computation is, that's what proofs are—physical phenomena. That enables me now to tie into the subsidiary question: "Does Nature Compute?" or "How Does Nature Compute?" So, well, in one sense: yes, of course, computations are physical processes and every physical process can be regarded as a computation if we just label, give abstract labels to all the input states and all the output states and then just let the system evolve under the laws of physics. But precisely because you could always do that—no matter what the world was like—calling everything a computation doesn't in itself gain us any understanding of the world, but the world also has that amazing property that I referred to at the beginning: more broadly, its computational universality. All those different computations embodied in physical processes are expressible in terms of a single finite set of elementary physical operations. They share a single, uniform, physical distinction between finite and infinite operations, and they can all be programmed to be performed on a single physical object: a universal computer, a universal quantum computer to be exact. And that's an object that can perform every computation that every other physically possible object can perform. And to the best of our knowledge the laws of physics do have that property of computational universality and its because of *that* that physical objects like ourselves can understand other physical objects (i.e., to do science). And it's also because of that same universality that mathematicians (like Hilbert) can build up an intuition of proof and then mistakenly think that it's independent of physics. It's not independent of physics, it's just universal in the physics that governs our world.

But in the class of all possible universes that cosmologists nowadays postulate—not the many universes from quantum theory but the cosmological ones with all different laws of physics—most of them have other, very different laws of physics than ours, and in most of those there's no computational universality. In a tiny subclass (but still infinite though) there is universality but with all sorts of different computable functions—and also, all sorts of different measures of what is simple and complex in those worlds, and all sorts of different criteria for what is finite and infinite in those worlds.

In some of those universes there are analogues of David Hilbert or Alan Turing each of them reaching towards a different conception of what counts as mathematically proving something, what counts as computing something. And there are also lots of Stephen Wolframs, each of them seeking to base a new kind of physics on what they respectively experience as inherently simple, inherently simple self-evident computational foundations beneath his Universe's contingent laws of physics. But, in reality there is no such distinction as simple versus complex, or finite versus infinite except as the laws of physics dictate. There is no mathematically preferred conception of computation, or computability, or finiteness, or simplicity. Absolutely nothing other than physics (and the cultural preferences that it conditions in us) singles out Turing computable functions or cellular automata or even quantum computation, or quantum cellular automata, as being fundamental, or special, or elementary in any way. In fact there is nothing that singles out the computational functions at all, nor bits, logical variables, as being the fundamental forms of data on which computations operate.

There is nothing deeper known about the physical world than the laws of physics. And, I think, there is nothing deeper known within physics than the quantum theory of computation. And for that reason I entirely agree that it's likely to be fruitful to recast our conception of the world and of the laws of physics and physical processes in computational terms, and to connect fully with reality it would have to be in quantum computational terms. But computers have to be conceived as being inside the universe, subject to its laws, not somehow prior to the universe, generating its laws.

The latter is the very misconception that led Zeno astray, and Hilbert and Kant, and many other thinkers throughout history, who haven't realized that while truth can be absolutely necessary and transcendent, all knowledge (even of such truths) is generated, computed, by physical processes, and the scope and limitations of such knowledge are conditioned by Nature's contingent laws. Thank you very much. (Applause)

## Questions & Answers

**Question**: Would you consider an experiment, a physical experiment, to be a computation?
**Answer**: Are you asking about a specifical experiment, or a specifical physical process?
**Q**: No, in general.
**A**: Yes, you can always regard any physical experiment as a computation. If the outcome is unknown then it's a computation where we are probing what the computer is doing, and if the outcome is known—that is, if the laws are known—then what we are doing is transforming the input into the output. But in order to make an experiment into a computation you have to label everything, you have to give labels to all the inputs, states and to all the outputs...
**Q**: Great. Let me then have a follow-up question on that.
**A**: Sure.

**Q**: So, suppose you perform an experiment[5] and you figure out that Nature tells you that the answer to your question, to your computation, takes a polynomial number of steps—so the answer is in fact polynomially complex[6].
**A**: Ah, wait—an experiment can't tell you that!
**Q**: Well, you do the experiment and you ask the question, for example, how many resources you need in order to determine (or measure) a very specifical physical quantity with a certain error...
**A**: Yes—but you can only do that for a finite number of times, so you can't tell if for large inputs it's going to be polynomial or not. In order to tell whether the resources are indeed polynomial you have to know how it behaves for arbitrarily large values of the inputs.
**Q**: That's right, that's right ...
**A**: ... and you can only do it for a finite number of experiments.
**Q**: I understand that. Now my question is this: suppose that with the well-known laws of nature (with quantum physics) I cannot find an algorithm that performs the same thing with the same complexity.
**A**: Yes.
**Q**: What would you say then, what would you conclude about the laws of nature as we know them? Would you conclude that quantum mechanics is not the right representation of how nature works, or ... what am I missing?
**A**: Well, I am not entirely sure that I understand what your proposed experiment is. But if you indeed find that ... I mean, the idea is that you guess, you conjecture what the laws of physics are, which tell us what to conjecture about what our computer will do. If we find that a real physical process can compute something—appears to be able to compute something—in polynomial time that no algorithm (as given by the laws of nature as we know them) will compute in polynomial time, we can then indeed conclude that the laws that you thought were operating the device are in fact not the true laws operating the device. So if that's the question you're asking then the

[5] This question may have originally aimed to ask whether quantum mechanics is in fact a good model of reality, maybe in the sense of one of Sir Roger Penrose's interpretations (see, e.g., `http://www.cs.indiana.edu/~dgerman/penrose.pdf`) or was perhaps aimed to be considered in the larger context of "simulating of reality" (with computers, in real time or any plausible kind of time).

[6] This is an additional indication that the question has something to do with the 1967 Feynman quote from "The Character of Physical Law" that was the motto of our conference: "It always bothers me that, according to the laws as we understand them today, it takes a computing machine an infinite number of logical operations to figure out what goes on in no matter how tiny a region of space, and no matter how tiny a region of time. How can all that be going on in that tiny space? Why should it take an infinite amount of logic to figure out what one tiny piece of space/time is going to do?"

answer is definitely: yes.
**Q**: Very good. Thank you very much.
**New Question**: Can you hear me or should I use the microphone?
**Answer**: I can hear you very well[7].
**Q**: I completely agree that maybe mathematical concepts have a root in reality and I also agree that some mathematicians forget it sometimes. Yet it seems that there's always an abstraction step that is needed to go from the root of the concept in reality to the concept itself. If we take a very simple concept, for example the concept of natural number—of course it's rooted in notion of calculability and the notion of calculability is rooted in the notion of physical object. But if we restrict to just what we see around us, the reality we (as finite human beings) can perceive around us we might take the point of view which says for example that there's a maximum natural number and no successor.
**A**: Yes, and that would be silly.
**Q**: In some sense these abstraction steps free us from this limitation that for some reason we have and that we don't even want to consider.
**A**: Yes.
**Q**: So in this respect, although it is rooted in reality the concept of natural number is not in reality itself—it's abstracted from reality.
**A**: Yes. Well, it is not in physical reality at any rate. I mean, one could argue that mathematical entities are real in a different sense, in that they have autonomous properties that we didn't necessarily cook into them. But yes—as I stressed in my talk—mathematical truth really is independent of physics, it is only knowledge of mathematics that is constrained by what the laws of physics say. So there's this enormous realm of mathematical truths which is independent of what the laws of physics are and there's a window on that—a tiny window—which has the truths that we can know. And that window is determined by the laws of physics.
**Q**: What is true of natural number is true of proofs too. So, for instance, the mathematical notion of proof abstracts from what can be found in the reality.

[7] We had four videotalks at the conference: Charles Bennett (IBM) and Seth Lloyd (MIT) spoke on the first day while this talk from Oxford followed by Lev Grover's (from Bell Labs) were on the second day. All audio and video connections were excellent but this particular connection to Oxford was somehow far better than all the other three, far better than the most optimistic of our expectations in the sense that Prof. Deutsch was able to hear what was spoken in our amphitheater a little bit better than we (that were in that room) seemed to be able to hear and understand each other. This may have factored slightly in how the first couple of questions were asked until the audience adjusted to the unexpected high quality of the audio and video link.

**A**: Yes, yes. So it goes like this: you can have physical intuitions about things which are really intuitions about what the laws of nature are and then you can abstract to make nicer mathematical quantities that don't have arbitrary restrictions on them. And subsequently you can guess that those mathematical quantities are also instantiated in real physical objects. So the class of mathematical truths that we can know *about* is much larger than the class that we can *know*. But there might be other things [that] we can't even know about! Presumably there are, because the infinities involved would be simply too large to allow knowing about them; well, I suppose we know about them *in that sense*.

So, yes: I think I agree. Possibly you're making a subtle point that I haven't picked up but, yes: proofs arise from a physical intuition. We then form a mathematical conception, a mathematical abstraction, which we call proofs and then we conjecture that that mathematical conception is actually genuinely true in physics. But we could be wrong about that last step! The mathematical object(s) we have set up and called the "proofs", or "laws of inference" and such may simply not correspond to reality and then—or we may not know the correspondent even approximately—and then it's the set of things that we really *can* prove with physics that are the provable things not the ones that that mathematical conceptions might end up proving, because that is not effective in that case.

**Q**: Thank you.

**New Question**: I liked your talk, I have one question about computability. You said in the beginning that most, in fact *all* functions in physics are computable. And I wonder if whether it's as you said (that it's a miracle) or whether there's some explanation for that. And maybe just a simple possible explanation is that all of physics is in fact expressed in terms of ordinary differential equations and there are fundamental and general existence theorems for these equations and so on, stating that in such and such case there will always be a solution to them and only one. And when one uses computable tools to model the world that's also when what one models appears as having been already computable all along. Because if you look at the history of physics—physics usually shies away from theories that are not deterministic or are heuristic and maybe this is the reason why when we express a lot of physics in such a way that it would be only deterministic and maybe this is the reason for which every function in physics is computable.

**Answer**: Well, it's not a matter of arbitrary choice on the part of physicists. Physicists are trying to make the lowest fit experiment and it turns out that the laws as we think they are have this property—for instance they are differential equations and not just any old differential equations, but

well-posed differential equations which have this property of computability. Now, I guess one can—this is a bit like the anthropic principle reasoning—I guess one can be amazed by this or not. I think I *am* amazed by it, because in the bigger picture of mathematics these things are very special. And we don't know of any mechanism or reason why the actual laws of physics should be expressed in terms of those special kinds of differential equations. And actually even among that small set of differential equations are the particularly simple ones that appear to be actually implemented. So we don't know why, and some cosmologists say it's an anthropic selection effect: observers that can ask such questions only exist in universes where evolution happens and evolution is the kind of computation that depends on the existence of simple computability. And I don't think that that's the full argument, myself. That would take us off on a tangent here but although that may be true it can't possibly be a sufficient reason for why the laws of physics are as they are because the set of all possible laws of physics doesn't even have a measure on it and so this reasoning about "most of them do one thing and the rest do another" hasn't really got any basis.
**New Question**: I would like to—I like very much your position and your talk but I would like to point out a possible similarity between mathematics and physics. You know, if you look at mathematics, say, 200 years ago—most functions were continuous. And people believed, that indeed *all* functions were continuous. And when they started understanding better and using better tools, they discovered that most functions are not continuous. So maybe this is just a historical moment/accident when physics looked for laws that can be expressed by computable functions and not something that is motivated by some good reason—but simply a historic effect.
**Answer**: Well, it can't be a purely historic effect because the laws of physics, as we currently believe they are, are extremely successful whether, you know, there may be corrections to them and there may be regimes in which they are completely wrong but they are—it is already a miracle that the physical world is as computable as it seems to be, even if in some other respects it may turn out to be non-computable or discontinuous functions or not be going by differential equations. It's already a miracle that it is as computable as it is. So there is definitely something out there! It's not just that we decided to look in one place—we decided to look in one place and *we found* computability and that's already a miracle.
**Q**: Well, it's a miracle also that for instance in engineering you now if you look at you know buildings, bridges planes they all are built with continuous functions! So you can say you know this is all in the imagination of mathematicians and discontinuities and you know all sort of sophisticated

deviations [in fact] appear. But if you look at the pragmatic view you know one could say that from the point of view of engineering only continuous functions come and exist.

**A**: Yes, but then engineering has been successful at least up to a point. Prior to the scientific age people tried all sorts of different kinds of explanations—anthropocentric type explanations, in which physical processes were governed by the intentions of supernatural beings, and that kind of thing... and they were looking for explanations in those terms and those explanations never were successful. So the fact that we can now build bridges using assumptions of continuous functions and so on, even if later turns out that it's just an approximation that's already something that we don't know how to explain! Why are computable functions available to build bridges with? It's not just that we looked for them, we also *found* them.

**New Question**: I would like to know a bit more about explanations related to physics being miraculously computable and would this make our Universe more [special] among other alternatives just because those are not computable and of course the condition of evolution being more likely...

**Answer**: As I said I don't think *that* can be the full explanation of this mystery. It could be that there are all sorts of Universes with different laws of physics and it could be that in most of them in some sense there are no observers. However there has to be a structure on this class of Universes—such that most of them "make sense". The set of all possible laws (which is not a set but a class) is too enormous for the concept of "most of them do one thing while a few of them do another" to make sense. You can't attack this problem purely with anthropic reasoning. The answer must be some kind of explanation of why the laws are as they are or at least why the measure of all possible laws is as it is—that kind of thing. So if you're asking is anthropic reasoning enough to explain all of this: I think not.

**Q**: Well, the interesting situation with computability is that obviously computable functions are of measure C and I imagine that in all alternative universes [that there is a] favorable choice [with a different complexity measure] is very unlikely.

**A**: Yes, quite so. So there's something to explain—it's not just selection.

**Moderator**: We can take only one more question—it's all the time we have[8]. Let's choose someone that hasn't asked any question yet. Go ahead...

**Last Question**: In your position that proofs are things that are computable in physical reality and mathematical proofs are [idealizations] I was interested in [knowing] how this relates with the difference between classical

[8]Lev Grover was already connected from Bell Labs and ready for his video talk

proofs and constructive proofs [as in] classical logic and constructor logic.
**Answer**: Well, I know next to nothing about that but I think I can tell you what the situation basically is. Like I said proof theory is a science. And one of the changes that one has to make already, you know, nevermind these exotic laws of physics that might exist the change that you already have to make with quantum computation is that we can no longer say, we can no longer define a proof as an object—that is, traditionally we thought of a proof as something that we can present on a piece of paper that satisfies certain laws: that in the beginning there must be axioms, and that it proceeds in lines and then each line must follow from the axioms according to a certain set of laws of inference and then the last line is the conclusion—and we say that the conclusion is proof from the axioms. Now in quantum theory that is no longer sufficient, there are other kinds of proofs that cannot be expressed in that way at least not in [a] tractable—not in a polynomial number of steps. There are proofs where there is not enough paper in the universe to express them in that way—but a quantum computer could nevertheless prove them and for that reason you have to change your [entire perspective]. In classical physics and in classical theory of computation the idea of a proof as an object obeying several laws and the idea of a proof as a process where you execute one operation after another are equivalent—that is, there is a one to one correspondence between those two conceptions but in quantum computation there isn't a one to one correspondence and it's possible that some things that can be done in a small number of single steps can't be expressed in an exponentially large object. So those distinctions between different kinds of proofs are induced by the laws of physics and if the laws of physics are different the classification of proofs will be different as well.

Figure 8: In Rawles Hall shortly before the talk: David Deutsch, Steve Egyhazi (front row, monitoring the connection) and Hector Zenil at the 2008 Midwest NKS Conference in Bloomington, IN.

**Moderator**: Unfortunately I think we are going to have to finish here. So let's thank Prof. David Deutsch one more time. (Applause) Thank you very much once again and we're going to disconnect now...
**David Deutsch**: Thanks for having me, and nice meeting you.